



On the Pre-Exponential Factor Comparing in Thermoluminescence (TL) Theory

Eugenio Chiaravalle¹, Michele Mangiacotti¹, Claudio Furetta², Giuliana Marchesani¹, Michele Tomaiuolo¹

¹Centro di Referenza Nazionale per la Ricerca della Radioattività nel Settore Zootecnico-Veterinario, Istituto Zooprofilattico Sperimentale della Puglia e della Basilicata, Foggia, Italy

²Independent Researcher, Rome, Italy

Email: c.furetta@alice.it

Received 3 May 2014; revised 13 June 2014; accepted 25 July 2014

Copyright © 2014 by authors and OALib.

This work is licensed under the Creative Commons Attribution International License (CC BY).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

The aim of the work has two folders: the first is to apply numerical and approximated methods to solve the integral comparing in the thermoluminescence theory, *i.e.* $\int_{T_0}^T \exp\left(-\frac{E}{kT'}\right) dT'$. The second point is to compare the influence of the two different ways of calculation on the values of the pre-exponential factor.

Keywords

Thermo

Subject Areas: Applied Physics, Condensed State Physics

1. Introduction

The thermoluminescent intensity, I , as a function of temperature, is obtained developing the rate-equations describing the different TL kinetic processes, *i.e.* the Randal-Wilkins or first order kinetics [1], the Garlick-Gibson model related to the second order [2] and the general order kinetics introduced by May-Partridge [3]. The equations describing the different processes are the following:

- for first order

$$I(T) = -n_0 s \exp\left(-\frac{E}{kT}\right) \exp\left[-\frac{s}{\beta} \int_{T_0}^T \exp\left(-\frac{E}{kT'}\right) dT'\right] \quad (1)$$

- for second order

How to cite this paper: Chiaravalle, E., Mangiacotti, M., Furetta, C., Marchesani, G. and Tomaiuolo, M. (2014) On the Pre-Exponential Factor Comparing in Thermoluminescence (TL) Theory. *Open Access Library Journal*, 1: e618.

<http://dx.doi.org/10.4236/oalib.1100618>

$$I(T) = (n_0^2 s') \exp\left(-\frac{E}{kT}\right) \left[1 + \frac{s' n_0}{\beta} \int_{T_0}^T \exp\left(-\frac{E}{kT'}\right) dT' \right]^{-2} \quad (2)$$

where $s' = s/N$.

- for general order, $1 < b < 2$, the TL intensity is given by

$$I(T) = n_0 s'' \exp\left(-\frac{E}{kT}\right) \left[1 + \frac{s''(b-1)}{\beta} \int_{T_0}^T \exp\left(-\frac{E}{kT'}\right) dT' \right]^{\frac{b}{b-1}} \quad (3)$$

where $s'' = s' n_0^{(b-1)}$.

In the previous equations the meaning of the symbols is the following:

- n_0 (cm^{-3}) is the number of electrons contained in the electron trap at temperature T_0 ,
- N is the trap concentration,
- E (eV) is the energy depth related to the trap below the bottom of the conduction band,
- k ($\text{eV}\cdot\text{K}^{-1}$) is the Boltzmann's constant,
- β is the linear heating rate dT/dt .

The quantity s , which appears in the case of Randal-Wilkins equation, is called frequency factor and it has a precise physical meaning: it should represent the number of times for second that a bound electron interacts with the lattice phonons, multiplied a transition probability factor. In the case of Garlick-Gibson second order model and May-Partridge general order, the quantities s' and s'' are called pre-exponential factors.

All equations contain the integral

$$F(T, E) = \int_{T_0}^T \exp\left(-\frac{E}{kT'}\right) dT' \quad (4)$$

which cannot be solved analytically.

The aim of this paper has two folders. The first one is to compare the solutions of (4) obtained by the approximation method and by its numerical solution. The second aspect is to investigate the influence of the two different ways of calculation on the values of the pre-exponential factor s'' .

2. Integral Approximation

A method which is usually followed for evaluating the value of the integral is by integration in parts, when the lower limit of integration is 0 instead of T_0 . So, a good approximation is provided by the asymptotic series

$$F(T, E) = \int_0^T \exp\left(-\frac{E}{kT'}\right) dT' = T \exp\left(-\frac{E}{kT}\right) \sum_{n=1}^{\infty} \left(\frac{kT}{E}\right)^n (-1)^{n-1} n! \quad (5)$$

The value of (4) is then given by

$$\int_{T_0}^T \exp\left(-\frac{E}{kT'}\right) dT' = F(T, E) - F(T_0, E)$$

Since $F(T, E)$ is a very strong increasing function of T , $F(T_0, E)$, considering $T_0 = 0$, is negligible compared to $F(T, E)$, the right hand side of Equation (5) can be considered to represent the integral value from T_0 as well.

In the most practical cases, a good approximation of the integral is given by the second order approximation, I_2 , of Equation (5) [4] [5]:

$$F(T, E) = \frac{kT^2}{E} \exp\left(-\frac{E}{kT}\right) \left(1 - \frac{2kT}{E}\right) \quad (6)$$

Equation (6) has been calculated as a function of E and T . The range of the activation energy E was from 0.5 eV to 2.25 eV in steps of 0.25 eV and the temperature was ranging from 300 to 500 K. **Table 1** gives the data obtained. I_2 indicates the second order approximation.

Table 1. Second order approximation of the Equation (6).

Temperature [K]	$I_2(T, E=0.5)$	$I_2(T, E=0.75)$	$I_2(T, E=1)$	$I_2(T, E=1.25)$	$I_2(T, E=1.5)$	$I_2(T, E=1.75)$	$I_2(T, E=2)$	$I_2(T, E=2.25)$
300	5.54E-08	2.42E-12	1.17E-16	5.96E-21	3.16E-25	1.72E-29	9.53E-34	5.36E-38
310	1.10E-07	6.58E-12	4.34E-16	3.02E-20	2.19E-24	1.63E-28	1.23E-32	9.48E-37
320	2.10E-07	1.68E-11	1.49E-15	1.39E-19	1.35E-23	1.34E-27	1.36E-31	1.40E-35
330	3.85E-07	4.07E-11	4.73E-15	5.83E-19	7.44E-23	9.75E-27	1.30E-30	1.77E-34
340	6.82E-07	9.35E-11	1.41E-14	2.25E-18	3.72E-22	6.32E-26	1.09E-29	1.92E-33
350	1.17E-06	2.05E-10	3.96E-14	8.05E-18	1.70E-21	3.69E-25	8.14E-29	1.82E-32
360	1.96E-06	4.32E-10	1.05E-13	2.69E-17	7.15E-21	1.95E-24	5.43E-28	1.53E-31
370	3.18E-06	8.76E-10	2.64E-13	8.43E-17	2.79E-20	9.46E-24	3.27E-27	1.15E-30
380	5.06E-06	1.71E-09	6.35E-13	2.49E-16	1.01E-19	4.23E-23	1.80E-26	7.75E-30
390	7.85E-06	3.23E-09	1.46E-12	6.98E-16	3.45E-19	1.75E-22	9.05E-26	4.75E-29
400	1.19E-05	5.93E-09	3.23E-12	1.86E-15	1.11E-18	6.76E-22	4.21E-25	2.66E-28
410	1.78E-05	1.06E-08	6.87E-12	4.72E-15	3.36E-18	2.45E-21	1.82E-24	1.37E-27
420	2.60E-05	1.83E-08	1.41E-11	1.15E-14	9.67E-18	8.35E-21	7.35E-24	6.56E-27
430	3.75E-05	3.10E-08	2.81E-11	2.68E-14	2.65E-17	2.69E-20	2.78E-23	2.92E-26
440	5.31E-05	5.13E-08	5.42E-11	6.04E-14	6.96E-17	8.23E-20	9.92E-23	1.21E-25
450	7.41E-05	8.31E-08	1.02E-10	1.31E-13	1.75E-16	2.40E-19	3.35E-22	4.74E-25
460	1.02E-04	1.32E-07	1.86E-10	2.76E-13	4.24E-16	6.68E-19	1.07E-21	1.75E-24
470	1.39E-04	2.05E-07	3.31E-10	5.62E-13	9.89E-16	1.78E-18	3.27E-21	6.10E-24
480	1.86E-04	3.14E-07	5.77E-10	1.11E-12	2.23E-15	4.57E-18	9.54E-21	2.02E-23
490	2.48E-04	4.73E-07	9.82E-10	2.15E-12	4.86E-15	1.13E-17	2.66E-20	6.39E-23
500	3.25E-04	7.01E-07	1.64E-09	4.04E-12	1.03E-14	2.69E-17	7.15E-20	1.93E-22

3. Numerical Solution of the Integral

The integral (4) has been also calculated, as a function of E and T , by means of the Matlab program. Special attention has been paid at the low temperature region where the exponential assumes initially very low values and then it changes very fast as the temperature increases.

The lower limit of the integration interval is the smallest floating point number, *i.e.* `realmin` in Matlab, while the upper limit value changes from 300 K to 700 K.

The integration limit has been divided in two subintervals: the first for $T \leq 400$ K and the second for $T > 400$ K in order to define the first region where the integration function assumes very low values and increases strongly with the temperature.

In the following the numerical solution of the integral (4) is indicated by $Q(T, E)$. **Table 2** shows the data obtained.

4. Condition at the Maximum

Considering the equation for general order kinetics, the condition at the maximum is obtained by Equation (3) as following. The logarithm of $I(T)$ is:

$$\ln[I(T)] = \ln(s''n_0) - \frac{E}{kT} - \frac{b}{b-1} \ln \left[1 + \frac{s''(b-1)}{\beta} \int_{T_0}^T \exp\left(-\frac{E}{kT'}\right) dT' \right] \quad (7)$$

then, the condition at the maximum is obtained quoting its derivative to zero:

$$\left. \frac{d(\ln I)}{dT} \right|_{T=T_M} = \frac{E}{kT_M^2} - \frac{b}{b-1} \left[1 + \frac{s''n_0(b-1)}{\beta} \int_{T_0}^{T_M} \exp\left(-\frac{E}{kT'}\right) dT' \right]^{-1} \frac{s''(b-1)}{\beta} \exp\left(-\frac{E}{kT_M}\right) = 0$$

Table 2. Calculated values of $Q(T, E)$ for given values of E and T .

Temperature [K]	$Q(T, E = 0.5)$	$Q(T, E = 0.75)$	$Q(T, E = 1)$	$Q(T, E = 1.25)$	$Q(T, E = 1.5)$	$Q(T, E = 1.75)$	$Q(T, E = 2)$	$Q(T, E = 2.25)$
300	5.62E-08	2.44E-12	1.17E-16	5.98E-21	3.16E-25	1.72E-29	9.54E-34	5.37E-38
310	1.12E-07	6.62E-12	4.35E-16	3.03E-20	2.19E-24	1.63E-28	1.23E-32	9.48E-37
320	2.13E-07	1.69E-11	1.49E-15	1.39E-19	1.35E-23	1.34E-27	1.36E-31	1.40E-35
330	3.92E-07	4.10E-11	4.75E-15	5.84E-19	7.46E-23	9.77E-27	1.30E-30	1.77E-34
340	6.95E-07	9.43E-11	1.42E-14	2.26E-18	3.73E-22	6.33E-26	1.09E-29	1.92E-33
350	1.20E-06	2.07E-10	3.98E-14	8.08E-18	1.70E-21	3.69E-25	8.15E-29	1.83E-32
360	2.00E-06	4.37E-10	1.05E-13	2.70E-17	7.17E-21	1.96E-24	5.44E-28	1.53E-31
370	3.26E-06	8.85E-10	2.66E-13	8.46E-17	2.80E-20	9.48E-24	3.28E-27	1.15E-30
380	5.18E-06	1.73E-09	6.39E-13	2.50E-16	1.02E-19	4.24E-23	1.80E-26	7.76E-30
390	8.04E-06	3.27E-09	1.47E-12	7.00E-16	3.46E-19	1.75E-22	9.07E-26	4.76E-29
400	1.22E-05	6.00E-09	3.25E-12	1.86E-15	1.11E-18	6.78E-22	4.22E-25	2.67E-28
410	1.83E-05	1.07E-08	6.92E-12	4.74E-15	3.37E-18	2.45E-21	1.82E-24	1.38E-27
420	2.68E-05	1.86E-08	1.42E-11	1.15E-14	9.70E-18	8.37E-21	7.36E-24	6.57E-27
430	3.86E-05	3.15E-08	2.83E-11	2.70E-14	2.66E-17	2.70E-20	2.79E-23	2.92E-26
440	5.48E-05	5.21E-08	5.46E-11	6.07E-14	6.99E-17	8.26E-20	9.94E-23	1.21E-25
450	7.66E-05	8.44E-08	1.03E-10	1.32E-13	1.76E-16	2.41E-19	3.35E-22	4.75E-25
460	1.06E-04	1.34E-07	1.87E-10	2.77E-13	4.26E-16	6.70E-19	1.07E-21	1.75E-24
470	1.44E-04	2.09E-07	3.34E-10	5.66E-13	9.93E-16	1.79E-18	3.28E-21	6.11E-24
480	1.93E-04	3.20E-07	5.82E-10	1.12E-12	2.24E-15	4.58E-18	9.56E-21	2.03E-23
490	2.57E-04	4.81E-07	9.92E-10	2.16E-12	4.88E-15	1.13E-17	2.67E-20	6.40E-23
500	3.39E-04	7.14E-07	1.66E-09	4.06E-12	1.03E-14	2.70E-17	7.17E-20	1.93E-22

from which we obtain

$$\frac{kT_M^2 b s'' n_0}{\beta E} \exp\left(-\frac{E}{kT_M}\right) = 1 + \frac{s''(b-1)}{\beta} \int_{T_0}^{T_M} \exp\left(-\frac{E}{kT'}\right) dT' \quad (8)$$

From the last equation it is possible to obtain the expression for the pre-exponential factor. Rearranging Equation (8), we obtain:

$$s'' = \frac{1}{n_0} \cdot \left[\frac{kT_M^2 b \exp\left(-\frac{E}{kT_M}\right)}{\beta E} - \frac{(b-1) \int_{T_0}^{T_M} \exp\left(-\frac{E}{kT'}\right) dT'}{\beta} \right]^{-1} \quad (9)$$

or, considering $s = s'' n_0$

$$s = \left[\frac{kT_M^2 b \exp\left(-\frac{E}{kT_M}\right)}{\beta E} - \frac{(b-1) \int_{T_0}^{T_M} \exp\left(-\frac{E}{kT'}\right) dT'}{\beta} \right]^{-1} \quad (10)$$

Using the second order approximation and $T_0 = 0$, we get from Equation (9)

$$s'' = \frac{1}{n_0} \cdot \left[\frac{kT_M^2 \exp\left(-\frac{E}{kT_M}\right)}{\beta E} \left(1 + \frac{2kT_M(b-1)}{E}\right) \right]^{-1} \quad (11)$$

and finally

$$s'' = \frac{\beta E \exp\left(\frac{E}{kT_M}\right)}{n_0 kT_M^2} \left[1 + \frac{2kT_M(b-1)}{E}\right]^{-1} \quad (12)$$

or, considering $s = s''n_0$,

$$s = \frac{\beta E \exp\left(\frac{E}{kT_M}\right)}{kT_M^2} \left[1 + \frac{2kT_M(b-1)}{E}\right]^{-1} \quad (13)$$

expressed in sec^{-1} .

Observing the previous equations, it is clear that both the heating rate, β , and the initial dose, n_0 , are multiplication factors: for this reason it is possible to compare directly the Equations (10) and (13) and to find a possible differences in the pre-exponential factor values when it is obtained by second order approximation, s_2 , *i.e.* Equation (13), or using the numerical solution of the integral, s , in Equation (10). In both cases only one parameter has been varied in each simulation, *i.e.* the kinetics order b , the heating rate β , the temperature at the maximum T_M or the initial delivered dose n_0 .

Tables 3-6 show the frequency factor as a function of the kinetic order b . The activation energy changes from 0.5 eV to 2.0 eV in steps of 0.5 eV. T_M and β have been kept constant and the values are 500 K and 10 K/sec respectively. **Figures 1-4** show the behaviour of s as a function of the kinetic order b for various values of the activation energy, *i.e.* E equal to 0.5 eV, 1.0 eV, 1.5 eV and 2.0 eV; T_M and β are kept constant, respectively equal to 500 K and 10 K/sec.

Table 3. Frequency factor as a function of the kinetic order b , for a value of the activation energy $E = 0.5$ eV. s has been evaluated by means of numerical integration, Equation (10), s_2 by means of the second order approximation, Equation (13); s_2/s is the ratio of the two values. T_M and heating rate β are respectively fixed to 500 K and 10 K/s.

Activation Energy $E = 0.5$ eV			
b (kinetics order)	s	s_2	s_2/s
1	3.108E+05	3.108E+05	1.000
1.2	3.036E+05	3.022E+05	0.995
1.4	2.967E+05	2.940E+05	0.991
1.6	2.901E+05	2.862E+05	0.987
1.8	2.838E+05	2.788E+05	0.983
2	2.777E+05	2.718E+05	0.979

Table 4. Frequency factor as a function of the kinetic order b , for a value of the activation energy $E = 1.0$ eV. s has been evaluated by means of numerical integration, Equation (10), s_2 by means of the second order approximation, Equation (13); s_2/s is the ratio of the two values. T_M and heating rate β are respectively fixed to 500 K and 10 K/s.

Activation energy $E = 1$ eV			
b (kinetics order)	s	s_2	s_2/s
1	5.57E+09	5.57E+09	1.00
1.2	5.49E+09	5.48E+09	1.00
1.4	5.41E+09	5.39E+09	1.00
1.6	5.33E+09	5.30E+09	0.99
1.8	5.25E+09	5.21E+09	0.99
2	5.18E+09	5.13E+09	0.99

Table 5. Frequency factor as a function of the kinetic order b for a value of the activation energy $E = 1.5$ eV. s has been evaluated by means of numerical integration, Equation (10), s_2 by means of the second order approximation, Equation (13); s_2/s is the ratio of the two values. T_M and heating rate β are respectively fixed to 500 K and 10 K/s.

Activation energy $E = 1.5$ eV			
b (kinetics order)	s	s_2	s_2/s
1	9.163E+14	9.163E+14	1.000
1.2	9.067E+14	9.059E+14	0.999
1.4	8.972E+14	8.957E+14	0.998
1.6	8.880E+14	8.857E+14	0.997
1.8	8.790E+14	8.760E+14	0.997
2	8.701E+14	8.665E+14	0.996

Table 6. Frequency factor as a function of the kinetic order b for a value of the activation energy $E = 2.0$ eV. s has been evaluated by means of numerical integration, Equation (10), s_2 by means of the second order approximation, Equation (13); s_2/s is the ratio of the two values. T_M and heating rate β are respectively fixed to 500 K and 10 K/s.

Activation energy $E = 2.0$ eV			
b (kinetics order)	s	s_2	s_2/s
1	1.339E+20	1.339E+20	1.000
1.2	1.328E+20	1.327E+20	0.999
1.4	1.317E+20	1.316E+20	0.999
1.6	1.307E+20	1.305E+20	0.998
1.8	1.297E+20	1.294E+20	0.998
2	1.287E+20	1.283E+20	0.998

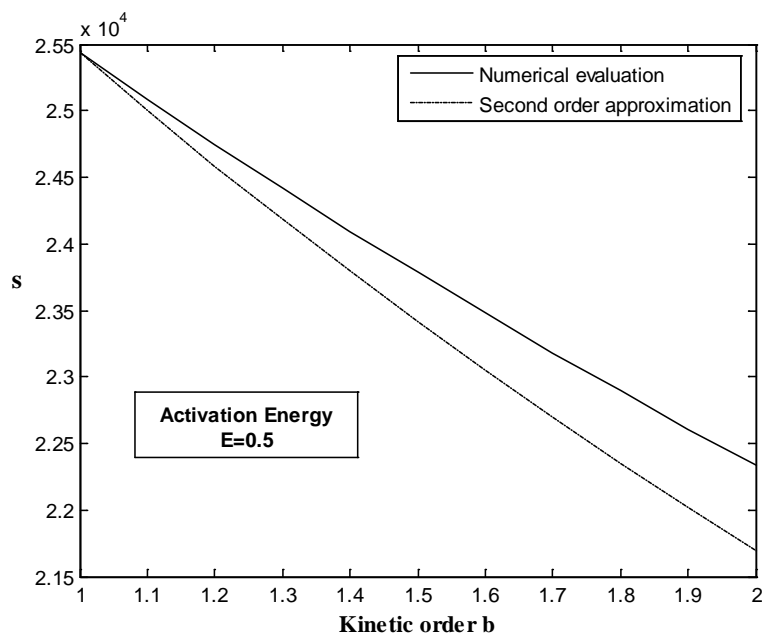


Figure 1. Behaviour of s as a function of b for a given activation energy $E = 0.5$ eV.

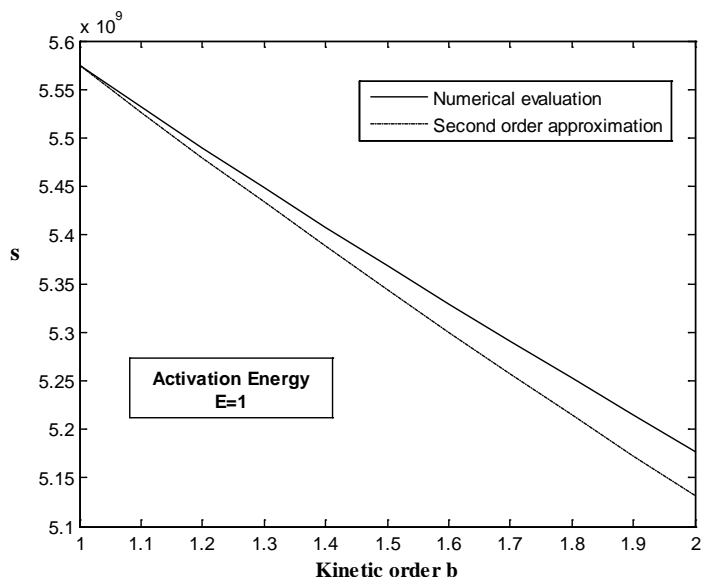


Figure 2. Behaviour of s as a function of b for a given activation energy $E = 1.0$ eV.

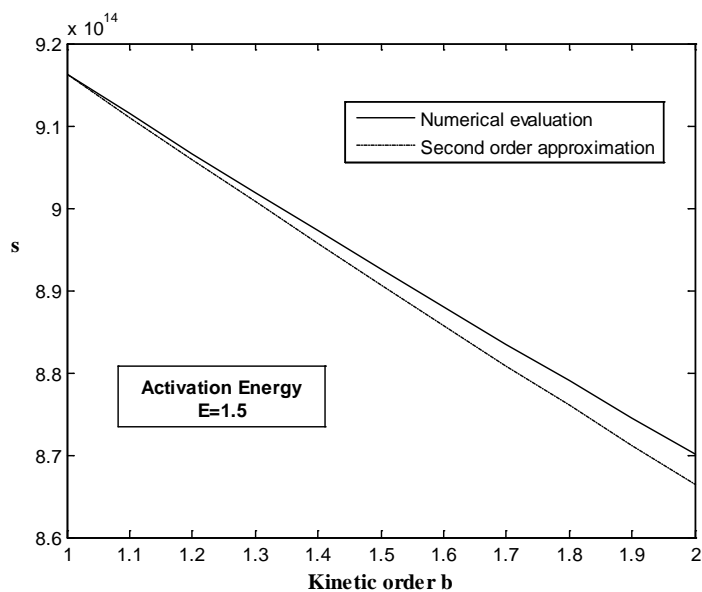


Figure 3. Behaviour of s as a function of b for a given activation energy $E = 1.5$ eV.

The following **Figure 5** shows the frequency factor as a function of T_M for a value of the activation energy, *i.e.* $E = 0.5$ eV.

Figure 6 shows s and s_2 calculated using Equation (9), as a function of the initial charge concentration n_0 ; the other parameters, *i.e.* E , b , T_M and β are constant.

Figure 7 shows the s and s_2 values as a function of various heating rates, from 10 K/sec to 50 K/sec in steps of 10 K/sec.

5. Higher Order Approximation

In this paragraph higher order approximations are presented and the results are compared with the second order approximation.

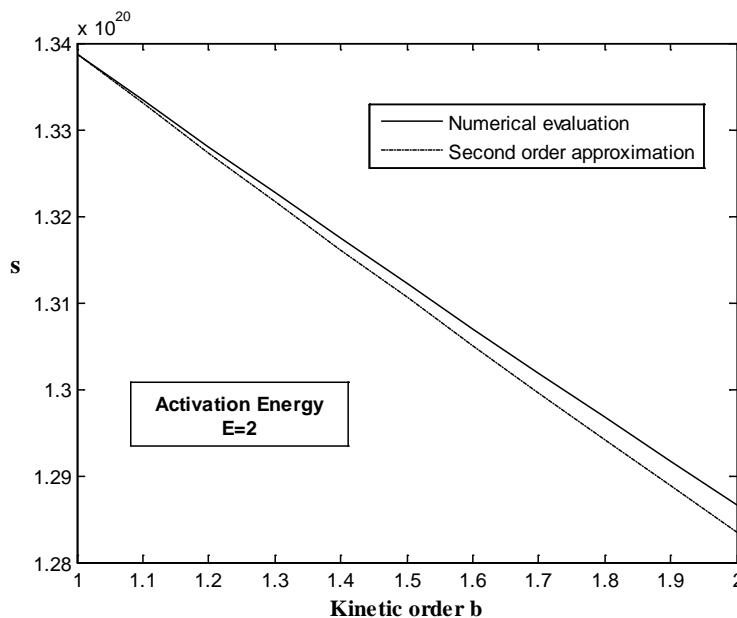


Figure 4. Behaviour of s as a function of b for a given activation energy $E = 2.0$ eV.

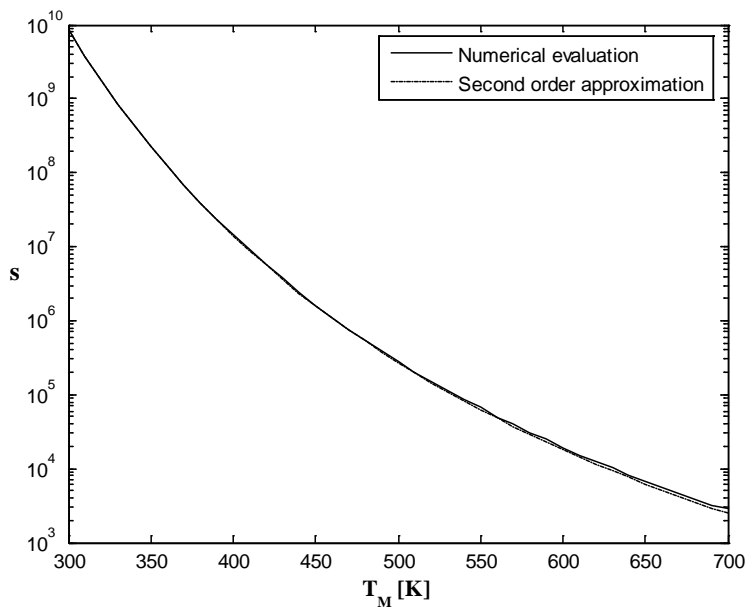


Figure 5. Frequency factor as a function of the temperature of the maximum T_M . The activation energy value has been fixed at 0.5 eV and the kinetics order at 2. T_M ranges from 300 to 700 K.

By means of the asymptotic series, the expression of s becomes:

$$s = \frac{\beta E}{kT_M^2} e^{\frac{E}{kT_M}} \left\{ 1 + (b-1) \sum_{n=1}^{+\infty} (-1)^{n+1} (n+1)! \left(\frac{kT_M}{E} \right)^n \right\}^{-1} \tag{14}$$

where the second order approximation is obtained for $n = 2$; the third order approximation for $n = 3$, and so on.

In particular, the third order expression is the following

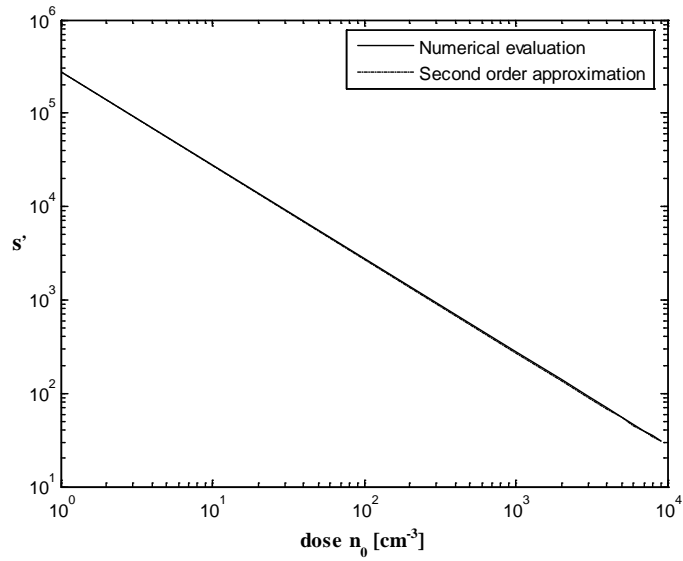


Figure 6. The pre-exponential factor s'' as a function of the dose n_0 evaluated numerically and by means of the second order approximation. Activation energy is fixed at 0.6 eV, the temperature of the maximum T_M is fixed at 500 K and the heating rate at 10 K/s. s'' is expressed in $[s^{-1} \cdot cm^{3(b-1)}]$.

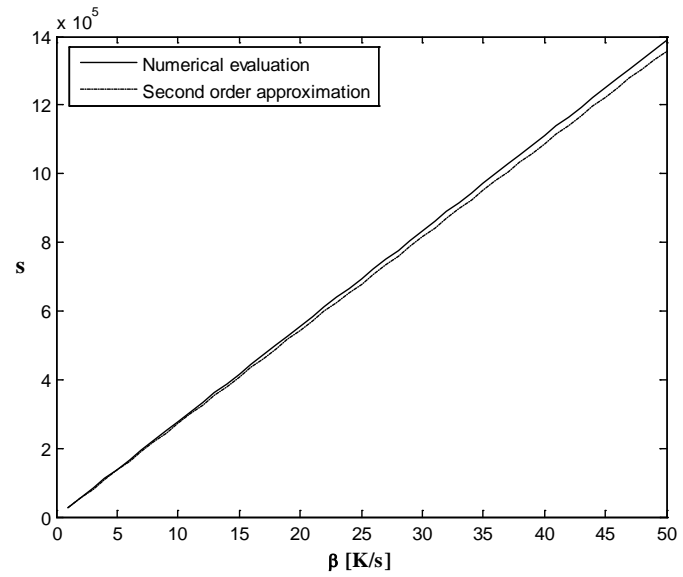


Figure 7. Frequency factor s as a function of the heating rate β . The activation energy is fixed at 0.6 eV and the temperature of the maximum, T_M , at 500 K.

$$s_3 = \frac{\beta E}{kT_M^2} e^{\frac{E}{kT_M}} \left\{ 1 + (b-1) \left[\frac{2kT_M}{E} - 6 \left(\frac{kT_M}{E} \right)^2 \right] \right\}^{-1} \quad (15)$$

and the fourth order approximation is given by

$$s_4 = \frac{\beta E}{kT_M^2} e^{\frac{E}{kT_M}} \left\{ 1 + (b-1) \left[\frac{2kT_M}{E} - 6 \left(\frac{kT_M}{E} \right)^2 + 24 \left(\frac{kT_M}{E} \right)^3 \right] \right\}^{-1} \quad (16)$$

Considering $s'' = s/n_0^{b-1}$, the expressions for the exponential factor in higher order approximation are obtained.

The calculated values are reported in **Table 7** as a function of the kinetic order; the other parameters are constant as before. **Figure 8** shows the frequency factor evaluated according to the various approximations used. It seems evident that the increase of the order of approximation does not produce a real benefit in the final result; a very little difference in the results can be observed as the kinetic order increases from 1 to 2.

6. Conclusions

Figures 1-4 show that the disagreement between the data, obtained using the numerical evaluation and the ones calculated with a second order approximation, becomes larger as the value of the kinetics order, b , increases; the difference is more evident for low values of the activation energy, *i.e.* $E = 0.5$ eV; this difference tends to disappear as the activation energy increases: at $E = 2.0$ eV no difference can be observed.

The behaviour of the frequency factor as a function of the peak temperature at the maximum, T_M , is given in **Figure 5** and **Figure 6**: the numerical evaluation gives the same results of the numerical approximation in the range of used temperature: from 300 K to 700 K and an activation energy value of 0.5 eV, and from 450 K to 700 K with an activation energy equal to 1.5 eV.

Table 7. s numerical evaluation; $s2$ second order approximation; $s3$ third order approximation; $s4$ fourth order approximation; $s2/s$, $s3/s$, $s4/s$ are respectively the ratios between second order approximation $s2$ and s , third order approximation $s3$ and s and finally fourth order approximation $s4$ and s .

Activation energy $E = 0.6$ eV							
b (kinetics order)	s	$s2$	$s2/s$	$s3$	$s3/s$	$s4$	$s4/s$
1	3.108E+05	3.108E+05	1.000	3.108E+05	1.000	3.108E+05	1.000
1.2	3.036E+05	3.022E+05	0.995	3.040E+05	1.001	3.035E+05	1.000
1.4	2.967E+05	2.940E+05	0.991	2.974E+05	1.003	2.964E+05	0.999
1.6	2.901E+05	2.862E+05	0.987	2.912E+05	1.004	2.897E+05	0.999
1.8	2.838E+05	2.788E+05	0.983	2.851E+05	1.005	2.833E+05	0.998
2	2.777E+05	2.718E+05	0.979	2.794E+05	1.006	2.771E+05	0.998

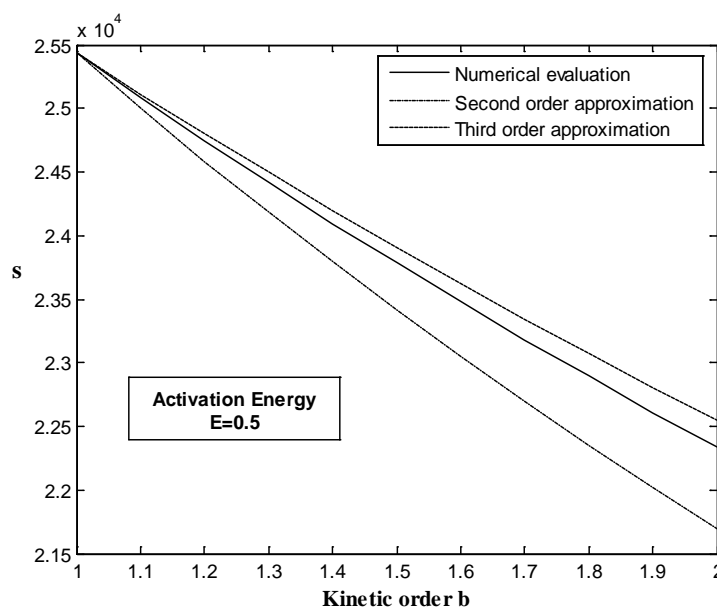


Figure 8. Frequency factor as a function of the kinetic order, evaluated using different methods.

No difference can be observed between numerical and second order calculation for the pre-exponential factor as a function of the irradiation dose (**Figure 7**). A difference appears for high values of the heating rate (**Figure 8**); the difference increases as the value of the kinetic order increases too.

References

- [1] Randall, J.T. and Wilkins, M.H.F. (1945) Phosphorescence and Electron Traps. *Proceedings of the Royal Society*, **A184**, 366.
- [2] Garlick, G.F.J. and Gibson, A.F. (1948) The Electron Trap Mechanism of Luminescence in Sulphide and Silicate Phosphors. *Proceedings of the Physical Society*, **60**, 574. <http://dx.doi.org/10.1088/0959-5309/60/6/308>
- [3] May, C.E. and Partridge, J.A. (1964) Thermoluminescent Kinetics of Alpha-Irradiated Alkali Halides. *The Journal of Chemical Physics*, **40**, 1401. <http://dx.doi.org/10.1063/1.1725324>
- [4] Chen, R. and McKeever, S.W.S. (1997) *Theory of Thermoluminescence and Related Phenomena*. World Scientific, Singapore City.
- [5] Chen, R. and Kirsh, Y. (1981) *Analysis of Thermally Stimulated Processes*. Pergamon Press, Oxford.