



Stronger Self-Focusing of Hermite-Cosh-Gaussian (HChG) Laser Beam in Plasma

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Abstract

The relativistic self-focusing of HChG beam in the plasma has been investigated theoretically using WKB and paraxial ray approximation for mode indices 0, 1 and 2 for different values of pulse amplitude, normalized plasma density and decentred parameter. The plot between beam width parameter f and the normalized length ξ for different value of normalized density has been reported and indicates the enhancement of the self-focusing phenomena of the laser beams in the plasmas. Also the focusing and defocusing of HChG beam in the plasma having dependency on decentred parameter and frequency of incident laser beam is reported.

Keywords

Self-Focusing, Plasma Density, Nonlinearity, Hermite-Cosh-Gaussian Beam

Subject Areas: Plasma, Theoretical Physics

1. Introduction

The interaction of light with matter is one of the basic phenomena in nature. Moreover the advancement in the short pulse laser technology has enabled experiments using laser pulses focused on extremely high intensity of the order of 10^{20} W/cm². Self-focusing of laser beams in plasmas [1]-[5] becomes one of the most interesting and fascinating field of research for several decades due to its various applications like the generation of inertial fusion energy driven by lasers [6], optical harmonic generation [7], the production of quasi mono-energetic electron bunches [8], laser driven fusion [9], x-ray lasers and the laser driven accelerators [10]. These applications need the laser pulse to propagate over several Rayleigh lengths in the plasmas without loss of energy.

The dielectric constant of plasma modifies greatly with the increase in intensity of the laser beam and it leads

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to self-focusing of the beam. The variation in the electron density in plasmas is caused by the propagating laser pulse of extremely high intensities of the order ranging from 10^{17} - 10^{20} W/cm². Such high intensity laser pulses provide sufficient energy to the constituent particles of the plasmas which suddenly increase the speed of electrons nearly equal to the speed of light enhancing the mass of the electron and hence give rise to the relativistic effect [2]. Relativistic self-focusing and self-channeling of Gaussian laser beam has been, recently, reported by Singh *et al.* [5] by applying moment theory approach to solve the non-linear differential equation for beam width parameter and then solve it numerically by Runge-Kutta method.

Recently, theoretical investigators have focused their attention on paraxial wave family of laser beams. Hermite-cosh- Gaussian beam is one of the solutions of paraxial wave equation and such HChG beam can be obtained in the laboratory by the superposition of two decentered Hermite-Gaussian beams as Cosh-Gaussian ones. Propagation of HChG beams in plasmas has been studied theoretically earlier by Belafhal *et al.* [3] and Patil *et al.* [11]. The focusing of HChG laser beams in magneto-plasma by considering ponderomotive nonlinearity has been theoretically examined by Patil *et al.* [12] and they reported the effect of mode index and decentered parameter on the self-focusing of the beams.

The present work is dedicated to the study of the relativistic self-focusing of HChG beam in plasma. We derive the equations for beam width parameter for HChG beam and solve them numerically by applying Wentzel-Kramers-Brillouin (WKB) approximation and Paraxial approximation [13] [14] for mode index 0, 1 and 2 and observe the enhancement in the self-focusing of the laser beams.

2. Nonlinear Dynamics

The field distribution of Hermite-Cosh-Gaussian (HChG) laser beams propagating in plasma along z -axis is of the following form:

$$E(r, z) = \frac{E_0}{f(z)} \left[H_m \left(\frac{\sqrt{2}r}{r_0 f(z)} \right) \right] e^{\frac{b^2}{4}} \left\{ e^{-\left(\frac{r}{r_0 f(z)} + \frac{b}{2}\right)^2} + e^{-\left(\frac{r}{r_0 f(z)} - \frac{b}{2}\right)^2} \right\} \quad (1)$$

where E_0 is the amplitude of HChG laser beam for the central position at $r = z = 0$, " $f(z)$ " is the dimensionless beam width parameters, " H_m " is the Hermite polynomial of m^{th} order, " r_0 " is the spot size of the beam and " b " is the decentered parameter of the beam.

The dielectric constant for the non-linear medium (collision-less plasma) is of the following form:

$$\varepsilon = \varepsilon_0 + \phi(EE^*) \quad (2)$$

where $\varepsilon_0 = 1 - \omega_{p0}^2 / \gamma \omega^2$, $m = m_0 \gamma$, $\gamma = 1 / \sqrt{1 - v^2 / c^2}$ and $\omega_{p0} = (4\pi e^2 n_0 / m)^{1/2}$ is the plasma frequency at equilibrium. Here " ε_0 " and " ϕ " represent the linear and non-linear parts of the dielectric constant respectively, m_0 , & ω are the electronic charge, rest mass of the electron and frequency of the incident laser respectively and " n_0 " is the equilibrium electron density.

The non-linearity induced due to ponderomotive forces in the dielectric constant in collisionless plasma and hence non-linear part of the dielectric constant can be written as [15]

$$\phi(EE^*) = \left(\frac{\omega_{p0}^2}{\gamma \omega^2} \right) \left[1 - \exp \left(- \frac{3m_0 \gamma \alpha_1 EE^*}{4M} \right) \right] \quad (3)$$

where $\alpha_1 = e^2 M / 6m_0^2 \gamma^2 \omega^2 k_B T_0$, here " M " is the mass of the scatterer in the plasma, " K_B " is the Boltzmann constant and " T_0 " is the plasma temperature at equilibrium.

3. Self-Focusing Equations

Wave equation can be deduced by using Maxwell's equations and can be written as

$$\nabla^2 \mathbf{E} - \frac{\varepsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} + \nabla \left(\mathbf{E} \frac{\nabla(\varepsilon)}{\varepsilon} \right) = 0 \quad (4)$$

Consider a plane polarized wave with electric field vector given by

$$\mathbf{E} = \mathbf{A}(r, z) \exp i(\omega t - kz) \tag{5}$$

where $A(r, z)$ is the complex amplitude of the electric field. Thus,

$$\partial^2 \mathbf{E} / \partial t^2 = -\omega^2 \mathbf{E} \tag{6}$$

Hence Equation (4) becomes

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} + \nabla \left(\mathbf{E} \frac{\nabla(\epsilon)}{\epsilon} \right) = 0 \tag{7}$$

where $k = (\omega/c) \epsilon^{1/2}$. For $(1/k^2) \nabla^2 (\ln \epsilon) \ll 1$.

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} + \frac{\partial^2 \mathbf{E}}{\partial r^2} - \frac{1}{r} \frac{\partial \mathbf{E}}{\partial r} + \frac{\epsilon \omega^2}{c^2} \mathbf{E} = 0 \tag{8}$$

We know that the solution of Equation (8) is of the form,

$$\mathbf{E} = \mathbf{A}(r, z) \exp i \left(\omega t - \frac{z}{c} \sqrt{\omega^2 - \frac{\omega_{p0}^2}{\gamma}} \right) \tag{9}$$

Substituting this value in Equation (8), we get

$$2ik \frac{\partial A}{\partial z} = \frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{k^2 A}{\epsilon_0} \phi(AA^*) \tag{10}$$

The solution of Equation (10) is of the form,

$$A(r, z) = A_{0p}(r, z) \exp[-ikS(r, z)] \tag{11}$$

Here “ A_{0p} ” and “ S ” are the real function of r and z .

Using Equation (9) in Equation (8) and separating real and imaginary parts, we get
Real part is

$$2 \frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r} \right)^2 = \frac{1}{k^2 A_{0p}} \left(\frac{\partial^2 A_{0p}}{\partial r^2} + \frac{1}{r} \frac{\partial A_{0p}}{\partial r} \right) + \frac{\phi(A_{0p}^2)}{\epsilon_0} = 0 \tag{12}$$

Imaginary part is

$$2 \frac{\partial A_{0p}}{\partial z} + \frac{1}{r} A_{0p} \frac{\partial S}{\partial r} + A_{0p} \frac{\partial^2 S}{\partial r^2} + 2 \frac{\partial A_{0p}}{\partial r} \frac{\partial S}{\partial r} = 0. \tag{13}$$

The solution of Equations (12) & (13) are of the form,

$$A_{0p}^2 = \frac{E_0^2}{f^2(z)} \left[H_m \left(\frac{\sqrt{2}r}{r_0 f} \right) \right] e^{\frac{b^2}{2} \left\{ e^{-2\left(\frac{r}{r_0 f} + \frac{b}{2}\right)^2} + e^{-2\left(\frac{r}{r_0 f} - \frac{b}{2}\right)^2} + 2e^{-\left(\frac{2r^2}{r_0^2 f^2} + \frac{b^2}{2}\right)} \right\}} \tag{14}$$

And

$$S = \frac{r^2}{2} \beta(z) + \phi(z) \tag{15}$$

with $\beta(z) = (1/f) \partial f / \partial z$, represents the inverse of the radius of curvature of the wavefront of the laser beam.

Here $\phi(z)$ is an arbitrary function of “ z ”. Using these values in Equation (12), we get,

For $m = 0$

$$\frac{\partial^2 f}{\partial \xi^2} = \frac{1}{f^3} \left[4 - 2b^2 - 12b^4 + 2b^6 - \frac{6\alpha E_0^2 r_0^2 m_0 \omega_{p0}^2}{\gamma^2 c^2 M} e^{\frac{b^2}{2}} e^{-\frac{-3\alpha E_0^2 m_0}{M \gamma f^2}} \right] \tag{16}$$

For $m = 1$

$$\frac{\partial^2 f}{\partial \xi^2} = \frac{1}{f^3} \left[4 + 2b^2 - 20b^4 + 2b^6 - \frac{12\alpha E_0^2 r_0^2 m_0 \omega_{p0}^2}{\gamma^2 c^2 M} e^{\frac{b^2}{2}} (b^2 - 2) \right] \quad (17)$$

For $m = 2$

$$\frac{\partial^2 f}{\partial \xi^2} = \frac{1}{f^3} \left[-28 + 78b^2 - 4b^4 + 22b^6 + \frac{24\alpha E_0^2 r_0^2 m_0 \omega_{p0}^2}{\gamma^2 c^2 M} e^{\frac{b^2}{2}} e^{\frac{-12\alpha E_0^2 m_0}{M \gamma f^2}} (5 - 2b^2) \right] \quad (18)$$

where beam width parameter f is a function of ξ , here $\xi = z/R_d$ and R_d is the diffraction length. Similarly Equation (14) gives the condition, $\partial f / \partial z = 0, f = \text{constant}$.

4. Results and Discussion

The variation of beam width parameter f with normalized propagation distance ξ for different values of relative plasma frequency; $\omega_{p0}/\omega = 0.7, 0.8$ and 0.9 has been seen for different parameters given as $b = 0.67, r_0 = 3 \times 10^{-6}$ m, $\omega = 9.4 \times 10^{14}$ rad/s and $\alpha E_0^2 = 0.3$. From **Figure 1**, it is clear that with the increase in plasma density, the beam width parameter f decreases greatly. Hence self-focusing of laser beam becomes more and more strong which supports the results of Sadighi-Bonabi *et al.* [8]. The results obtained are also in good agreement with the results reported by Singh *et al.* [5]. In **Figure 2**, the variation of beam width parameter f with dimensionless normalized propagation distance (ξ) for different values of relative plasma density; $(\omega_{p0}/\omega) = 0.5, 0.6$ and 0.7 has been observed. The plot obtained clarifies that for $\alpha E_0^2 = 0.2$ and $\gamma = 0.99$, beam width parameter decreases with the increase in relative plasma frequency (ω_{p0}/ω) . Hence self-focusing of laser beam becomes stronger and earlier as well. Also for higher values of relative plasma density, self-focusing phenomenon occurs earlier with the normalized propagation distance. The results obtained are in good agreement with the results reported in ref. [5].

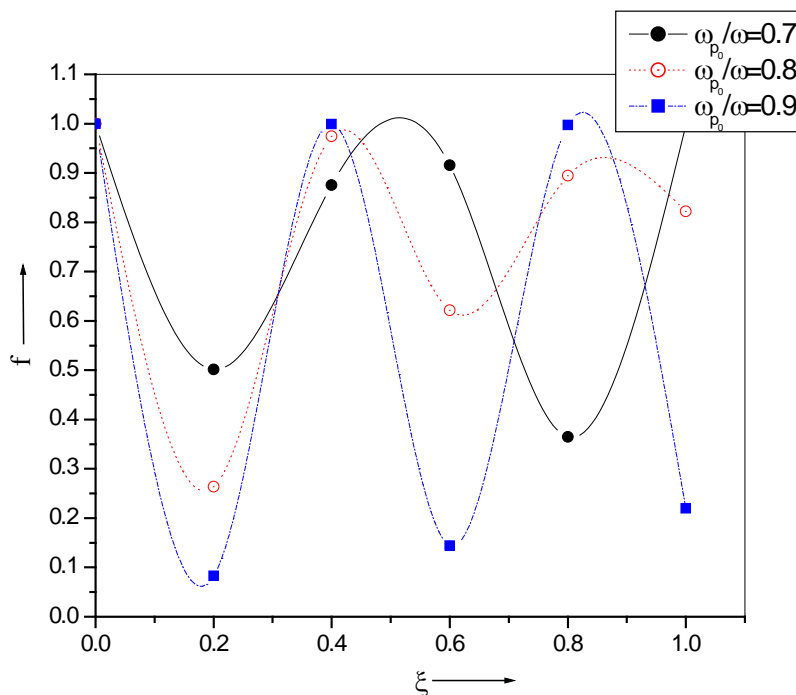


Figure 1. Variation of beam width parameter (f) with normalized propagation distance ξ for relative density $\omega_{p0}/\omega = 0.7$ (Black curve) $\omega_{p0}/\omega = 0.8$ (Red curve) and $\omega_{p0}/\omega = 0.9$ (Blue curve) for various parameter taken as $m = 0, b = 0.67, r_0 = 3 \times 10^{-6}$ m, $\omega = 9.4 \times 10^{14}$ rad/s and $\alpha E_0^2 = 0.3$.

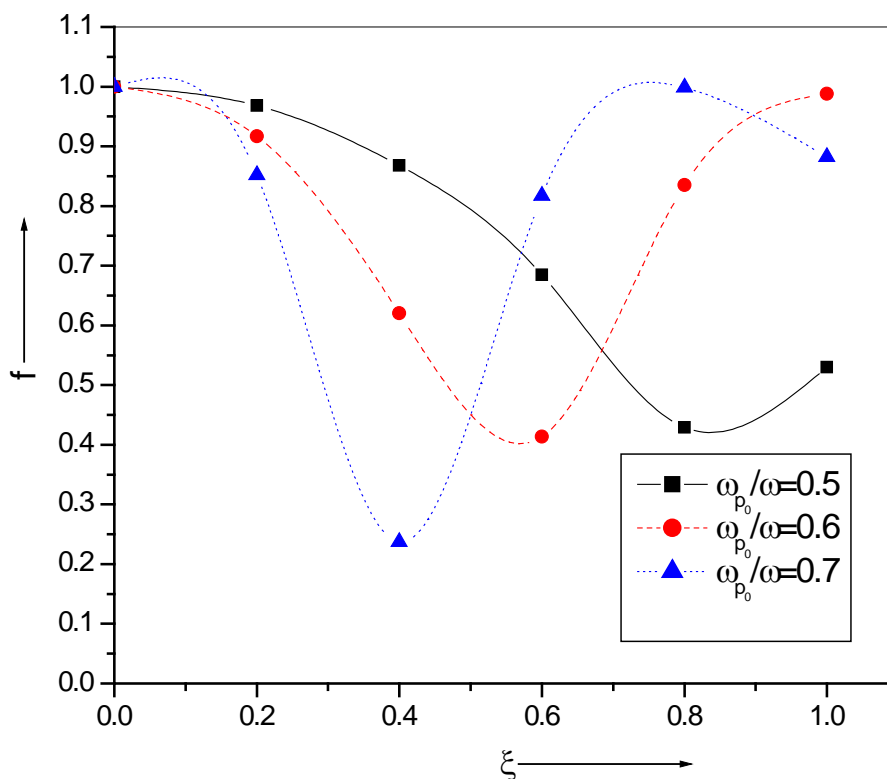


Figure 2. Variation of beam width parameter f with normalized distance ξ for $\omega_{p0}/\omega = 0.5$ (Black curve) $\omega_{p0}/\omega = 0.6$ (Red curve) $\omega_{p0}/\omega = 0.7$ (Blue curve) for parameters $\gamma = 0.99$, $m = 0$, $b = 0$, $r_0 = 3 \times 10^{-5}$ m, $\omega = 3.5 \times 10^{14}$ rad/s and $\alpha E_0^2 = 0.2$.

Figure 3 represents the variation of beam width parameter f with dimensionless normalized propagation distance ξ , for mode index $m = 1$, at different values of frequency of incident laser beam viz. $\omega = 9.3 \times 10^{14}$ rad/s (Black curve) $\omega = 9.4 \times 10^{14}$ rad/s (Red curve) $\omega = 9.5 \times 10^{14}$ rad/s (Blue curve) for considering various parameters as $b=1$, $r_0 = 3 \times 10^{-6}$ m, relative mass $(m_0/M) = 0.5$ and $\alpha E_0^2 = 0.1$. From **Figure 3**, one can easily conclude that for $\omega = 9.3 \times 10^{14}$ rad/s (Black curve), strong focusing of laser beam occurs and for $\omega = 9.4 \times 10^{14}$ rad/s (Red curve) and $\omega = 9.5 \times 10^{14}$ rad/s (Blue curve), laser beam gets defocused for decentered parameter $b = 1$. It happens because the increase in the value of frequency of incident beam varies the density of the plasma greatly. Hence focusing and de-focusing nature of HChG beam for mode index $m = 1$ is decided by the choice of the frequency of the laser beam propagating through the plasma medium. Patil *et al.* [16] investigate analytically HChG laser beam propagation in non-degenerate germanium having space charge neutrality for first three mode indices (m) and conclude that decentered parameter (b) changes the nature of self focusing/de-focusing significantly. In addition to that we conclude that frequency of incident laser beam (ω) give additional freedom to choose a parameter which can control the nature of self-focusing/de-focusing of HChG laser beam for mode indices $m = 1$. These results are also in agreement with the results obtained by Kant *et al.* [17].

Figure 4, represents the variation of beam width parameter (f) with normalized propagation distance (ξ) for mode index $m = 2$ for decentered parameter $b = 0, 1$ and 2 with $\omega = 9.4 \times 10^{14}$ rad/s, $r_0 = 3 \times 10^{-6}$ m, $\omega_{p0}/\omega = 0.5$, $m_0/M = 0.25$ and $\alpha E_0^2 = 0.1$. For decentered parameter $b = 0$, self-focusing of the HChG beam occurs and for decentered parameter $b = 1$ and 2 , defocusing of the HChG beam take place. The second non-linear term of Equation (17) dominates over the first term for $b = 0$ and cause the self-focusing of HChG beam ($m = 2$). For $b = 0$ and 1 , the beams propagates without focusing or defocusing nearly up to $\xi = 0.5$ and then a small effect of self-focusing is observed. The results obtained are in agreement with the results reported by Patil *et al.*, [11] in case of propagation of Hermite-Cosh-Gaussian laser beams in n-InSb. Various studies have been done to achieve stronger self-focusing by using various laser profiles [18]-[22].

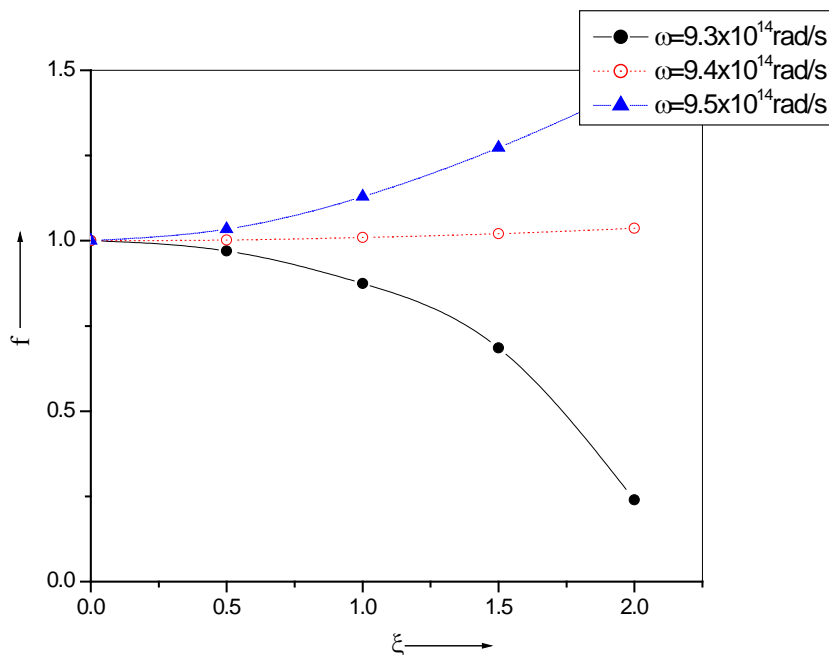


Figure 3. Variation of beam width parameter f with normalized propagation distance ξ for propagating laser frequency $\omega = 9.3 \times 10^{14}$ rad/s (Black curve) $\omega = 9.4 \times 10^{14}$ rad/s (Red curve) $\omega = 9.5 \times 10^{14}$ rad/s (Blue curve) for parameter $m = 1$, $b = 1$ $r_0 = 3 \times 10^{-6}$ m, relative mass $(m_0/M) = 0.5$ and $\alpha E_0^2 = 0.1$.

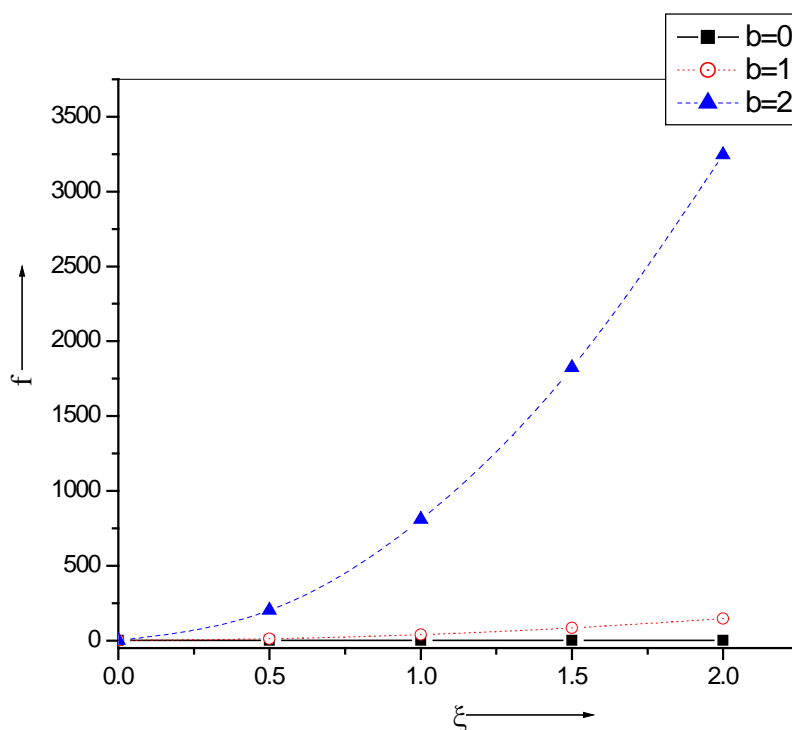


Figure 4. Variation of beam width parameter f with normalized propagation distance ξ for mode index $m = 2$ for decentered parameter $b = 0$ (Black curve), $b = 1$ (Red curve) and $b = 2$ (Blue curve) for parameter $\omega = 9.4 \times 10^{14}$ rad/s, $r_0 = 3 \times 10^{-6}$ m, $\omega_{p0}/\omega = 0.5$, $m_0/M = 0.25$ and $\alpha E_0^2 = 0.1$.

5. Conclusion

From the above results, we conclude that focusing/defocusing phenomena of HChG beams for mode indices $m = 0, 1$ and 2 , will be controlled with the frequency of the laser beam propagating through the plasma and decentered parameter. Moreover, self-focusing becomes stronger and earlier with the increase in plasma density for mode index $m = 0$. For $m = 2$, Hermite-Cosh-Gaussian beam exhibits self-focusing nearly up to normalized propagation distance $\xi = 0.5$ for decentered parameter $b = 0$ and 1 while for higher values of decentered parameter *i.e.* $b = 2$, purely defocusing of the beam occurs. The results obtained in this study adds some more choices of parameters viz. mode indices (m), decentered parameter (b) and frequency of incident laser beam (ω) for investigators working in the field of self-focusing of laser beams of various profiles. Present study may be useful for the scientist working on laser induced fusion.

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