Simulation and Comparative Analysis of SS-LMS & RLS Algorithms for Electronic Dispersion Compensation

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Abstract

In this paper electronic feed forward equalization is performed to mitigate the link chromatic dispersion. The equalizer coefficients are computed by a decision-directed process based on the sign-sign least mean square and the recursive least square algorithm. Therefore, this paper evaluates the performance of these algorithms in chromatic dispersion compensation at bit rate of 10 Gb/s. This paper compares these two adaptation algorithms for receiver based on analogue electronic dispersion equalizers by simulation and experiment. This paper concluded that recursive least-square algorithm is computationally more complex than sign-sign least mean square algorithm since matrix inversion is required, but achieves faster convergence.

Keywords

Bit Error Rate (BER), Electronic Dispersion Compensation (EDC), Feed-Forward Equalizer (FFE), Recursive Least Square (RLS), Sign-Sign Least Mean Square (SS-LMS)

Subject Areas: **Optical Communications, Simulation/Analytical Evaluation of Communication Systems**

1. Introduction

In optical fiber, the group velocity of the propagating signal is frequency dependent and optical pulses hence spread in time. This results in chromatic dispersion (CD), thus limiting the transmission distance and/or data rate [1]-[6]. Without proper compensation, the performance of the fiber communication systems will be severely limited. The CD is traditionally compensated using optical devices with opposite dispersion but such approaches cannot be easily tuned/improved to accommodate different fiber spans/properties and quality measures [5] [6]. The electronic equalizer can be used to mitigate the chromatic dispersion because it can be dynamically tuned at

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high speed, has much smaller form factor and much lower cost. In particular, digital signal processing (DSP) can be employed to realize compensators with high functionality and reproducibility. An electronic equalizer can be integrated into a single chip using the high-speed Silicon Germanium (SiGe) or Indium Phosphorus (InP) technology [7]. Further cost reduction is possible if electronic equalizer and other circuits on the receiver are integrated on the same chip.

This work presents and compares the performance of a prototype adaptive electronic dispersion compensation (EDC) receiver using Sign-Sign Least Mean Square (SS-LMS), and Recursive Least Square (RLS) algorithm, coupled with a directly modulated laser (DML), operating at the OC-192 rate. The remainder of this paper is set as follows. Second section presents the theory of dispersion penalty. Third section explains the experimental setting. Fourth section presents the use of feed forward equalizer in receiver. Fifth section explains sign-sign least mean square algorithm. Sixth section explains recursive least square algorithm. A section seventh provides the comparison between algorithms using experimental results. The conclusion is presented in section eighth.

2. Dispersion Penalty

Dispersion induced pulse broadening affects the performance in two ways. First a part of the pulse energy spreads beyond the allocated bit slot and leads to intersymbol interference. **Figure 1** shows the dependence of pulses overlap on transmission rate means with the increase in bit rate the dispersion increases therefore intersymbol interference has more effect at higher bit rates. According to current standards (ITU-T G984.1), 2.5 Gb/s transmitters must support distances up to 20 Km. However, research efforts are underway to extend operating rates up to 10 Gb/s for the same value of transmission reach [8]. Second, the pulse energy within the bit slot is reduced when the optical pulse broadens. Such a decrease in the pulse energy reduces the signal to noise ratio (SNR) at the decision circuit. Since the SNR should remain constant to maintain the system performance, the receiver requires more average power. This is the origin of dispersion induced power penalty (δ_d).

An exact calculation of δ_d is difficult, as it depends on many details, such as the extent of pulse shaping at the receiver. So, the dispersion penalty δ_d can be defined as the increase (in dB) in the received power that would compensate the peak power reduction, and is given by following equation [10]:

$$\delta_d = 10\log_{10} f_b \tag{1}$$

where f_b is the pulse broadening factor.

When the pulse broadening is due to a wide source spectrum at the transmitter, the pulse broadening f_b is given by following equation [10]:

$$f_b = \sigma/\sigma_0 = \left[1+\left(DL\sigma_\lambda/\sigma_0\right)\right]^{1/2} \tag{2}$$

where σ_0 is the RMS width of the optical pulse at the fiber input and σ_λ is the RMS width of the source spectrum which is Gaussian. Another formula for dispersion penalty is given in following equation [11]:

$$\delta_d = 5\log_{10}\left(1+2\pi\left(BD\Delta\lambda\right)^2 L^2\right) \tag{3}$$

where δ_d is dispersion penalty, $\Delta\lambda$ is the range of wavelengths emitted by a source, B is the bit rate and L is the fiber length.

3. Experimental Setting

The experimental setting at the receiver, for processing of received signal to achieve better performance of optical fiber communication system is shown in **Figure 2**.

It consists of a multi-tap feed forward equalizer (FFE), an error calculator, weight update signal generator and a digital controller. The feed forward equalizer section comprises of a parallel implementation with five tap delay lines with delay value 100 ps and tap spacing at the sample rate *i.e.*, 80,000. The equalizer filter structure is a finite impulse response (FIR) filter that takes the sampled values of the electrical input signal as its input (after optical to electrical conversion). In the simulation of this work, Gaussian pulse wave is used as the input pulse. An error calculator is used to calculate error and is worked in conjunction with weight update signal generator to generate filter coefficients. The digital controller computes, updates and monitors the filter coefficient values. In weight update signal generator various algorithms are used to update weights so that mean square error is reduced.

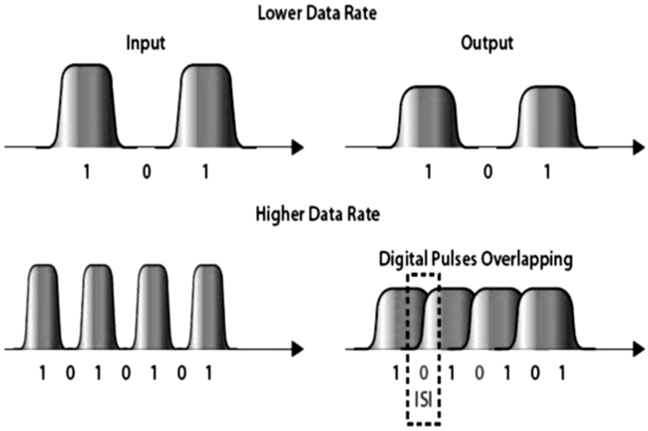

Figure 1. The dependence of pulses overlap on transmission rate [9].

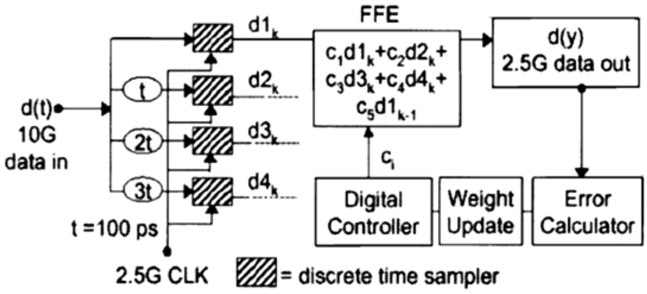

Figure 2. Experimental setting for EDC with feed forward equalizer [12].

4. Feed-Forward Equalizer in Receiver

A feed-forward equalizer is the simplest type of equalizer and its output is produced by summing the current and past values of the received signal which linearly weighted by the filter coefficients.

The basic structure of an adaptive equalizer is shown in **Figure 3**, where the subscript k is used to denote a discrete time index. Note that in **Figure 3** there is a single input y_k into the equalizer at any time instant. The value of y_k depends upon the instantaneous state of the channel. The adaptive equalizer has N delay elements, $N+1$ taps, and $N+1$ tunable complex multipliers, called weights or coefficients. These weights are updated continuously by the adaptive algorithm. The adaptive algorithm is controlled by the error signal e_k. This error signal is derived by comparing the output of the equalizer $\hat{d}_k$, with some signal d_k which is either an exact replica of the transmitted signal x_k or which represents a known property of the transmitted signal.

The adaptive algorithm uses e_k to minimize a cost function which is mean square error (MSE) between the desired signal and the output signal of the equalizer based on the classical equalization theory [13] [14]. The MSE is denoted by $\left[e(k)e^*(k)\right]$, and a known training sequence must be periodically transmitted when a replica of the transmitted signal is required at the output of the equalizer.

5. Sign-Sign Least Mean Square Algorithm

The equalizer coefficients are computed by the sign-sign least mean square (SS-LMS) method, because it demonstrates the simplicity and robustness needed for realization in very high speed circuits [15]. The flowchart for Sign-Sign Least Mean Square (SS-LMS) algorithm shown in **Figure 4** has been summarized as follows [16]:

Step 1: The very first step was to set the initial filter weights, minimum mean square error.

Step 2: After that the i no. of time delayed versions of received signal using 100 ps time delay was multiplied with these weights and got actual output which was summation of all these terms.

Step 3: Then error signal was calculated as given in following equation [15];

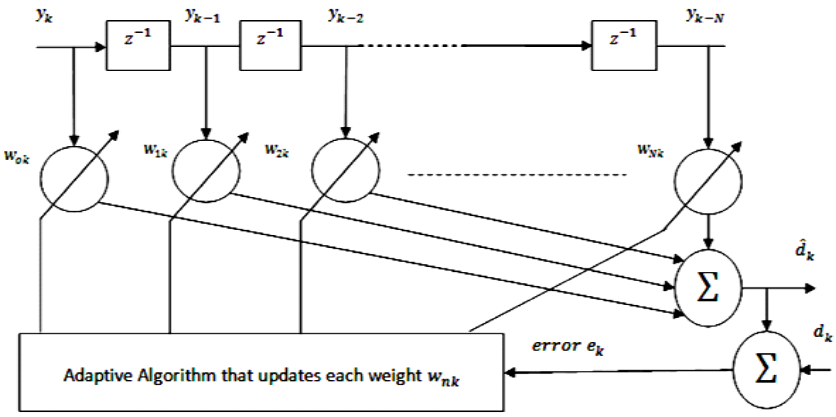

Figure 3. A basic linear equalizer during training [13].

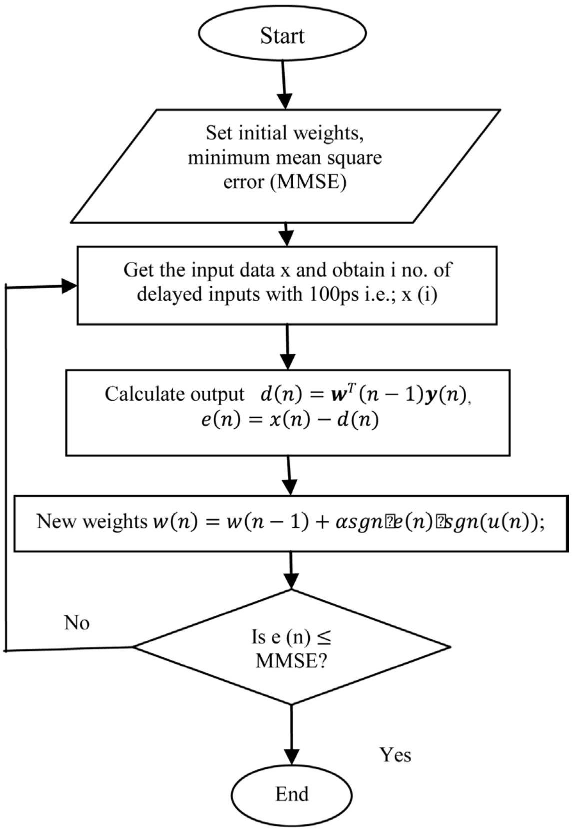

Figure 4. Flow chart of Sign-Sign LMS algorithm.

$$e(n) = u(n) - y(n) \tag{4}$$

where n is number of inputs, $u(n)$ is desired output signal, $y(n)$ actual output and $e(n)$ is error signal.

Step 4: Then the filter weights was updated using sign-sign least mean square method as given in following equation [15]:

$$w(n) = w(n-1) + \alpha \operatorname{sgn}(e(n)) \operatorname{sgn}(u(n)) \tag{5}$$

where $w(n)$ is updated weights, $w(n-1)$ is previous weights, $e(n)$ is error signal , $u(n)$ is actual input signal and α is the step size which controls the convergence rate and stability of algorithm. The value of α is chosen from [17]:

$$0 < \alpha < 2 \Big/ \sum_{i=1}^{N} \lambda_i \tag{6}$$

Since λ_i is the ith eigenvalue of the covariance matrix $\boldsymbol{R}_{NN}$.

Step 5: This procedure was repeated until the limit of minimum mean square error was achieved.

6. Recursive Least Square Algorithm

The convergence rate of the gradient based Least mean square algorithm is very slow, in order to achieve faster convergence, complex algorithms which involve additional parameters to control the adaptation rate are used. Recursive least square algorithm is based on a least squares approach, which significantly improves the convergence of adaptive equalizers [17] [18]. The flowchart for RLS algorithm shown in **Figure 5** has been summarized as follows [13]:

Step 1: First of all $w(0) = k(0) = x(0) = 0$ and $\boldsymbol{R}^{-1}(0) = \delta \boldsymbol{I}_{NN}$, was initialized.
where $\boldsymbol{I}_{NN}$ is an $N \times N$ identity matrix, and δ is a large positive constant.

Step 2: Then value of $d(n)$ was calculated using the following equation [16]:

$$d(n) = \boldsymbol{w}^{\mathrm{T}}(n-1)\boldsymbol{y}(n) \tag{7}$$

where $\boldsymbol{w}^{\mathrm{T}}(n-1)$ is the transpose of previous weights and $\boldsymbol{y}(n)$ are the actual outputs.

Step 3: After that value of error signal $e(n)$ was calculated using following equation [16]:

$$e(n) = x(n) - d(n) \tag{8}$$

where $x(n)$ is the desired output.

Step 4: Then value of $k(n)$ and $\boldsymbol{R}^{-1}(n)$ were calculated using following equations [19]:

$$k(n) = \frac{\boldsymbol{R}^{-1}(n-1)\boldsymbol{y}(n)}{\lambda + \boldsymbol{y}^{\mathrm{T}}(n)\boldsymbol{R}^{-1}(n-1)\boldsymbol{y}(n)} \tag{9}$$

$$\boldsymbol{R}^{-1}(n) = \frac{1}{\lambda}\left[\boldsymbol{R}^{-1}(n-1) - \boldsymbol{k}(n)\boldsymbol{y}^{\mathrm{T}}(n)\boldsymbol{R}^{-1}(n-1)\right] \tag{10}$$

where λ is weighting coefficient.

Step 5: By using the value of these above equations new weights were calculated given by [19]:

$$\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + \boldsymbol{k}(n)e^{*}(n) \tag{11}$$

Step 6: This weight update procedure was repeated until the value of mean square error was less than or equal to minimum MSE value.

The λ is the weighing coefficient that can change the performance of the equalizer. Usually this factor vary from $0.8 < \lambda < 1$. The value of λ has no influence on the rate of convergence, but does determine the tracking ability of the RLS equalizers. The smaller is the value of λ, the better the tacking ability of the equalizer. However if λ is too small, the equalizer will be unstable [20].

7. Results and Discussions

The results obtained with Sign-Sign Least Mean Square and Recursive Least Square algorithm by performing various experiments, have been summarized in **Figures 6-9**.

Figure 6 shows the Bit error rate (BER) versus Received power in dBm without EDC for different values of fiber length at typical dispersion value of 17 ps/nm-km. The required value of received power at the input of optical fiber receiver is −25.7 dBm, −22.36 dBm, −18.23 dBm, −15.28 dBm, and −13.24 dBm for fiber length of 0 km, 5 km, 10 km, 15 km and 20 km respectively to maintain bit error rate of 1.974×10^{-9}. The percentage increase in required received power is 14.94, 40.97, 68.19 and 94.11 without EDC for fiber length 5 km, 10 km, 15 km and 20 km respectively.

Figure 7 shows the Bit error rate (BER) versus Received power in dBm with EDC using SS-LMS for different values of fiber length at typical dispersion value of 17 ps/nm-km. The required value of received power at the input of optical fiber receiver is −25.7 dBm, −24.22 dBm, −22.58 dBm, −21.21 dBm, and −20.09 dBm for

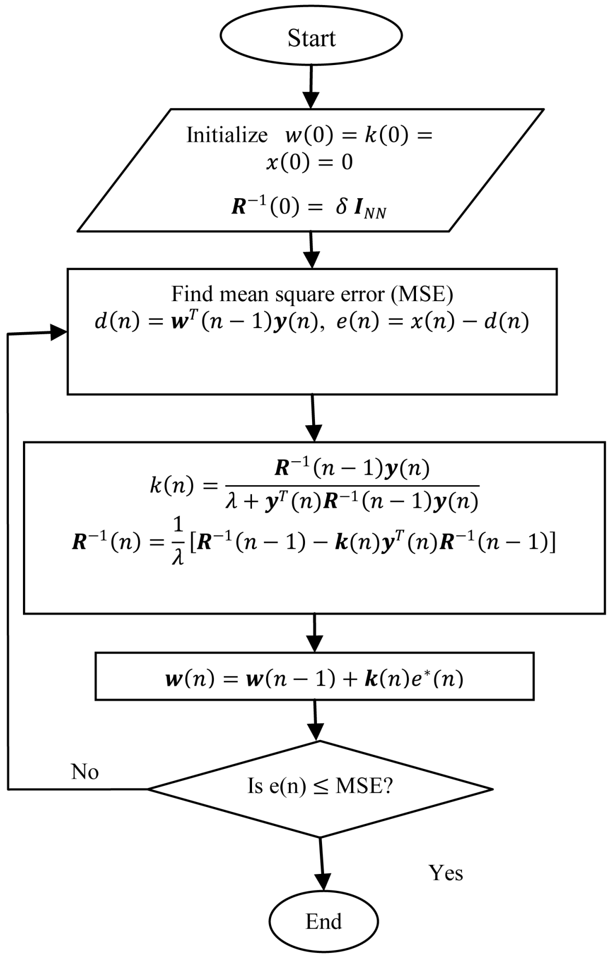

Figure 5. Flow chart of RLS algorithm.

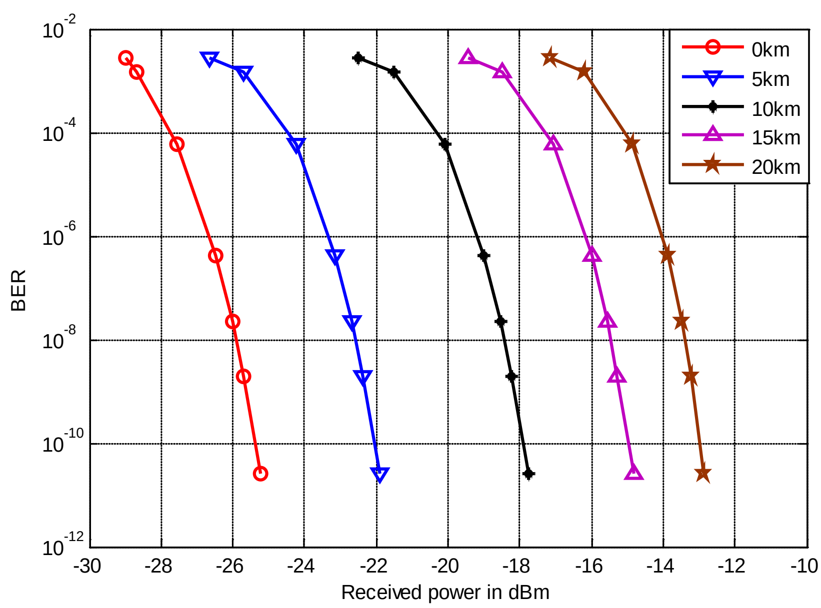

Figure 6. BER versus Received power in dBm without EDC for different values of fiber length at typical dispersion value of 17 ps/nm-km.

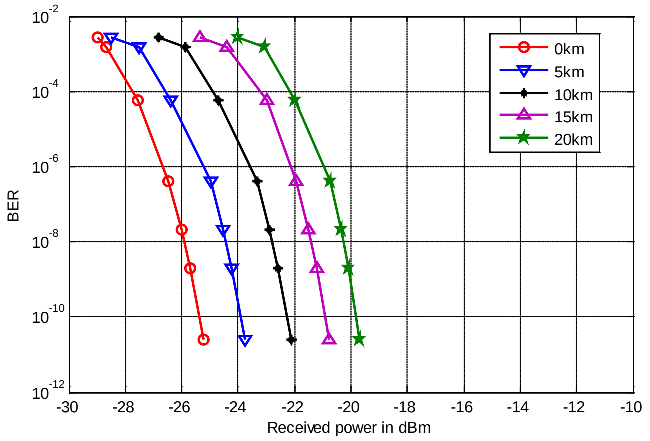

Figure 7. BER versus Received power in dBm with EDC using SS-LMS for different values of fiber length at typical dispersion value of 17 ps/nm-km.

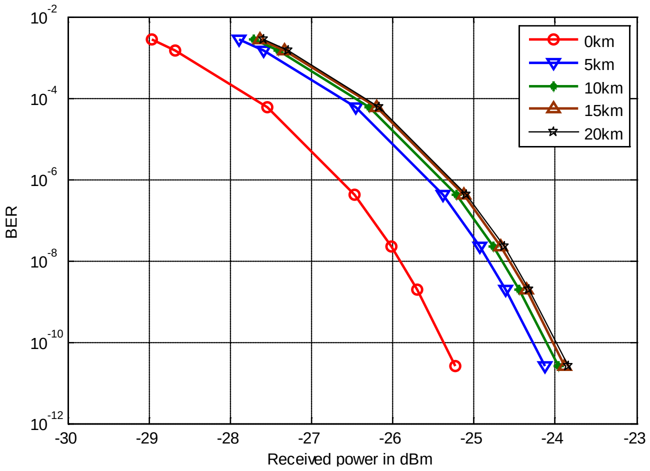

Figure 8. BER versus Received power in dBm with EDC using RLS for different values of fiber length at typical dispersion value of 17 ps/nm-km.

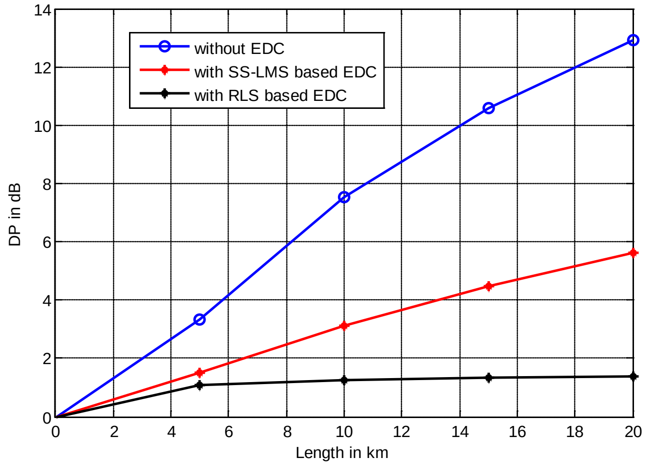

Figure 9. Dispersion penalty (DP) in dB versus fiber length in km without and with EDC using SS-LMS and RLS algorithms.

Table 1. Measured dispersion penalty (dB), defined at BER = 1.97×10^{-9} at typical dispersion value of 17 ps/nm-km.

Length of Fiber (Km)	Without EDC	With EDC	
		SS-LMS	RLS
5	3.343	1.485	1.092
10	7.511	3.12	1.259
15	10.58	4.487	1.342
20	12.91	5.642	1.384

fiber length of 0 km, 5 km, 10 km, 15 km and 20 km respectively to maintain bit error rate of 1.974×10^{-9}. The percentage increase in required received power is 6.11, 13.82, 21.16, and 27.92 with SS-LMS algorithm for fiber length 5 km, 10 km, 15 km and 20 km respectively.

Figure 8 shows the Bit error rate (BER) versus Received power in dBm with EDC using RLS for different values of fiber length at typical dispersion value of 17 ps/nm-km. The required value of received power at the input of optical fiber receiver is −25.7 dBm, −24.61 dBm, −24.44 dBm, −24.36 dBm, and −24.22 dBm for fiber length of 0 km, 5 km, 10 km, 15 km and 20 km respectively to maintain bit error rate of 1.974×10^{-9}. The percentage increase in required received power is 4.43, 5.15, 5.5, and 6.11 with RLS algorithm for fiber length 5 km, 10 km, 15 km and 20 km respectively.

Figure 9 shows Dispersion penalty (DP) in dB versus fiber length in km without and with EDC using SS-LMS and RLS algorithms. Here the percentage increase in fiber length of 300, the dispersion penalty has increased by 286% without EDC, 279.93% with SS-LMS based EDC and 26.73% with RLS based EDC at typical value of 17 ps/nm-km. The maximum value of dispersion penalty is 12.91, 5.642 and 1.384 without EDC, with SS-LMS based EDC and with RLS algorithm based EDC at 20 km fiber length and fiber dispersion of 17 ps/nm-km. This figure shows the EDC with SS-LMS algorithm roughly doubles the transmission length and the EDC with RLS algorithm roughly ten times the transmission length for the same value of dispersion penalty.

Table 1 shows the value of dispersion penalty which is obtained without EDC and with EDC by using various algorithms at BER of 1.97×10^{-9} and typical dispersion value of 17 ps/nm-km, for different value of fiber length.

Table 1 shows that for length of 20 km the value of dispersion penalty without EDC is 12.91 and dispersion penalty with EDC using SS-LMS and RLS algorithm is 5.642 and 1.384.

8. Conclusion

It has been found from this study that the performance of two algorithms is different because the convergence rate of Sign-Sign LMS algorithm depends only on single parameter but RLS algorithm convergence rate depends on many parameters. Sign Least Mean Square Algorithm approximately doubles the usable fiber length for a given value of dispersion penalty, and this algorithm is simplest and requires less memory to store equalizer coefficients. Another conclusion from this study is that the EDC using Feed-forward Equalizer with Recursive Least Square algorithm approximately achieves ten times the usable fiber length for given value of dispersion penalty means achieves faster convergence, but this algorithm is very complex and requires more memory because matrix inversion is performed in this algorithm. So, if the system cost is major factor then Sign-Sign Least Mean Square algorithm is the preferred algorithm and if usable fiber length is the major aspect then Recursive Least Square Algorithm is used.

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