

On El Naschie's Fractal-Cantorian Space-Time and Dark Energy—A Tutorial Review

Leila Marek-Crnjac

Department of Mathematics, Technical School Center of Maribor, Maribor, Slovenia

Email: leila.marek@guest.arnes.si

Received 15 November 2015; accepted 6 December 2015; published 10 December 2015

Copyright © 2015 by author and Scientific Research Publishing Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

This tutorial review is dedicated to the work of the outstanding Egyptian theoretical physicist and engineering scientist Prof. Mohamed El Naschie. Every physics student knows the well-known Einstein's mass-energy equation, $E = mc^2$, but unfortunately for physics, few know El Naschie's modification, $E(O) = mc^2/22$, and El Naschie's dark energy equation $E(D) = mc^2(21/22)$ although this new insight has truly far reaching implications. This paper gives a short tutorial review of El Naschie's fractal-Cantorian space-time as well as dark energy. Emphasis is put on the fundamental concept of Cantor set, fractal dimensions, zero set, empty set, and Casimir effect.

Keywords

E-Infinity Theory, Dark Energy, Casimir Effect, Quantum Wave, Quantum Particle, Quantum Entanglement, Nanotechnology, Black Holes Information Paradox

1. Introduction

Modern theoretical physics has a truly fascinating and marvellous story to tell and teach everyone, particularly physics students, regarding its logical structure and development [1] [2]. We could start the story with Newton's smooth three-dimensional space although to make a long story short, it is advisable to start with relativity [1] [2]. Einstein's theory of relativity, which he designed in a smooth four dimensional space-time, was the first major modern revolution in theoretical physics since Newton and Maxwell [1]. Not only that but the work of Einstein was positioned somehow in many respects between relativity and the next revolution, namely quantum mechanics. This is so despite Einstein's reluctance to embrace quantum entanglement [3] and the fundamental changes

in our philosophy which this new theory implies [4] [5]. These fascinating aspects of Einstein’s work and the implication of El Naschie’s extension [6] are the subject of the present tutorial. In addition, we included many important references for work related to El Naschie’s E-infinity theory.

2. Einstein’s Mass-Energy Equation and Beyond

Einstein’s well-known iconic equation of relativity relating total energy to mass and the speed of light is given by

$$E = mc^2 \quad (1)$$

It was derived for a photon. It does not model the energy of our real world. Einstein’s theory was established on the assumption of absolute smooth 3 + 1 dimensional space-time. However, space and time are not at all smooth. To demonstrate the discontinuity of space-time [7], we consider a TV screen (see **Figure 1**) that is smooth at any ordinary observable scales. However, when the scale becomes smaller and smaller, until to a very small one, the surface reveals an unsmooth face consisting of many arrayed pixels with fractal dimensions larger than 2, the dimension fluctuation is of great importance to understand El Naschie’s fractal space-time, which will be explained in details in the forthcoming sections. Time is also discontinuous when it is extremely small. A film gives 24 or more slips per second. This gives a continuous movement (see **Figure 2**). However, in the case of 20 or less slips per second, the movement becomes discontinuous.

Now as we mentioned earlier on, Einstein’s mass-energy equation is derived under the assumption of absolute smooth space-time. In reality space-time is intrinsically discontinuous when it tends to a quantum scale [6] and a



Figure 1. The “smooth” TV screen is not smooth at small scales. The idea of presenting this fact in this form is due to Prof. Lee Smolin and Prof. Ji-Huan He.

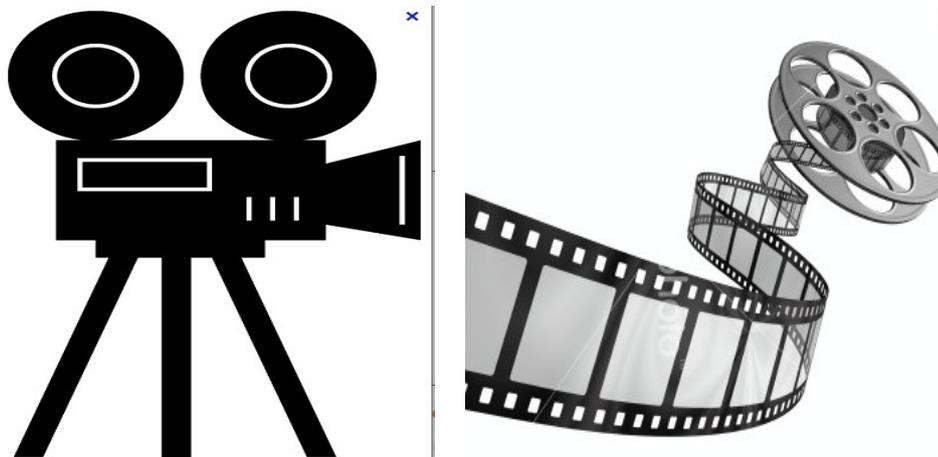


Figure 2. “Continuous” movement in a film consists of millions of discontinuous flashes of static single photos. The idea of this figure is due to Prof. Ji-Huan He.

Hilbert cube can excellently model the actual fractal space-time [8]. When space-time becomes discontinuous at very high resolution corresponding to very small scales, Newton-Leibniz calculus ceases to be valid and fractal theory must be adopted to describe all phenomena. According to El Naschie's E-infinity theory [6], space-time is a random Cantor set with Hausdorff dimension of $\phi = (\sqrt{5} - 1)/2$ instead of $\ln 2/\ln 3$. (See **Figure 3**).

When we construct a Cantor set, whether deterministic or random, we end up with two Cantor sets, the zero set and the empty set, the former consists of infinite points, and the latter is the left of the unit interval [9], see **Figure 3**. These two Cantor sets are extremely important for understanding El Naschie's theory [10]. The zero set represents the quantum particle and its Hausdorff dimension is the golden mean, $\phi = (\sqrt{5} - 1)/2$. The empty set models the quantum wave but it also models quantum space-time itself [11], its Hausdorff dimension is $1 - \phi = \phi^2$.

We consider an extremely large plan with discontinuous boundary at an extremely small scale, see **Figure 4**. The average Hausdorff dimension of the plan given in **Figure 4** is

$$1 + \frac{\ln 4}{\ln 3} = 2 + 0.2618 \tag{2}$$

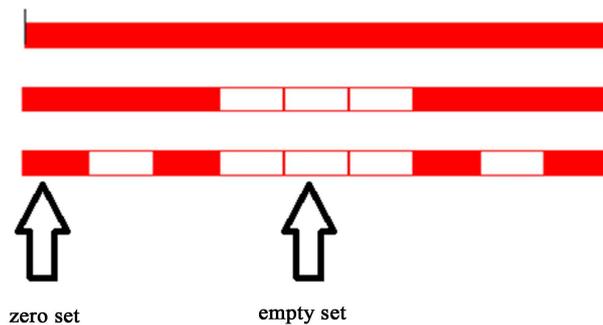


Figure 3. The construction of a deterministic Cantor set with Hausdorff dimensions of $\ln 2/\ln 3$. When uniform randomness is added, the Hausdorff dimension is slightly reduced to $\phi = (\sqrt{5} - 1)/2$. In both cases the measure, *i.e.* length is zero.

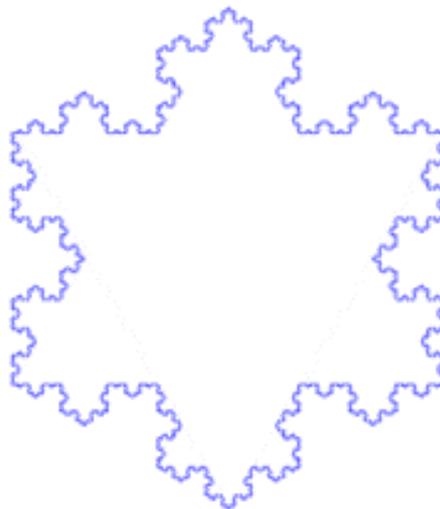


Figure 4. Fractal boundary with fractal dimensions of $\ln 4/\ln 3$. Dark energy, which is a hypothetical form of energy that permeates all of space and tends to accelerate the expansion of the universe, is hidden on the universe's fractal boundary. According to E-infinity theory, Prof. El Naschie revealed that dark energy currently accounts for about 95.5% of the total mass-energy of the universe and is hidden at the universe's fractal boundary as per Dvořetzký's theorem.

where 0.2618 is the fluctuation of plan dimensions. The fluctuation dimensions for our space-time at quantum scale is $\phi^3 = 0.236\dots$, where $\phi = (\sqrt{5} - 1)/2$ [6].

The average Hausdorff dimension of our space-time is [6]

$$D_4 = 4 + \phi^3 = 4 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4 + \dots}}} = \frac{1}{\phi^3} = 4.236 \tag{3}$$

The general El Naschie's dimensional form is [6]

$$D_n = \phi^{-(n-1)}. \tag{4}$$

For our real $n = 4$ space-time, Equation (4) gives Equation (3).

The fractal explanation of Equation (3) is given in Figure 5 [8] and looks like a Russian doll with self-similarity in all scales.

It is instructive to relate in a visually impressive way how the dimensionality of space progressed from Newton to El Naschie via Einstein following a proposal by Prof. Ji-Juan He:

$$\begin{aligned} \text{Dimensions} = & 3 + 1 + \phi^3 \\ & \boxed{\quad} \text{Newton} \\ & \boxed{\quad} \text{Einstein} \\ & \boxed{\quad} \text{El Naschie} \end{aligned} \tag{5}$$

Similarly the five dimensional Kaluza-Klein space-time [12] and Witten's fractal M-theory [13] [14] can be represented as follows:

$$\begin{aligned} \text{Dimensions} = & 4 + 1 + \phi^3 \\ & \boxed{\quad} \text{5-dimensional Kaluza-Klein spacetime} \\ & \boxed{\quad} \text{El Naschie's modification} \end{aligned} \tag{6}$$

$$\begin{aligned} \text{Dimensions} = & 11 + \phi^5 \\ & \boxed{\quad} \text{Witten's M-theory} \\ & \boxed{\quad} \text{El Naschie \& Ji-Huan He's modification} \end{aligned} \tag{7}$$

Further more El Naschie was able to show that Einstein's iconic $E = mc^2$ is in fact the sum of two basically quantum parts [15]-[23], namely that of the quantum particle energy

$$E(O) = mc^2/22 \tag{8}$$

and that of the quantum wave energy

$$E(D) = mc^2 (21/22) \tag{9}$$

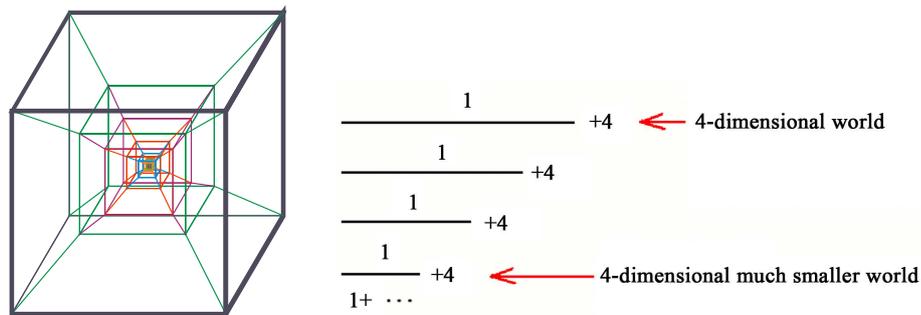


Figure 5. El Naschie-Ji-Huan He fractal space-time model with self-similarity [8].

so that Einstein, without realizing, did indeed hit the nail on its head, quantumly speaking [24]. For an easily understandable explanation of these facts, we consider the 5 dimensional Kaluza-Klein space-time [7] [12]. When it tends to quantum scale, the average Hausdorff dimension of fractal Kaluza-Klein space-time is $5 + \phi^3$. Consider a quasi-Hausdorff hyper volume of the 5 dimensional Kaluza-Klein space-time, which consists of the zero set of quantum particles, and the empty set of quantum waves [7]-[10].

The quantum particles are not occupied within a $1 \times 1 \times 1 \times 1 \times 1$ hyper volume but a $\phi \times \phi \times \phi \times \phi \times \phi$ fractal hyper volume. That means that the quantum particles have energy equal to [7]-[10]

$$E = \lim_{v \rightarrow c} \frac{\phi \times \phi \times \phi \times \phi \times \phi}{1 \times 1 \times 1 \times 1 \times 1} \frac{1}{2} mv^2 = \frac{1}{2} \phi^5 mc^2 \tag{10}$$

This is El Naschie's meantime equation [11] predicting that 95.4915028% of the energy in the cosmos is the missing hypothetical dark energy. El Naschie's theory combines Newton's mechanics with quantum mechanics. A mini-symposium on dark energy in 4th international symposium on nonlinear dynamics was held in Shanghai, China on October 30, 2012, for celebrating El Naschie's greatest finding, see Figure 6.

We recall first Einstein's relativistic mass equation, which is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{11}$$

In the above equation, the velocity v relates to that of the photon. Now we consider a photon moving in x-direction, while a particle moves on a 2 dimensional plane with same projection velocities in x- and y-directions, that is the velocity in x-direction is $v/\sqrt{2}$. Similarly in N -dimensional world, the mass equation can be modified to

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2/N}{c^2}}} \approx m_0 \left(1 + \frac{1}{2N} \frac{v^2}{c^2} \right) \tag{12}$$

and its energy can be approximately written in the form

$$E = \lim_{v \rightarrow c} \left(\frac{1}{2N} mv^2 \right) \tag{13}$$

Using Mitten's 11 dimensional M-theory [13], we have $N = 11$, and

$$E = \frac{1}{22} mc^2 \tag{14}$$

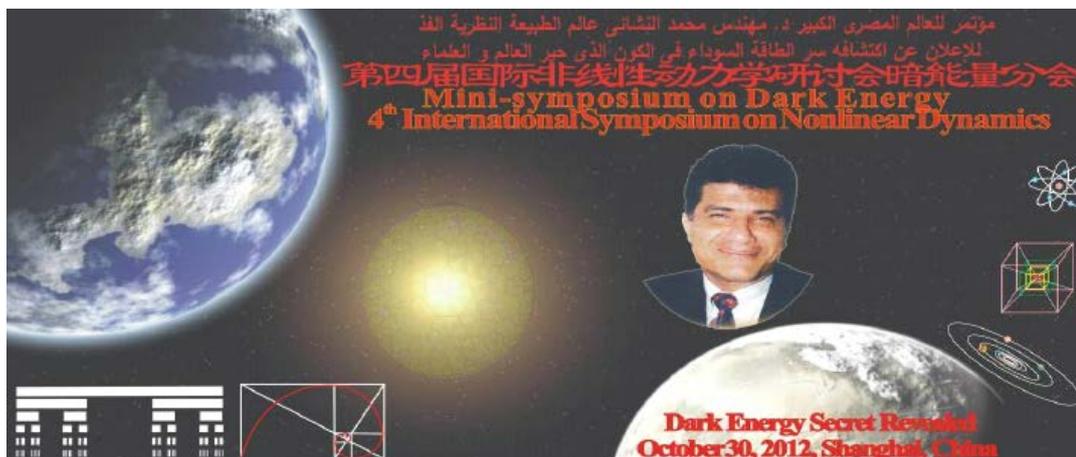


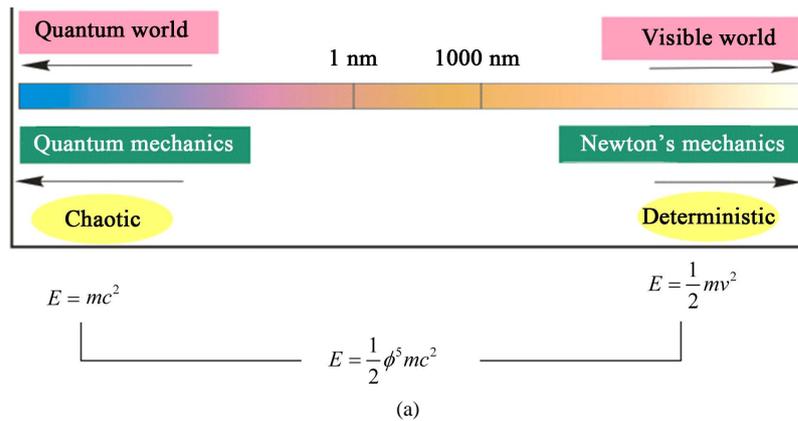
Figure 6. El Naschie's photo appearing in a historical conference poster with Cantor set, the symbolic golden mean and the fractal Hibert cube space-time model. (Kind permission of Prof. El Naschie and Prof. He).

Using fractal M-theory [13] [14], $N = 11 + \phi^5$, we have

$$E = \frac{1}{2(11 + \phi^5)} mc^2 = \frac{\phi^5}{2} mc^2 \tag{15}$$

which is exactly the same as El Naschie's equation, Equation (10). We believe that El Naschie's theory gives a bridge joining our visible world with the invisible quantum world, as illustrated in Figure 7(a). which was proposed for the first time by Prof. Ji-Huan He.

It is fair to say that only a few would place the field of deterministic chaos and fractals as the next mile stone or revolution after quantum mechanics. However, the work of G. Cantor and his transfinite theory are by far the most fundamental mathematics which quantum physics requires and this fact at long last becomes known via the work of the pioneer of nonlinear dynamics, chaos and fractals, notably Lorenz, Ruelle, Feigenbaum, Mandelbrot, Takens, York and El Naschie to mention only a few [25]-[30]. However apart of the immensely important work of L. Hardy [3], the quantum mechanics connection to chaos and fractals was another intensive effort which took much longer to bear fruit due to the genius of people like R. Feynman and the dedicated efforts of numerous scientists, particularly the quantum chaos pioneers Chirikov, Casati, Ford, Gutzwiller and Berry as well as the fractal space-time pioneers Ord and Nottale in addition to El Naschie's E-infinity Cantorian space-time theory [6]. Stripped to the bare core, nonlinear dynamics is about the discovery of Cantor sets for mechanics so we are justified in asking what do fractals bring to physics in general and quantum mechanics in particular? Without going into the discussion of why we will restrict our explanation to a specific fundamental fractal, namely random Cantor set [6] for which, by a well-known theorem, the Hausdorff dimension is the most irrational number $\phi = (\sqrt{5} - 1)/2$ [6] we will look next at one of the most important aspects of quantum mechanics,



(b)

Figure 7. From Newton and Einstein to El Naschie. The transfinite corrections of E-Infinity theory are nothing else but quantum entanglement corrections of the Hardy type. The probability of quantum entanglement is experimentally found to be 9.017%, exactly as El Naschie's and L. Hardy's theoretical prediction which is $\phi^5 = 9.0169945\%$. (Kind permission of Prof. El Naschie and Prof. He).

namely the Heisenberg uncertainty principle [31]. Now this principle precludes the use of one of the most powerful tools of mechanics, namely the phase space method of analysing dynamics and stability of mechanical sets [1] [32]. Needless to say the method proposed by Wiegner to overcome this limitation is not anywhere as widely used as the Hilbert space approach or as the path integral method [32]. That is where Cantor sets come to the rescue. With a Cantor set phase space, *i.e.* a “point-less” phase space with non-standard points, there is no problem to do quantum calculations without violating the uncertainty principles [31] [32]. El Naschie did similar things in the past in real space-time and he integrated Hilbert space into the E-infinity larger picture [33]. From here we can then point in a systematic way to the undreamed of possibilities of chaotic fractals and random Cantor set to tackle quantum physics starting from a comprehensive picture up to an exact solution [34]-[36]. This brings us to the next important point in the present discussion, namely Hardy’s quantum entanglement. Hardy was seeking an exact solution to a basic particles entanglement [3]. Using orthodox quantum mechanics in ket and bra formalism of Dirac he found the quantum probability for entanglement of two quantum particles to be about 9 percent [3]. However what he really did not suspect was what Prof. D. Mermin published a little later showing that this 9 percent was an exact value equal to ϕ to the power of five [37]. That was probably the first exact result linking without a trace of a doubt the random Cantor set with quantum mechanics and the fundamental dimensional function of von Neumann-Connes continuous and noncommutative geometry [38] of which Penrose fractal tiling [1] is a generic space mimicking E-infinity theory [6]. Remembering that Penrose tiling depends crucially upon a golden mean proportionality, we see that the hunch that this golden mean is fundamental is far more than a hunch as, as we will see in the present work, it is a fact discussed on numerous previous occasions [39]-[44]. Maybe it is at this point that we should stress the marvel hidden in a golden mean based number system and E-infinity is indeed embedded in a golden mean binary constituting *de facto* a golden mean computer rivalling even a transfinite version of Turing’s machine without hardware save for a modern pocket calculator on the side to perform the elementary manipulation of the adding, multiplying and dividing the golden mean and its power [42] [45]. Having solved the measure technical computational problem as well as the fundamental contradiction between the discrete and the continuous by building a method which unites both opposed concepts into a transfinite discretum which has the cardinality of the continuum, we realize that we have not only a much better understanding of the vacuum fluctuation but we found a handle on it which can be used to the extent of building mini fractal universes in the laboratory from which we can extract clean energy in the form of a Casimir energy reactor [46]. Before that however we show that ordinary energy is identical to Casimir energy and that the cosmological dark energy is the complimentary energy of the Casimir energy. For an instructive simple and exact picture of quantum space-time, the reader is invited to examine **Figure 2** of Ref. [34]. How this all fits together is the subject of the coming sections.

3. Building Elements of E-Infinity Diagrams

The main two elements or building blocks of E-infinity diagrams are the zero set and the empty set [7]. From these irreducibly simple set theoretical elements we can virtually build an entire space-time and more. For a Cantor set as illustrated in **Figure 3**, the fractal dimensions of the zero set for infinite “points” are $\ln 2 / \ln 3$, while the fractal dimensions of the empty set is $1 - \ln 2 / \ln 3$. For a random Cantor set, we have [11].

a) The zero set $D(H) = \phi$

b) The empty set $D(H) = 1 - \phi = \phi^2$

where $D(H)$ is the Hausdorff dimensions and $\phi = (\sqrt{5} - 1) / 2$. From the above we obtain the latent Casimir space-time set representing the latent topological energy of space-time as the difference of ϕ and ϕ^2 in symbolic diagram (see **Figure 7(a)**) reflecting the essence of the famous Casimir experiment with two uncharged but perfecting conducting plates [47].

From the diagrams of **Figure 8** we can generalize to two limiting cases (see **Figure 8** and **Figure 9**).

- 1) When the distance between the two plates of **Figure 8** tends to real zero, this is then the totally empty set $\phi^{(-\infty)} = \text{zero}$. In the surrounding space we have a non-empty set with the average latent pressure everywhere equal ϕ^3 while the average space-time density is basically the fractal five dimensional average $5 + \phi^3$ instead of 5 dimensional Kaluza-Klein space-time. Consequently the density of the latent Casimir pressure is simply $\phi^3 / (5 + \phi^3)$. This is exactly equal $\phi^5 / 2$, which is our well known ordinary measurable energy of the cosmos and in astounding agreement with cosmic measurements and observation [32]. The immediate rather profound conclusion is that the measured real energy density of the cosmos is nothing more but nothing

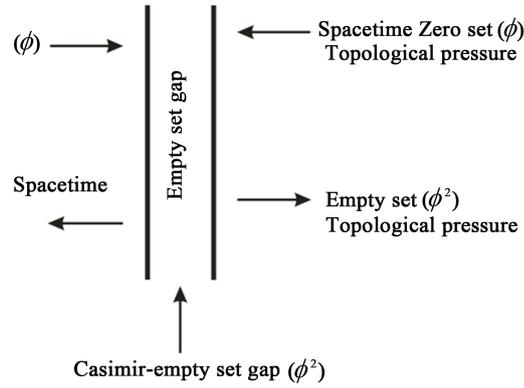


Figure 8. Symbolic representation of the E-infinity Casimir diagram for nano scales. The difference between ϕ and ϕ^2 gives the latent space-time topological pressure ϕ^3 . The inverse of ϕ^3 on the other hand is the average Hausdorff dimension of space-time $4 + \phi^3$.

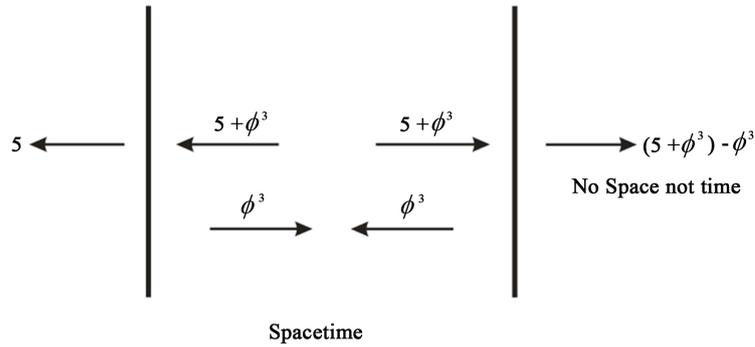


Figure 9. Symbolic representation of the E-infinity Casimir diagram for Hubble scales. The boundary of the holographic boundary of the universe is one sided Möbius-like so that we have no outside. In other words, the outside is the totally empty set. This is the mathematical definition of the philosophical concept of non-existence or nothingness.

less than the Casimir latent energy of space-time and at this scale it is not related in any way to the Riemann curvature of space-time but to the chaotic fractality of space-time in full agreement with our picture which we adopted based on Feynman’s conjecture that gravity is similar to van der Waals fluctuation of micro space-time frequently termed Feynman-El Naschie van der Waals quantum gravity conjecture [6] [39].

- 2) Now, we look at the other extreme where the distance between the Casimir plates is equal to the diameter of our universe as shown schematically in **Figure 2**. The repulsive Casimir-like pressure is in this case equal $5 + \phi^3$ minus the latent Casimir topologic pressure ϕ^3 which is working in the opposite direction so that the net repulsive topological pressure pushing the boundary of the universe outwardly is given by $(5 + \phi^3) - \phi^3 = 5$. Therefore the density of this repulsive topological pressure is $5 / (5 + \phi^3)$. This happens to be equal to $1 - [\phi^3 / (5 + \phi^3)]$ which means it is simply identical to what we calculated for the dark energy of the cosmos $5\phi^2 / 2$. We recall that $5\phi^2 / 2$ was interpreted as the energy of the pre-quantum wave, *i.e.* the energy of the empty set albeit in five dimensional Kaluza-Klein space [48]. The corresponding diagram is shown in **Figure 2**.

4. E-Infinity Hierarchy for Quantum Space-Time

We start from our exact picture of quantum space-time [35]. This picture consists of three concentric circles, see **Figure 10**. When a Cantor set as illustrated in **Figure 3** continue dividing ad infinitum, only “points” for the zero set are obtained, each point has zero dimensions, but each point can still be divided with fractal dimension:

$$D_0 = \phi^{-(0-1)} = \phi \tag{16}$$

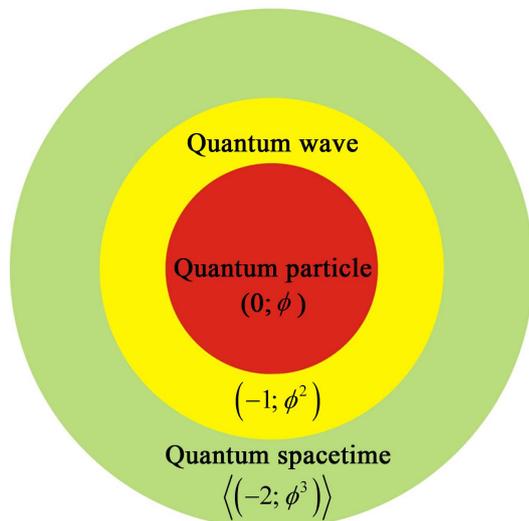


Figure 10. The quantum space-time E-infinity hierarchy [34]-[36]. (Kind permission of Prof. El Naschie).

The first is the zero set of the quantum pre-particle $(0; \phi)$. The boundary of the points in the zero set has a negative dimension, -1 [49]. To show this we consider a square with 2 dimensions, its boundary is a line with 1 dimension. The boundary of a line is two points with zero dimension, so it is easy to be concluded that the boundary of a point has a negative one dimension [49]. By Equation (3), the Hausdorff dimensions for $n = -1$ reads

$$D_{-1} = \phi^{-(-1-1)} = \phi^2 \tag{17}$$

Around this pre-particle we have a topological surface representing the empty set pre-quantum wave $(-1; \phi^2)$. The boundary of negative one the empty set has dimensions of negative two dimensions, and Hausdorff dimensions for $n = -2$ are

$$D_{-2} = \phi^{-(-2-1)} = \phi^3 \tag{18}$$

Finally, we have the third layer which happens to be an expectation value $\langle\langle 2; \phi^3 \rangle\rangle$ representing the average value of an infinite number of empty sets constituting the pre-quantum space-time.

Now, ϕ , ϕ^2 and ϕ^3 are not only Hausdorff dimensions but they can also be understood as a topological frequency or critical parameters of corresponding limit cycles. Therefore we can use the well-known comparison theorem of eigenvalue due to Dunkerley formula, which is a well-known approximation for the fundamental frequency of vibration of certain elastic structures, and figure the combined critical value or joint Hausdorff dimension as follows:

$$\frac{1}{p} = \frac{1}{\phi} + \frac{1}{\phi^2} + \frac{1}{\phi^3} \tag{19}$$

Consequently we have

$$\frac{1}{p} = \frac{\phi + \phi^2 + \phi^3}{\phi^6} = \frac{1 + \phi^3}{\phi^6} = 2 \frac{1}{\phi^5} = (2)(11 + \phi^5) = 22 + k \tag{20}$$

where ϕ^5 is Hardy's quantum entanglement, $11 + \phi^5$ is the dimension of Witten's fractal M-theory and $k = \phi^3(1 - \phi^3)$ [6] [31] [34]-[36]. This is a remarkable result on more than one count. On the first count we found the density of ordinary measurable energy to be obviously equal p and to be at the same time the energy density of the latent Casimir energy of the cosmos [34]-[36]

$$E(O) = \frac{1}{p} = \frac{1}{22 + k} \approx 95.5\% \tag{21}$$

5. The E-Infinity Casimir-Dark Energy Diagram for Topological Interactions in Cantorian Space-Time

Similar to the quasi probability of Wigner’s quantum mechanics in phase space [32], our topological probability approach to quantum mechanics as enshrined in E-infinity [31] [34]-[36] represents an alternative extremely simple fourth formulation for quantum mechanics which is not directly connected to Hilbert space, path integral nor the density matrix approach [32]. Naturally it bears a resemblance to all the afore mentioned formulations but is by no means identical and in fact it adds a few new points to our understanding of the deep mathematical structure of orthodox quantum mechanics hiding behind complex numbers and analytical continuation [1] [6].

The internal logic of our approach rivals that of Weyl-Wigner quantum mechanics in phase space and its logical underpinning by Groenewold and Moyal [32]. For instance as we enter into the negative dimensions regime and find that the Hausdorff dimension ϕ^n is getting increasingly small, it is clear that this means the “Cantor set” is becoming thinner and tending to real nothingness or the absolute empty set [6]. The situation is thus paralleling the density of the density matrix. Similarly when we are summing in E-infinity theory over all dimensions to reach our expectation Hausdorff dimension of the core of E-infinity space-time $\langle d_c^{(n)} \rangle = 4 + \phi^3$ then we are using essentially a similar mathematical concept as that of Feynman path integral but exchanging “paths” for “dimensions” [1]. We could go on describing our approach but could never explain it as good as by treating a fundamental example and nothing could be better and more fundamental than looking at the Casimir effect and dark energy from the perspective of our E-infinity topological probability quantum field theory [6].

6. The Mageuijo-Smolin Quantum Gravity [43]

To obtain $E(O)$ from $E(D)$ and vice versa we have the following transformation [43]

$$\frac{\text{fractal part}}{\text{solid part}} = \frac{\phi^3}{5} \rightarrow \frac{\phi^3}{5} + 1 \rightarrow \frac{1}{\frac{\phi^3}{5} + 1} = E(D) = 95.5\% = \frac{\text{solid part}}{\text{fractal part}} \quad (22)$$

$$\frac{\text{solid part}}{\text{fractal part}} = \frac{5}{\phi^3} \rightarrow \frac{5}{\phi^3} + 1 \rightarrow \frac{1}{\frac{5}{\phi^3} + 1} = E(O) = 4.5\% = \frac{\text{fractal part}}{\text{solid part}} \quad (23)$$

The same applies to a fractal de Sitter universe [1] [50]

$$D = 5 + \phi^3 = (\text{solid part} = 5) + (\text{fractal part} = \phi^3). \quad (24)$$

That means ordinary energy is

$$\frac{\text{fractal part}}{\text{total}} = \frac{\phi^3}{5 + \phi^3} \quad (25)$$

and dark energy is

$$\frac{\text{solid part}}{\text{total}} = \frac{5}{5 + \phi^3} \quad (26)$$

7. The Advantage of being Transfinite Cantorian for Quantum Mechanics

Let us start with Heisenberg’s uncertainty [1] [31]. In E-infinity space-time [6] there is no ordinary points. It is exactly as in von Neumann’s space [6] which he describes in jest as “pointless”. Consequently in Cantorian space-time this fundamental principle is taken care of automatically *ab initio*. In fact writing ϕ^3 as $(\phi)(\phi^2)$ reveals that neither ϕ as a quasi-coordinate nor ϕ^2 as its derivative are fixed numbers which can be written in decimal expansion in a finite way nor the quasi phase space spanned by ϕ and ϕ^2 . The same is true for the product ϕ^3 which is ambiguous in a fundamental way because ϕ could be seen as a distance or a mean velocity while ϕ^2 could be interpreted as acceleration or mean velocity squared. However, for the union of the two sets, the zero set ϕ and the empty set ϕ^2 we have the remarkable collapse in a simple integer [38] [45]

$$(\phi = 0.618033989) + (\phi^2 = 0.3819660) = \text{one} \quad (27)$$

It would be a mistake to think this result is trivial. However, written in symbolic form $\phi = (\sqrt{5} - 1)/2$ and $\phi^2 = 4/(\sqrt{5} + 1)^2$ this is of course trivial computation but not when written in the chaotic decimal expansion. In other words E-infinity transfinite Cantorian quantum mechanics has at its disposal and in a natural way, a transfinite golden computer [42] [45] with a real efficiency which borders on the surreal because simple finite arithmetic operations can handle uncountably infinite numbers and find at the end a neat compact answer [45]. In fact nothing can attest to this coincidentia oppositorum like that first rate orderly character of the continued fraction expansion of ϕ , namely [25]-[30]

$$\phi = \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} \rightarrow 0.61833989 \quad (28)$$

or

$$\phi = 1/\sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}, \quad (29)$$

when compared to its totally chaotic decimal expansion [25]-[30]

$$\phi = 0.618033989 \quad (30)$$

and find on the top of that a well-known fact, namely that ϕ is the Hausdorff dimension of a random triadic Cantor set.

8. The Disadvantage of Being Transfinite Cantorian-Quantum Theory

At the risk of appearing facetious, we would like to seriously propose that in no minor measure a draw back of E-infinity Cantorian space-time theory is that it is excessively simple. A theory should be simple but not excessively so. Being excessively simple puts a theory at risk of being called trivial as an easy shot by members of the “voluntary opposition”. In his early days working in applied mechanics, the author was in awe vis-à-vis the work of a great scholar Prof. Cliff Truesdell who coined the word rational mechanics [51] [52]. The competence of Truesdell is beyond any doubt. However, his mathematical vanity was equally beyond doubt. He was able to smooth anything mathematically to the point that the author felt he was facing a superman of mechanical science. This is what impressed Prof. El Naschie as a naïve young post graduate student until he grew out of it with the help of real engineering scientists like W.T. Köiter, J.M.T. Thompson, J. Croll, S. Nemat Nasser and A.W. Walker to mention only a few [51] [52]. There is science and there is selling science within science politics and the part related to funding is more frequently than not unrelated to science and depends on salesmanship and media more than on logic. The problem with E-infinity is that it fulfills the criteria, which says that maximal accuracy is attained in the ideal limit at maximal simplicity. When simplicity is interpreted wrongly as triviality, then it is tragic. We could find nothing better to close this section with than the wise words of a Socrates of modern science, Prof. J.A. Wheeler [53] in his classic book “Information, Physics, Quantum: The Search for Links” where he wrote “*Surely some day we can believe, we will grasp the central idea of it all as so simple, so beautiful, so compelling that we will say to each other, ‘Oh, how could it have been otherwise!’ . How could we all have been so blind so long!*”.

9. The VAK and Quantum Phase Space from the View Point of E-Infinity

In the following, we show a simple deep connection between a few fundamental aspects of E-infinity and Weyl-Wigner theory [54].

- If we take ϕ to mean a coordinate then ϕ^2 is the quasi derivative and $\phi - \phi^2$ becomes a phase space with $(\phi)(\phi^2) = \phi^3$ being simply a cell of this phase space.
- Seen that way, then $(\phi)(\phi)(\phi)(\phi^2) = \phi^5$ of Hardy may be recognized as the unit cell of 3D Cantorian-quantum phase space.

- c) Since the VAK is a Hamiltonian “strange” attractors conjectured by Rene Thom to represent the equilibrium states of quantum mechanics [3] [55], we see how the VAK, E-infinity and Wigner quantum phase quasi probability are intimately connected. We could proceed further to black hole information [1] and the Hawking and Beekenstein theory [1] but space limitation will not permit that.

10. El Naschie’s Recent Work on the Black Hole Information Paradox [66]-[69]

In very recent work El Naschie used the entire mathematical machinery of E-infinity theory [55]-[73] to re-attack the nagging information paradox of black holes from new view point [64]-[69]. It would be intuitively reasonable to suppose that in the case of black holes with high dimensionality, *i.e.* extra dimensions, the horizon *i.e.* the hyper quasi surface of the black hole horizon will also be of higher dimensionality. Therefore it would seem to follow that the information that resides only on the surface will be, so to speak, diluted because the Bekenstein limit is supposed to remain the same. In other words the net effect is that the information density will decrease or so it would seem initially [64].

In the present work we show that due to the well-known theorem on measure concentration the above conclusion is fallacious [64]. The said theorem due to the legendary Ukrainian-Israeli mathematician and past time President of the famous Weizmann Institute, I. Dvoretzky leads to the definite conclusion that in sufficiently high dimensional spaces such as our quantum space-time, about 96% of the volume resides on the surface or very near to it while a near to only 4% remains in the deceptive bulk. An almost identical result may be obtained using E-infinity theory [55]-[73] with regard to energy where the 96% energy residing on the surface is identified with the supposedly missing so called dark energy. Noting the well know connection between information, entropy and thus thermodynamics and energy we see that our conclusion has an indirect actual cosmic measurement and observational justification, in fact, confirmation. In addition, we note parenthetically that the Beekenstein wonderful result upon which we are basing ourselves still needs an extension to a fractal version by means of which the reduction in the information density will also be excluded. It is thought that in this form the Beekenstein real limit on information will become topological and measure theoretical universality which dispels the black hole information paradox in an unheard of simplicity conserving the most important feature of the theory of Hawking on the one side and ‘tHooft-Susskind on the other without violating any fundamental laws of physics. In the present analysis we will follow two converging roads to show the counterintuitive results of measure concentration due to very high dimensionality. We start first by Dvoretzky’s theorem [64] then we do the same using the wave-particle duality of E-infinity theory.

10.1. Dvoretzky’s Theorem

The moral which we can learn, in fact relearn from this theorem is a well-known wisdom from many counterintuitive results of geometry in higher dimensions, namely that we should in general never generalize an obvious conclusion from a low dimensional space to a higher one. For instance on a flat two dimensional space any two lines will intersect in a point unless they are parallel. However, the spectacular failure of this simple obvious result in three dimensional space is embarrassingly clear. Now let us start with a Euclidean ball [64] [65]

$$B^n = \left\{ x \in R^n : \sum_1^n x_i^2 \leq 1 \right\}. \quad (31)$$

Working in the usual way to find the volume of this n dimensional ball we arrive via gamma function and Stirling formula to [64] [65]

$$V_n \cong \left(\sqrt{\frac{2\pi e}{n}} \right)^n. \quad (32)$$

That means for $V = 1$ the radius is a very large one equal approximately to

$$\sqrt{\frac{n}{2\pi e}}. \quad (33)$$

Now, we proceed to the distribution of the mass, *i.e.* how the “volume” of this ball is distributed. To do that, we estimate first the $(n - 1)$ volume of a slice through the centre of the unit ball. Since the radius of the ball is

$$r = V_n^{(1/n)} \tag{34}$$

then the volume of the slice $(n - 1)$ dimensional ball is given by [64] [65]

$$(V_{n-1})(r^{n-1}) = V_{n-1} \left(\frac{n}{V_n} \right)^{(n-1)/n} \tag{35}$$

Using the Stirling formula again we find that the slice has the volume \sqrt{e} for very large n . The next question is what is the $(n - 1)$ dimensional volume of a parallel slice? The slice at distance x from the centre is an $(n - 1)$ dimensional ball with radius $\sqrt{r^2 - x^2}$ so that the volume of the smaller slice is given approximately by

$$V_{n-1}(\text{smaller slice}) \cong (\sqrt{e}) \left(\frac{1-x^2}{r^2} \right)^{(n-1)/n} \tag{36}$$

Since r is approximately

$$r \cong \sqrt{\frac{n}{2\pi e}} \tag{37}$$

one finds

$$V_{n-1}(\text{smaller slice}) \cong (\sqrt{e}) \exp(-\pi e x^2) \tag{38}$$

That means we obtain “mass” distribution that is almost Gaussian, with variance which surprisingly does not depend upon n :

$$\text{var} \cong \frac{1}{2\pi e} \tag{39}$$

That way we conclude the following remarkable result, namely that almost all the “volume” stays within a flab of fixed width and our result announced in the introduction of the present paper follows that about 96% of the “mass”, *i.e.* the volume lies in the slab [64]

$$\left\{ x \in R^n : -\frac{1}{2} \leq x_i \leq \frac{1}{2} \right\} \tag{40}$$

That means 96% is concentrated near the subspace $(n - 1)$ which may be regarded as the hyper surface of an n dimensional black hole. This is a clear failure of our low dimensional intuition to anticipate what happens in high dimensional cases.

10.2. E-infinity Particle-Wave Duality

In E-infinity theory, the pre-quantum particle as well as the pre-quantum wave follows from the fundamental equation fixing the invariants of the noncommutative E-infinity space-time [38]

$$D(a, b) = a + b\phi \tag{41}$$

where $a, b \in \mathbb{Z}$ and $\phi = (\sqrt{5} - 1)/2$. Setting $a = b = 0$ one finds the absolutely empty set $D = 0$. By contrast for $a = 0$ and $b = 1$ one finds the zero set $E(O) = \phi$ which models the particle while its cobordism, *i.e.* the surface is nothing but the empty set [34]

$$D = 1 - \phi = \phi^2 \tag{42}$$

which models the quantum wave. Transferring this result to Kaluza-Klein “quantum” space-time we note that the “inner” volume must be correlated, *i.e.* intersectional which is appropriate for a volume and leads to [70]

$$V(O) = \phi^5 \tag{43}$$

where $D(\text{Kaluza-Klein}) = 5$. The outer surface, *i.e.* the quantum wave on the other hand is additive and non-correlated so that the union operation is what leads to the volume

$$V(D) = 5\phi^2 \tag{44}$$

A typical volume representative for both would be clearly the arithmetic mean

$$\frac{\phi^5 + 5\phi^2}{2} = 1 \quad (45)$$

In turn looking at the above as energy density we see that [70]

$$V(O) = \phi^5/2 = E(O) \quad (46)$$

for $m = c = 1$ while

$$V(D) = 5\phi^2/2 = E(D) \quad (47)$$

In other words $E(O)$ is our familiar ordinary measureable energy density of the quantum particle

$$E(O) = (\phi^5/2)mc^2 \approx mc^2/22. \quad (48)$$

while $E(D)$ is our dark energy density of the quantum wave which we cannot measure [12]-[17]

$$E(D) = (5\phi^2/2)mc^2 \approx mc^2(21/22) \quad (49)$$

Adding both together we obtain the celebrated result [70]

$$E = E(O) + E(D) = E(\text{Einstein}) = mc^2. \quad (50)$$

Now remembering that energy and information are directly related via entropy, the preceding result is confirmation of what we obtained earlier on using Dvoretzky's theorem, namely that 96% of the information is drawn to the surface higher dimensionality rather than "diluted" by it. Needless to say, the preceding results remain valid for a rotating Kerr black hole [71].

11. Conclusions

We presented in this relatively short trivial review a general theory for quantum space-time and zero point energy fluctuation based on E-infinity and related mathematical concepts. The main results and conclusions may be summarized in the following rather important points:

- 1) El Naschie's E-infinity theory is a pointless self-referential geometrical-topological space-time theory related to von Neumann-Connes' noncommutative geometry of Penrose fractal tiling universe.
- 2) Casimir energy and ordinary energy density of the cosmos are not only identical conceptually but identical numerically.
- 3) Casimir energy and cosmic dark energy are complimentary in the most strict mathematical and physical meaning.
- 4) The difference between Casimir energy and dark energy is a difference of boundary condition where the boundary of the holographic boundary of the universe is a one-sided Möbius-like manifold (see Figure 2 of Ref. [56]).
- 5) Using a heap of space filling fractal nano spheres, we can build in principle a mini universe and use it as an energy reactor (see Figure 1 of Ref. [57]).
- 6) The main conclusion is a natural consequence of mirror symmetry and Witten's T-duality (see Figures 2-4 of Ref. [58]) in addition to Dvoretzky's theorem, which explains why energy is concentrated at the edge of the universe similar to electrical charge of a Faraday cage and information of a black hole.
- 7) The famous Einstein equation $E = mc^2$ is in fact the sum of two quantum relativity parts $E(O) = mc^2/22$ of the quantum particle and $E(D) = mc^2(21/22)$ of the quantum wave. In particular, $E(O)$ is 4.5% of the total E and is the measured ordinary energy density of the cosmos while $E(D)$ is 95.5% of the total E and represents the dark energy and dark matter density of the cosmos [70].
- 8) The formal logic proposed by M.S. El Naschie is the only ingredient missing from Hawking's theory and that of 'tHooft and Susskind [66]-[69]. This is the case because as explained by El Naschie formal logic, *i.e.* self referentiality and Gödel theorem implies self-similar pointless geometry [66]-[69] [74]-[76]. In turn, this solves the puzzling density of information in a black hole and the rest is only connected to measure concentration due to Dvoretzky's theorem [64] [65] [74]-[76].

Acknowledgements

The author is truly grateful to Prof. El Naschie for providing generous guidance and supplying important diagrams and figures to illustrate the present paper. I also thank Prof. Ji-Huan He for many helpful contributions to this paper.

References

- [1] Penrose, R. (2004) *The Road to Reality. A Complete Guide to the Laws of the Universe*. Jonathan Cape, London.
- [2] Rindler, W. (2004) *Relativity (Special, General and Cosmological)*. Oxford, University Press, Oxford.
- [3] Hardy, L. (1993) Nonlocality of Two Particles without Inequalities for Almost All Entangled States. *Physical Review Letters*, **71**, 1665-1668. <http://dx.doi.org/10.1103/PhysRevLett.71.1665>
- [4] Rössler, O.R. (1998) *Endophysics*. World Scientific, Singapore City. <http://dx.doi.org/10.1142/3183>
- [5] El Naschie, M.S. (2001) On a General Theory for Quantum Gravity. In: Diebner, H., Druckrey, T. and Weibel, P., Eds., *Science of the Interface*, Genista Verlag, Tübingen.
- [6] El Naschie, M.S. (2004) A Review of E-Infinity and the Mass Spectrum of High Energy Particle Physics. *Chaos, Solitons & Fractals*, **19**, 209-236. [http://dx.doi.org/10.1016/S0960-0779\(03\)00278-9](http://dx.doi.org/10.1016/S0960-0779(03)00278-9)
- [7] He, J.-H. (2014) A Tutorial Review on Fractal Space-Time and Fractional Calculus. *International Journal of Theoretical Physics*, **53**, 3698-3718. <http://dx.doi.org/10.1007/s10773-014-2123-8>
- [8] He, J.H. (2009) Hilbert Cube Model for Fractal Space-Time. *Chaos Solitons and Fractals*, **4**, 2754-2759. <http://dx.doi.org/10.1016/j.chaos.2009.03.182>
- [9] El Naschie, M.S. (2002) Quantum Loops, Wild Topology and Fat Cantor Sets in Transfinite High-Energy Physics. *Chaos, Solitons & Fractals*, **13**, 1167-1174. [http://dx.doi.org/10.1016/S0960-0779\(01\)00210-7](http://dx.doi.org/10.1016/S0960-0779(01)00210-7)
- [10] El Naschie, M.S. (2003) The VAK of Vacuum Fluctuation: Spontaneous Self-Organization and Complexity Theory Interpretation of High Energy Particle Physics and the Mass Spectrum. *Chaos, Solitons and Fractals*, **18**, 401-420. [http://dx.doi.org/10.1016/S0960-0779\(03\)00098-5](http://dx.doi.org/10.1016/S0960-0779(03)00098-5)
- [11] El Naschie, M.S. (2013) A Unified Newtonian-Relativistic Quantum Resolution of the Supposedly Missing Dark Energy of the Cosmos and the Constancy of the Speed of Light. *International Journal of Modern Nonlinear Theory and Application*, **2**, 43-54. <http://dx.doi.org/10.4236/ijmnta.2013.21005>
- [12] El Naschie, M.S. (2013) Dark Energy from Kaluza-Klein Space-Time and Noether's Theorem via Lagrangian Multiplier Method. *Journal of Modern Physics*, **4**, 757-760. <http://dx.doi.org/10.4236/jmp.2013.46103>
- [13] El Naschie, M.S. (2005) A Few Hints and Some Theorem about Witten's M Theory and T-Duality. *Chaos, Solitons & Fractals*, **25**, 545-548. <http://dx.doi.org/10.1016/j.chaos.2005.01.009>
- [14] He, J.-H. (2007) The Number of Elementary Particles in a Fractal M-Theory of 11.2360667977 Dimensions. *Chaos, Solitons & Fractals*, **32**, 346-351. <http://dx.doi.org/10.1016/j.chaos.2006.06.005>
- [15] El Naschie, M.S. (2014) Cosmic Dark Energy Density from Classical Mechanics and Seemingly Redundant Riemannian Finitely Many Tensor Components of Einstein's General Relativity. *World Journal of Mechanics*, **4**, 153-156. <http://dx.doi.org/10.4236/wjm.2014.46017>
- [16] El Naschie, M.S. (2014) Capillary Surface Energy Elucidation of the Cosmic Dark Energy-Ordinary Energy Duality. *Open Journal of Fluid Dynamics*, **4**, 15-17. <http://dx.doi.org/10.4236/ojfd.2014.41002>
- [17] El Naschie, M.S. (2014) Einstein's General Relativity and Pure Gravity in a Cosserat and De Sitter-Witten Spacetime Setting as the Explanation of Dark Energy and Cosmic Accelerated Expansion. *International Journal of Astronomy and Astrophysics*, **4**, 332-339. <http://dx.doi.org/10.4236/ijaa.2014.42027>
- [18] El Naschie, M.S. (2014) Electromagnetic-Pure Gravity Connection via Hardy's Quantum Entanglement. *Journal of Electromagnetic Analysis and Applications*, **6**, 233-237. <http://dx.doi.org/10.4236/jemaa.2014.69023>
- [19] El Naschie, M.S. (2014) Cosmic Dark Energy from 't Hooft's Dimensional Regularization and Witten's Topological Quantum Field Pure Gravity. *Journal of Quantum Information Science*, **4**, 83-91. <http://dx.doi.org/10.4236/jqis.2014.42008>
- [20] El Naschie, M.S. (2014) Entanglement of E8E8 Exceptional Lie Symmetry Group Dark Energy, Einstein's Maximal Total Energy and the Hartle-Hawking No Boundary Proposal as the Explanation for Dark Energy. *World Journal of Condensed Matter Physics*, **4**, 74-77. <http://dx.doi.org/10.4236/wjcmp.2014.42011>
- [21] El Naschie, M.S. (2014) The Meta Energy of Dark Energy. *Open Journal of Philosophy*, **4**, 157-159. <http://dx.doi.org/10.4236/ojpp.2014.42022>
- [22] El Naschie, M.S. (2014) Pinched Material Einstein Space-Time Produces Accelerated Cosmic Expansion. *Internation-*

- al Journal of Astronomy and Astrophysics*, **4**, 80-90. <http://dx.doi.org/10.4236/ijaa.2014.41009>
- [23] El Naschie, M.S. (2014) From Chern-Simon, Holography and Scale Relativity to Dark Energy. *Journal of Applied Mathematics and Physics*, **2**, 634-638. <http://dx.doi.org/10.4236/jamp.2014.27069>
- [24] El Naschie, M.S. (2014) Why E Is Not Equal to mc^2 . *Journal of Modern Physics*, **5**, 743-750. <http://dx.doi.org/10.4236/jmp.2014.59084>
- [25] Addison, P.S. (1997) *Fractals and Chaos: An Illustrated Course*. Institute of Physics, Bristol. <http://dx.doi.org/10.1887/0750304006>
- [26] Ruelle, D. (1991) *Chance and Chaos*. Princeton University Press, Princeton.
- [27] Graham, L. and Kantor, J.-M. (2009) *Naming Infinity*. The Belknap Press of Harvard University Press, Cambridge, Massachusetts-London.
- [28] Ott, E. (1993) *Chaos in Dynamical Systems*. Cambridge University Press, Cambridge, UK.
- [29] Kapitaniak, T. (Ed.) (1992) *Chaotic Oscillators (Theory and Applications)*. World Scientific, Singapore. <http://dx.doi.org/10.1142/9789814360258>
- [30] Casati, G., Guarneri, I. and Smilansky, U. (1993) *Quantum Chaos*. North Holland, Amsterdam.
- [31] El Naschie, M.S. (1995) Young Double-Slit Experiment, Heisenberg Uncertainty Principle and Correlation in Cantorian Space-Time. In: El Naschie, M.S., Rössler, O.E. and Prigogine, I., Eds., *Quantum Mechanics, Diffusion and Chaotic Fractals*, Pergamon Press/Elsevier, Oxford, 93-100.
- [32] Kim, Y.S. and Noz, M.E. (1991) *Phase Space Picture of Quantum Mechanics*, World Scientific, Singapore. <http://dx.doi.org/10.1142/1197>
- [33] El Naschie, M.S. (2007) Hilbert Space, Poincaré Dodecahedron and Golden Mean Transfiniteness. *Chaos, Solitons & Fractals*, **31**, 787-793. <http://dx.doi.org/10.1016/j.chaos.2006.06.003>
- [34] El Naschie, M.S. (2015) Hubble Scale Dark Energy Meets Nano Scale Casimir Energy and the Rational of Their T-Duality and Mirror Symmetry Equivalence. *World Journal of Nano Science and Engineering*, **5**, 57-67. <http://dx.doi.org/10.4236/wjnse.2015.53008>
- [35] El Naschie, M.S. (2015) An Exact Mathematical Picture of Quantum Space-Time. *Advances in Pure Mathematics*, **5**, 560-570. <http://dx.doi.org/10.4236/apm.2015.59052>
- [36] El Naschie, M.S. (2015) From Fusion Algebra to Cold Fusion or from Pure Reason to Pragmatism. *Open Journal of Philosophy*, **5**, 319-326. <http://dx.doi.org/10.4236/ojpp.2015.56040>
- [37] Mermin, D. (1994) Quantum Mysteries Refined. *American Journal of Physics*, **62**, 880-887. <http://dx.doi.org/10.1119/1.17733>
- [38] Connes, A. (1994) *Noncommutative Geometry*. Academic Press, San Diego.
- [39] Marek-Crnjac, L. and El Naschie, M.S. (2013) Chaotic Fractal Tiling for the Missing Dark Energy and Veneziano Model. *Applied Mathematics*, **4**, 22-29. <http://dx.doi.org/10.4236/am.2013.411A2005>
- [40] Marek-Crnjac, L. and El Naschie, M.S. (2013) Quantum Gravity and Dark Energy Using Fractal Planck Scaling. *Journal of Modern Physics*, **4**, 31-38. <http://dx.doi.org/10.4236/jmp.2013.411A1005>
- [41] Auffray, J.-P. (2015) E-Infinity, the Zero Set, Absolute Space and Photon Spin. *Journal of Modern Physics*, **6**, 536-545. <http://dx.doi.org/10.4236/jmp.2015.65058>
- [42] El Naschie, M.S., Olsen, S., He, J.-H., Nada, S., Marek-Crnjac, L. and Helal, A. (2012) On the Need for Fractal Logic in High Energy Quantum Physics. *International Journal of Modern Nonlinear Theory and Application*, **1**, 84-92. <http://dx.doi.org/10.4236/ijmnta.2012.13012>
- [43] El Naschie, M.S., Marek-Crnjac, L., Helal, A. and He, J.-H. (2014) A Topological Magueijo-Smolín Varying Speed of Light Theory, the Accelerated Cosmic Expansion and the Dark Energy of Pure Gravity. *Applied Mathematics*, **5**, 1780-1790. <http://dx.doi.org/10.4236/am.2014.512171>
- [44] Marek-Crnjac, L. (2013) Modification of Einstein's $E = mc^2$ to $E = (1/22)mc^2$. *American Journal of Modern Physics*, **2**, 255-263. <http://dx.doi.org/10.11648/j.ajmp.20130205.14>
- [45] El Naschie, M.S., He, J.-H., Nada, S., Marek-Crnjac, L. and Helal, A. (2012) Golden Mean Computer for High Energy Physics. *Fractal Space-Time and Noncommutative Geometry in Quantum and High Energy Physics*, **2**, 80-92.
- [46] He, J.-H. and El Naschie, M.S. (2012) On the Monadic Nature of Quantum Gravity as a Highly Structured Golden Ring, Spaces and Spectra. *Fractal Space-Time and Noncommutative Geometry in Quantum and High Energy Physics*, **2**, 94-98.
- [47] El Naschie, M.S. (2014) Casimir-Like Energy as a Double Eigenvalue of Quantumly Entangled System Leading to the Missing Dark Energy Density of the Cosmos. *International Journal of High Energy Physics*, **1**, 55-63.

- <http://dx.doi.org/10.11648/j.ijhep.20140105.11>
- [48] El Naschie, M.S. (2015) The Casimir Topological Effect and a Proposal for a Casimir-dark Energy Nano Reactor. *World Journal of Nano Science and Engineering*, **5**, 26-33. <http://dx.doi.org/10.4236/wjnse.2015.51004>
- [49] He, J.-H. (2010) A Note on Elementary Cobordism and Negative Space. *International Journal of Nonlinear Sciences and Numerical Simulation*, **11**, 1093-1095. <http://dx.doi.org/10.1515/IJNSNS.2010.11.12.1093>
- [50] El Naschie, M.S. (2013) Topological-Geometrical and Physical Interpretation of the Dark Energy of the Cosmos as a “Halo” Energy of the Schrodinger Quantum Wave. *Journal of Modern Physics*, **4**, 591-596. <http://dx.doi.org/10.4236/jmp.2013.45084>
- [51] El Naschie, M.S. (1991) *Stress, Stability and Chaos in Structural Engineering: An Energy Approach*. McGraw-Hill, London.
- [52] Thompson, J.M.T. and Hunt, G. (1973) *A General Theory of Elastic Stability*. Wiley, London.
- [53] Wheeler, J.A. (1990) Information, Physics, Quantum: The Search for Links. In: Zurek, W., Ed., *Complexity, Entropy, and the Physics of Information*, Addison-Wesley, Redwood City, 309-336.
- [54] Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R.A., Nugent, P., Castro, P.G., Deustua, S., Fabbro, S., Goobar, A. and Groom, D.E. (1999) Measurements of Omega and Lambda from 42 High Redshift Supernova. *The Astrophysics Journal*, **517**, 565-585. <http://dx.doi.org/10.1086/307221>
- [55] El Naschie, M.S. (2011) Quantum Entanglement as a Consequence of a Cantorian Micro Spacetime Geometry. *Journal of Quantum Information Science*, **1**, 50-53. <http://dx.doi.org/10.4236/jqis.2011.12007>
- [56] El Naschie, M.S. (2015) A Casimir Dark Energy Nano Reactor Design—Phase One. *Natural Science*, **7**, 287-298. <http://dx.doi.org/10.4236/ns.2015.76032>
- [57] El Naschie, M.S. (2015) Casimir-Dark Energy Nano Reactor Design Proposal Based on Fractals. *International Journal of Innovation in Science and Mathematics*, **3**, 187-194.
- [58] Marek-Crnjac, L., El Naschie, M.S. and He, J.-H. (2013) Chaotic Fractals at the Root of Relativistic Quantum Physics and Cosmology. *International Journal of Modern Nonlinear Theory and Application*, **2**, 78-88. <http://dx.doi.org/10.4236/ijmnta.2013.21A010>
- [59] Auffray, J.-P. (2013) A Xon Signature in the Electron Spin. *Journal of Modern Physics*, **4**, 9-13. <http://dx.doi.org/10.4236/jmp.2013.411A1002>
- [60] Auffray, J.-P. (2014) Biology-Physics the Missing Link? *Journal of Modern Physics*, **5**, 359-363. <http://dx.doi.org/10.4236/jmp.2014.56046>
- [61] Auffray, J.-P. (2013) Is Dark Matter Made up of Xons? *Journal of Modern Physics*, **4**, 121-125. <http://dx.doi.org/10.4236/jmp.2013.48A011>
- [62] Auffray, J.-P. (2014) E-Infinity Dualities, Discontinuous Space-Times, Xonic Quantum Physics and the Decisive Experiment. *Journal of Modern Physics*, **5**, 1427-1436. <http://dx.doi.org/10.4236/jmp.2014.515144>
- [63] Auffray, J.-P. (2015) On an Intriguing Invention Albert Einstein Made Which Has Gone Unnoticed Hitherto. *Journal of Modern Physics*, **6**, 1478-1491. <http://dx.doi.org/10.4236/jmp.2015.611152>
- [64] El Naschie, M.S. (2015) The Counterintuitive Increase of Information Due to Extra Space-Time Dimensions of a Black Hole and Dvoretzky’s Theorem. *Natural Science*, **7**, 483-487. <http://dx.doi.org/10.4236/ns.2015.710049>
- [65] El Naschie, M.S. (2015) Application of Dvoretzky’s Theorem of Measure Concentration in Physics and Cosmology. *Open Journal of Microphysics*, **5**, 11-15. <http://dx.doi.org/10.4236/ojm.2015.52002>
- [66] El Naschie, M.S. (2015) The Self-Referential Pointless Universe Geometry as the Key to the Resolution of the Black Hole Information Paradox. *International Journal of Innovation in Science and Mathematics*, **3**, 254-265.
- [67] El Naschie, M.S. (2015) A Resolution of the Black Hole Information Paradox via Transfinite Set Theory. *World Journal of Condensed Matter Physics*, **5**, 249-260. <http://dx.doi.org/10.4236/wjcmp.2015.54026>
- [68] El Naschie, M.S. (2015) If Quantum “Wave” of the Universe Then Quantum “Particle” of the Universe: A Resolution of the Dark Energy Question and the Black Hole Information Paradox. *International Journal of Astronomy & Astrophysics*, **5**, 243-247. <http://dx.doi.org/10.4236/ijaa.2015.54027>
- [69] El Naschie, M.S. (2015) A Complementarity Resolution of the Black Hole Information Paradox. *American Journal of astronomy and Astrophysics*, **3**, 77-86. <http://dx.doi.org/10.11648/j.ajaa.20150305.11>
- [70] El Naschie, M.S. (2014) From $E = mc^2$ to $E = mc^2/22$ —A Short Account of the Most Famous Equation in Physics and Its Hidden Quantum Entangled Origin. *Journal of Quantum Information Science*, **4**, 284-291. <http://dx.doi.org/10.4236/jqis.2014.44023>
- [71] El Naschie, M.S. (2015) Kerr Black Hole Geometry Leading to Dark Matter and Dark Energy via E-Infinity Theory and the Possibility of Nano Space-Time Singularity Reactor. *Natural Science*, **7**, 210-225.

<http://dx.doi.org/10.4236/ns.2015.74024>

- [72] El Naschie, M.S. (2016) Cosserat-Cartan and de Sitter-Witten Space-Time Setting for Dark Energy. *Quantum Matter*, **5**, 1-4. <http://dx.doi.org/10.1166/qm.2016.1247>
- [73] Marek-Crnjac, L. (2015) On Quantum Entanglement in Connection with Dark Energy from Historical to Modern Theories. In: Reimer, A., Ed., *Horizons in World Physics*, Chap. 4, Nova Science Publishers, New York, 115-126.
- [74] El Naschie, M.S. (2006) Fractal Black Holes and Information. *Chaos, Solitons & Fractals*, **29**, 23-35. <http://dx.doi.org/10.1016/j.chaos.2005.11.079>
- [75] Kauffman, L. (1987) Self-Reference and Recursive Forms. *Journal of Social and Biological Structure*, **10**, 53-72. [http://dx.doi.org/10.1016/0140-1750\(87\)90034-0](http://dx.doi.org/10.1016/0140-1750(87)90034-0)
- [76] Kauffman, L. (2013) *Knots and Physics*. World Scientific, Singapore. <http://dx.doi.org/10.1142/8338>