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Natural Science

Relationship of nine constants

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ABSTRACT

Through the process of trial and error, four unitless equations made up of nine constants have been found with exact answers. The related constants are the Speed of Light [1], the Planck constant [2], Wien's displacement constant [3], Avogadro's number [4], the universal Gravity constant [5], the Ampere constant [6], the Faraday constant [7], the Gas constant [8] and Apery's constant [9].

Keywords: Planck; Wien; Avogadro; Faraday; Apery; Ampere

1. INTRODUCTION

At the end of the spring semester 2013, I had found an expression of a few physical constants that gave the correct value of the universal Gravity constant [5]. I shared my findings with my classmates and they all pointed out the units were incorrect.

This started the search for a unitless expression of physical constants similar in form to the Fine Structure constant but with more constants.

In the context of this paper, the term unitless is defined as all the exponents of the units on the left hand side of the equation are equal to zero and the right hand side of the equation is represented by only a numeric expression.

2. MAIN BODY

The first few equations were found by trial and error. One would literally examine a listing of physical constants and guess which set of constants multiplied together and divided by another set of multiplied constants produces an answer with units raised to the zero power.

I had the limited success of finding the Fine Structure constant over and over again. At this point I changed my strategy by writing a program that would try every combination of a set of constants within a certain integer range of exponents, with its dimensionality equal to a selected SI unit. This strategy worked in the sense that it produced a large set of equations, of the selected constants that had the required SI units of seconds or meters, etc.

The programming process and the testing the programs happened over a few weeks and various sets of physical constants where tried. Overall the physical constants that produced the most equations were selected to be in the final set of nine presented in this paper.

A few things happened concurrently that allowed me to find the equations presented in this paper. One was that I started using a unit of an ampere-mole as a range extender in my search programs. The derived unit could be removed from the final answers yet its presence in the program allowed more equations to be found.

The second was that it occurred to me that the structure of the programs that I had written; could search for unitless equations too. The third was that I added the Faraday constant to the primary set of search constants. I intended to use the Faraday constant as a more robust replacement for the derived ampere-mole constant, and was hoping for similar results.

A few minutes later, the first of many unitless equations appeared on the screen. Through the process of trial and error I had found a set of eight physical constants that produced unitless equations.

Once a pattern was found in the first few equations, a new program in the Cuda GPU language was written to find unitless combinations expressed as the powers of the constants. A program listing is included for completeness as Appendix I.

A set of 200 unitless equations are shown in **Table 1**, and **Eq.1** through **Eq.4** are the results of the reduced row echelon form of **Table 1**. The reduced row echelon operation on **Table 1**, results with two rows.

Eq.1 represents the first row and **Eq.2** represents the second row of the reduced row echelon form. **Eq.3** represents the multiplication of the **Eq.1** and **Eq.2** and **Eq.4** represents the quotient of **Eq.1** and **Eq. 2**. One can

Table 1. A family of unitless equations.

Index Number	Ln (c^0	h^0	A^0	R^0	w^0	G^0	N^0	F^0) =	value
1	0	0	0	0	0	0	0	0	0
2	−64	−70	−2	69	69	−1	−71	2	0.29664
3	−128	−140	−4	138	138	−2	−142	4	0.59328
4	193	211	6	−208	−208	3	214	−6	0.71252
5	−192	−210	−6	207	207	−3	−213	6	0.88992
6	129	141	4	−139	−139	2	143	−4	1.0092
7	65	71	2	−70	−70	1	72	−2	1.3058
8	1	1	0	−1	−1	0	1	0	1.6024
9	−63	−69	−2	68	68	−1	−70	2	1.8991
10	−127	−139	−4	137	137	−2	−141	4	2.1957
11	194	212	6	−209	−209	3	215	−6	2.315
12	−191	−209	−6	206	206	−3	−212	6	2.4924
13	130	142	4	−140	−140	2	144	−4	2.6116
14	66	72	2	−71	−71	1	73	−2	2.9082
15	2	2	0	−2	−2	0	2	0	3.2049
16	−62	−68	−2	67	67	−1	−69	2	3.5015
17	−126	−138	−4	136	136	−2	−140	4	3.7982
18	195	213	6	−210	−210	3	216	−6	3.9174
19	−190	−208	−6	205	205	−3	−211	6	4.0948
20	131	143	4	−141	−141	2	145	−4	4.214
21	67	73	2	−72	−72	1	74	−2	4.5107
22	3	3	0	−3	−3	0	3	0	4.8073
23	−61	−67	−2	66	66	−1	−68	2	5.1039
24	−125	−137	−4	135	135	−2	−139	4	5.4006
25	196	214	6	−211	−211	3	217	−6	5.5198
26	−189	−207	−6	204	204	−3	−210	6	5.6972
27	132	144	4	−142	−142	2	146	−4	5.8165
28	68	74	2	−73	−73	1	75	−2	6.1131
29	4	4	0	−4	−4	0	4	0	6.4097
30	−60	−66	−2	65	65	−1	−67	2	6.7064
31	−124	−136	−4	134	134	−2	−138	4	7.003
32	197	215	6	−212	−212	3	218	−6	7.1223
33	−188	−206	−6	203	203	−3	−209	6	7.2997
34	133	145	4	−143	−143	2	147	−4	7.4189
35	69	75	2	−74	−74	1	76	−2	7.7155
36	5	5	0	−5	−5	0	5	0	8.0122
37	−59	−65	−2	64	64	−1	−66	2	8.3088
38	−123	−135	−4	133	133	−2	−137	4	8.6055
39	198	216	6	−213	−213	3	219	−6	8.7247
40	−187	−205	−6	202	202	−3	−208	6	8.9021
41	134	146	4	−144	−144	2	148	−4	9.0213
42	70	76	2	−75	−75	1	77	−2	9.318
43	6	6	0	−6	−6	0	6	0	9.6146
44	−58	−64	−2	63	63	−1	−65	2	9.9113
45	−122	−134	−4	132	132	−2	−136	4	10.208
46	199	217	6	−214	−214	3	220	−6	10.327
47	−186	−204	−6	201	201	−3	−207	6	10.505
48	135	147	4	−145	−145	2	149	−4	10.624

Continued

49	71	77	2	−76	−76	1	78	−2	10.92
50	7	7	0	−7	−7	0	7	0	11.217
51	−57	−63	−2	62	62	−1	−64	2	11.514
52	−121	−133	−4	131	131	−2	−135	4	11.81
53	200	218	6	−215	−215	3	221	−6	11.93
54	−185	−203	−6	200	200	−3	−206	6	12.107
55	136	148	4	−146	−146	2	150	−4	12.226
56	72	78	2	−77	−77	1	79	−2	12.523
57	8	8	0	−8	−8	0	8	0	12.819
58	−56	−62	−2	61	61	−1	−63	2	13.116
59	−120	−132	−4	130	130	−2	−134	4	13.413
60	201	219	6	−216	−216	3	222	−6	13.532
61	−184	−202	−6	199	199	−3	−205	6	13.709
62	137	149	4	−147	−147	2	151	−4	13.829
63	73	79	2	−78	−78	1	80	−2	14.125
64	9	9	0	−9	−9	0	9	0	14.422
65	−55	−61	−2	60	60	−1	−62	2	14.719
66	−119	−131	−4	129	129	−2	−133	4	15.015
67	202	220	6	−217	−217	3	223	−6	15.134
68	−183	−201	−6	198	198	−3	−204	6	15.312
69	138	150	4	−148	−148	2	152	−4	15.431
70	74	80	2	−79	−79	1	81	−2	15.728
71	10	10	0	−10	−10	0	10	0	16.024
72	−54	−60	−2	59	59	−1	−61	2	16.321
73	−118	−130	−4	128	128	−2	−132	4	16.618
74	203	221	6	−218	−218	3	224	−6	16.737
75	−182	−200	−6	197	197	−3	−203	6	16.914
76	139	151	4	−149	−149	2	153	−4	17.034
77	75	81	2	−80	−80	1	82	−2	17.33
78	11	11	0	−11	−11	0	11	0	17.627
79	−53	−59	−2	58	58	−1	−60	2	17.923
80	−117	−129	−4	127	127	−2	−131	4	18.22
81	204	222	6	−219	−219	3	225	−6	18.339
82	−181	−199	−6	196	196	−3	−202	6	18.517
83	140	152	4	−150	−150	2	154	−4	18.636
84	76	82	2	−81	−81	1	83	−2	18.933
85	12	12	0	−12	−12	0	12	0	19.229
86	−52	−58	−2	57	57	−1	−59	2	19.526
87	−116	−128	−4	126	126	−2	−130	4	19.823
88	205	223	6	−220	−220	3	226	−6	19.942
89	−180	−198	−6	195	195	−3	−201	6	20.119
90	141	153	4	−151	−151	2	155	−4	20.238
91	77	83	2	−82	−82	1	84	−2	20.535
92	13	13	0	−13	−13	0	13	0	20.832
93	−51	−57	−2	56	56	−1	−58	2	21.128
94	−115	−127	−4	125	125	−2	−129	4	21.425
95	206	224	6	−221	−221	3	227	−6	21.544
96	−179	−197	−6	194	194	−3	−200	6	21.722
97	142	154	4	−152	−152	2	156	−4	21.841
98	78	84	2	−83	−83	1	85	−2	22.137

Continued

99	14	14	0	−14	−14	0	14	0	22.434
100	−50	−56	−2	55	55	−1	−57	2	22.731
101	−114	−126	−4	124	124	−2	−128	4	23.027
102	207	225	6	−222	−222	3	228	−6	23.147
103	−178	−196	−6	193	193	−3	−199	6	23.324
104	143	155	4	−153	−153	2	157	−4	23.443
105	79	85	2	−84	−84	1	86	−2	23.74
106	15	15	0	−15	−15	0	15	0	24.037
107	−49	−55	−2	54	54	−1	−56	2	24.333
108	−113	−125	−4	123	123	−2	−127	4	24.63
109	208	226	6	−223	−223	3	229	−6	24.749
110	−177	−195	−6	192	192	−3	−198	6	24.926
111	144	156	4	−154	−154	2	158	−4	25.046
112	80	86	2	−85	−85	1	87	−2	25.342
113	16	16	0	−16	−16	0	16	0	25.639
114	−48	−54	−2	53	53	−1	−55	2	25.936
115	−112	−124	−4	122	122	−2	−126	4	26.232
116	209	227	6	−224	−224	3	230	−6	26.351
117	−176	−194	−6	191	191	−3	−197	6	26.529
118	145	157	4	−155	−155	2	159	−4	26.648
119	81	87	2	−86	−86	1	88	−2	26.945
120	17	17	0	−17	−17	0	17	0	27.241
121	−47	−53	−2	52	52	−1	−54	2	27.538
122	−111	−123	−4	121	121	−2	−125	4	27.835
123	210	228	6	−225	−225	3	231	−6	27.954
124	−175	−193	−6	190	190	−3	−196	6	28.131
125	146	158	4	−156	−156	2	160	−4	28.251
126	82	88	2	−87	−87	1	89	−2	28.547
127	18	18	0	−18	−18	0	18	0	28.844
128	−46	−52	−2	51	51	−1	−53	2	29.14
129	−110	−122	−4	120	120	−2	−124	4	29.437
130	211	229	6	−226	−226	3	232	−6	29.556
131	−174	−192	−6	189	189	−3	−195	6	29.734
132	147	159	4	−157	−157	2	161	−4	29.853
133	83	89	2	−88	−88	1	90	−2	30.15
134	19	19	0	−19	−19	0	19	0	30.446
135	−45	−51	−2	50	50	−1	−52	2	30.743
136	−109	−121	−4	119	119	−2	−123	4	31.04
137	212	230	6	−227	−227	3	233	−6	31.159
138	−173	−191	−6	188	188	−3	−194	6	31.336
139	148	160	4	−158	−158	2	162	−4	31.455
140	84	90	2	−89	−89	1	91	−2	31.752
141	20	20	0	−20	−20	0	20	0	32.049
142	−44	−50	−2	49	49	−1	−51	2	32.345
143	−108	−120	−4	118	118	−2	−122	4	32.642
144	213	231	6	−228	−228	3	234	−6	32.761
145	−172	−190	−6	187	187	−3	−193	6	32.939
146	149	161	4	−159	−159	2	163	−4	33.058
147	85	91	2	−90	−90	1	92	−2	33.355
148	21	21	0	−21	−21	0	21	0	33.651
149	−43	−49	−2	48	48	−1	−50	2	33.948

Continued

150	−107	−119	−4	117	117	−2	−121	4	34.244
151	214	232	6	−229	−229	3	235	−6	34.364
152	−171	−189	−6	186	186	−3	−192	6	34.541
153	150	162	4	−160	−160	2	164	−4	34.66
154	86	92	2	−91	−91	1	93	−2	34.957
155	22	22	0	−22	−22	0	22	0	35.254
156	−42	−48	−2	47	47	−1	−49	2	35.55
157	−106	−118	−4	116	116	−2	−120	4	35.847
158	215	233	6	−230	−230	3	236	−6	35.966
159	−170	−188	−6	185	185	−3	−191	6	36.144
160	151	163	4	−161	−161	2	165	−4	36.263
161	87	93	2	−92	−92	1	94	−2	36.559
162	23	23	0	−23	−23	0	23	0	36.856
163	−41	−47	−2	46	46	−1	−48	2	37.153
164	−105	−117	−4	115	115	−2	−119	4	37.449
165	216	234	6	−231	−231	3	237	−6	37.569
166	−169	−187	−6	184	184	−3	−190	6	37.746
167	152	164	4	−162	−162	2	166	−4	37.865
168	88	94	2	−93	−93	1	95	−2	38.162
169	24	24	0	−24	−24	0	24	0	38.458
170	−40	−46	−2	45	45	−1	−47	2	38.755
171	−104	−116	−4	114	114	−2	−118	4	39.052
172	217	235	6	−232	−232	3	238	−6	39.171
173	−168	−186	−6	183	183	−3	−189	6	39.348
174	153	165	4	−163	−163	2	167	−4	39.468
175	89	95	2	−94	−94	1	96	−2	39.764
176	25	25	0	−25	−25	0	25	0	40.061
177	−39	−45	−2	44	44	−1	−46	2	40.358
178	−103	−115	−4	113	113	−2	−117	4	40.654
179	218	236	6	−233	−233	3	239	−6	40.773
180	−167	−185	−6	182	182	−3	−188	6	40.951
181	154	166	4	−164	−164	2	168	−4	41.07
182	90	96	2	−95	−95	1	97	−2	41.367
183	26	26	0	−26	−26	0	26	0	41.663
184	−38	−44	−2	43	43	−1	−45	2	41.96
185	−102	−114	−4	112	112	−2	−116	4	42.257
186	219	237	6	−234	−234	3	240	−6	42.376
187	−166	−184	−6	181	181	−3	−187	6	42.553
188	155	167	4	−165	−165	2	169	−4	42.672
189	91	97	2	−96	−96	1	98	−2	42.969
190	27	27	0	−27	−27	0	27	0	43.266
191	−37	−43	−2	42	42	−1	−44	2	43.562
192	−101	−113	−4	111	111	−2	−115	4	43.859
193	220	238	6	−235	−235	3	241	−6	43.978
194	−165	−183	−6	180	180	−3	−186	6	44.156
195	156	168	4	−166	−166	2	170	−4	44.275
196	92	98	2	−97	−97	1	99	−2	44.572
197	28	28	0	−28	−28	0	28	0	44.868
198	−36	−42	−2	41	41	−1	−43	2	45.165
199	−100	−112	−4	110	110	−2	−114	4	45.461
200	221	239	6	−236	−236	3	242	−6	45.581

use dimensional analysis to check that **Eq.1** through **Eq.4** are unitless equations.

$$\left(\frac{c^6F^2}{A^2GNRw}\right)^{\frac{1}{6}}=\left(\frac{49}{\pi^2}\right)^9\frac{\pi^{\frac{7}{12}}\times 5^{\frac{5}{2}}}{\zeta(3)^2} \quad (1)$$

$$\left(\frac{F^2R^5w^5}{A^2Gh^6N^7}\right)^{\frac{1}{6}}=\left(\frac{49}{\pi^2}\right)^{13}\frac{2^{\frac{2}{7}}}{5^2\pi^{\frac{7}{12}}} \quad (2)$$

$$\left(\frac{c^3F^2R^2w^2}{A^2Gh^3N^4}\right)^{\frac{1}{3}}=\left(\frac{49}{\pi^2}\right)^{22}\frac{2^{\frac{2}{7}}\times 5^{\frac{1}{2}}}{\zeta(3)^2} \quad (3)$$

$$\left(\frac{chN}{Rw}\right)=\frac{\pi^{\frac{55}{6}}\times 5^{\frac{9}{2}}}{2^{\frac{2}{7}}\times 7^8\times\zeta(3)^2}\approx\left(\frac{49}{\pi^2}\right) \quad (4)$$

Figure 1 is a plot of **Table 1**, and is intended to show that the system of equations in **Table 1** is not random but very periodic. The green line represents the natural log of the right hand sides of the equations and the other lines represent the exponent powers of the physical constants.

Figure 2 is also a plot of **Table 1**, where the equations of the table have been resorted based on the values of the ninth column of the table, instead of the tenth column.

Figures 1 and **2** should prove that the family of unitless equations contained in **Table 1** is not random but instead is a structure made up of periodic waveforms.

3. DISCUSSION

Once we know that the dimensionality of the left hand sides of the equations are correct, then our focus switches to the right hand side of the equations. One should note by definition all the physical constants on the left hand side are measurements and have limited accuracy.

Obviously the equations based physical measurements can not be more accurate than the measurements themselves. My method was to give the Maple software program the benefit of the doubt when computing the right hand sides of the equations.

For example while factoring and processing **Eq.3** with Maple's identify command, the Apery's constant [9] appears in the result. Apery's constant can be expressed as a series, which means we could convert the right hand side of **Eq.3** into a series just by redistributing some of the factors. For this reason I left Apery's constant in the answer which propagated to other the equations.

I view the form of the right hand sides of the equations as an idealized guess times an error term which was supplied by the reduced row echelon operation.

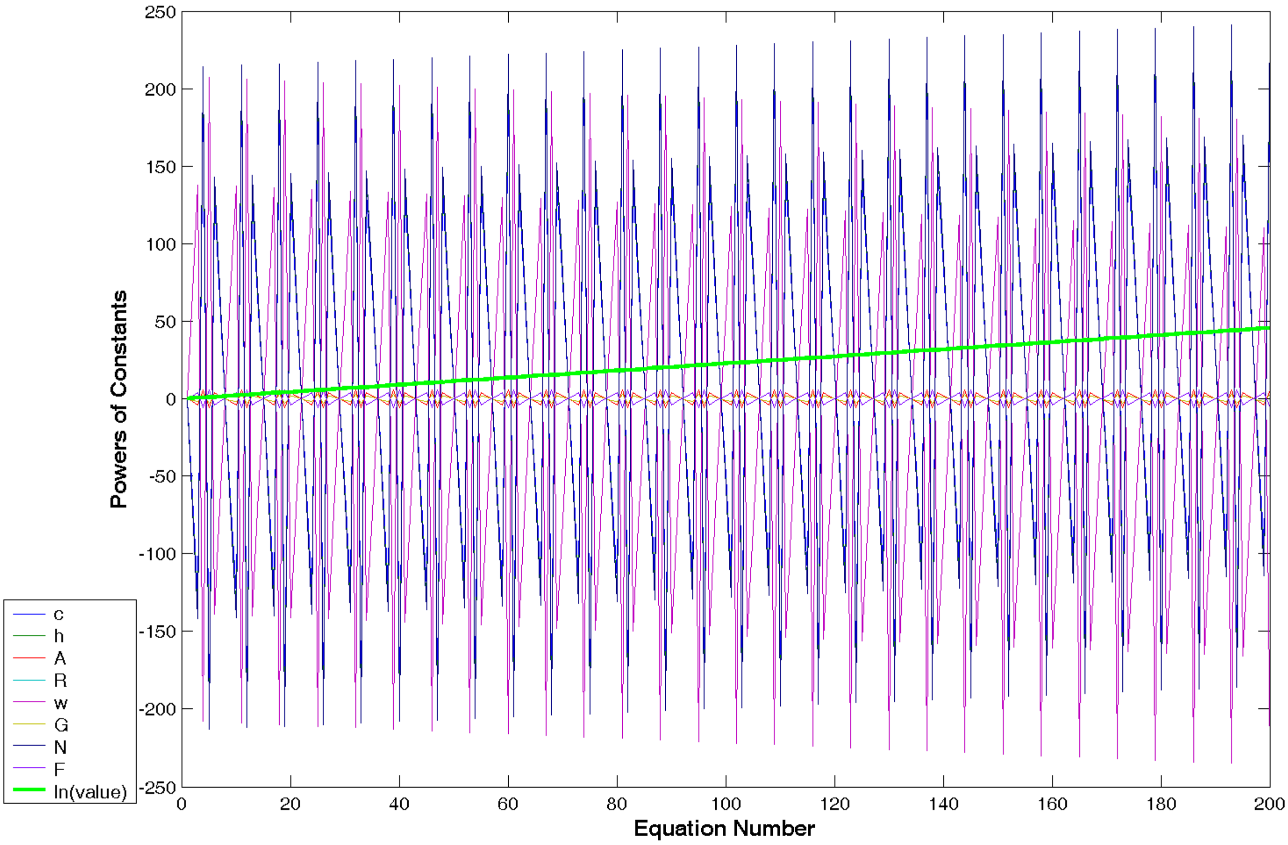

Figure 1. Plot of **Table 1** sorted by the right hand side values.

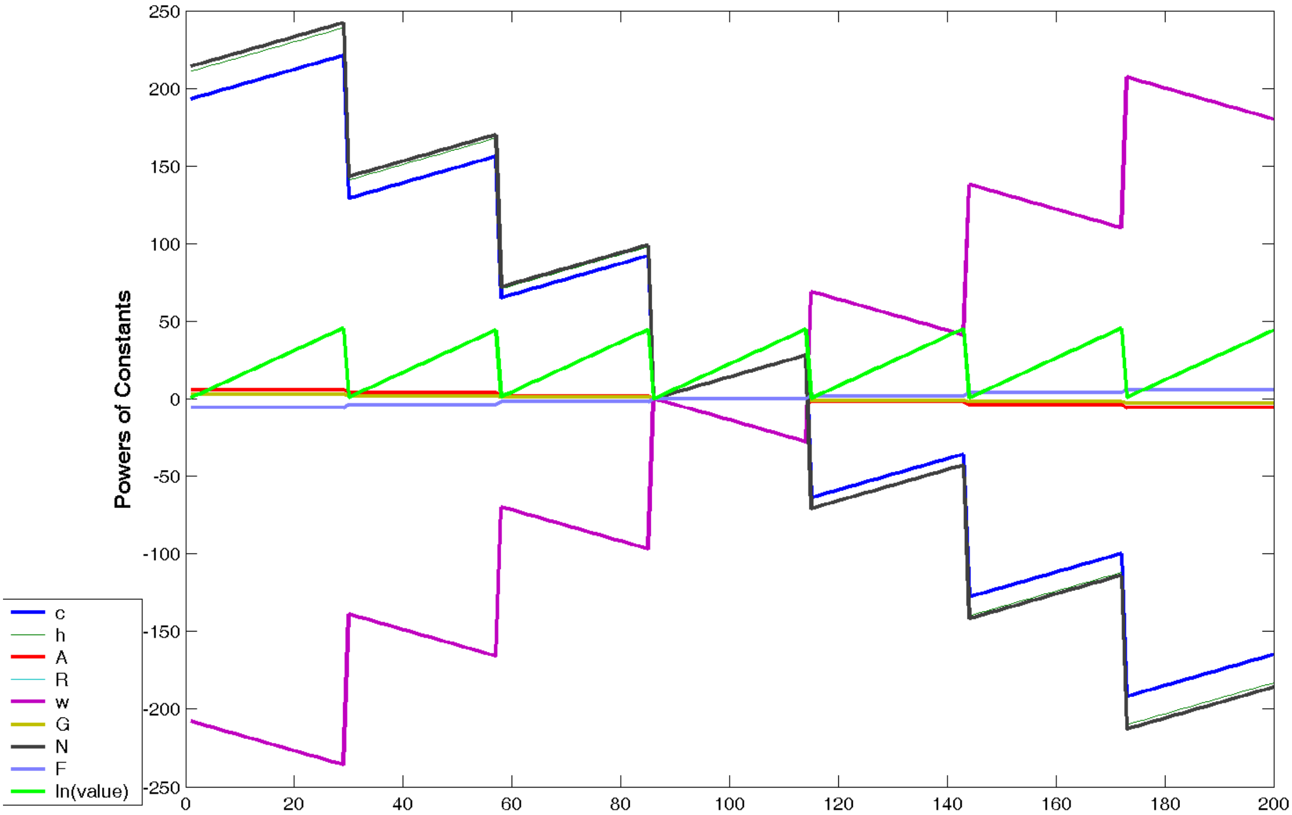

Figure 2. Plot of **Table 1** sorted by the exponent values of the Faraday constant term.

A problem is that the right hand side of the equations are inherently more accurate than the left hand side of the equations; which means any exact answer found by my method is merely a good guess.

On the other hand, these guesses appear to have over seven significant digits of accuracy. Practically speaking, the right hand sides of **Eq.1** through **Eq.4** are close enough to the "right answers" to solve most problems and if one wishes more accuracy one can always use the left hand side to directly compute a decimal value.

4. SUMMARY

In some ways, this paper is mundane. We have a family of similar equations where any single equation can be proven with dimensional analysis to be unitless.

Assuming that a suitable expression can be found for the right hand sides of the equations, then most of these equations could be used like a Swiss army knife to change from one physical constant to another.

On the mundane side we basically have a relationship between nine constants that connects the constants like a key ring. On the other hand, one could argue that the relationships shown in this paper existed before any of the physical constants were measured.

Obviously I can not address the range of philosophical issues that this paper may cause. To answer the reader's unspoken question, I do not know why these relationships exist; I only know that each time that I check them they seem to be correct. I invite other papers to address the deeper issues and physical interpretations of my equations.

A database of over 17,000 equations is available for download; the reader is encouraged to download the database and verify my work. By definition the terms of these equations tend to be self canceling, meaning if you make the wrong substitution, the whole left hand side can disappear and just leave a number. This has happened to me quite a few times in the last few months, which leads me to my final statement of the paper: "I claim nothing."

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APPENDIX I

```
-------------------------------------------------------------
#include "stdio.h"
#define searchsize 29

// Search Size Should be an Odd Interger Greater than 5
// nvcc helloworld08g29.cu -o world08g29 -arch=sm_21
-maxrregcount=24 -ccbin=gcc-4.4
// Note size 29 runs in about 15 seconds on a GTX560,
size 79 runs in a few hours.
// All rights reserved, M. Snyder June 18, 2013
// Looking for solutions of
(x4-x5)^2+(x3+x8)^2+(x4+x7+x8)^2+(x2+x4-x6)^2+(-2
*(x4+x6)+x8-x1-x2)^2+(2*(x2+x4)+x1+x5+3*x6)^2 = 0

__global__ void helloworld()
{
int x1,x2,x3,x4,x5,x6,x7,x8,rlow,rhgh;

rlow=-((gridDim.x-1)/2);
rhgh=((gridDim.x-1)/2);
x1=blockIdx.x+rlow;
x2=blockIdx.y+rlow;
x3=threadIdx.x+rlow;
x4=rlow;
x5=rlow;
x6=rlow;
x7=rlow;
x8=rlow;

while (x8<rhgh)
{
if (x4 + x7 == -x8){
if (x3 == -x8){
if ( (x2+x4) == x6){
if (2*(x2+x4) + x1 + x5 == -3*x6){
if (x4 == x5){
if (-2*( x4 + x6) + x8 == x1 + x2){
printf("%+4d,%+4d,%+4d,%+4d,%+4d,%+4d,%+4d,%+
4d \n", x1,x2,x3,x4,x5,x6,x7,x8);
}
}
}
}
}
}
x4=x4+1;
if (x4>rhgh){x5=x5+1;x4=rlow;
if (x5>rhgh){x6=x6+1;x5=rlow;
if (x6>rhgh){x7=x7+1;x6=rlow;
if (x7>rhgh){x8=x8+1;x7=rlow;
}
}
}
}
}
}

int main()
{
int rangeofsearch(searchsize),seconds;
dim3 grid,block;
grid.x=rangeofsearch;
grid.y=rangeofsearch;
block.x=rangeofsearch;
size_t buf=1e7;
cudaFuncSetCacheCon-
fig(helloworld,cudaFuncCachePreferL1);
cudaDeviceSetLimit(cudaLimitPrintfFifoSize, buf);
seconds = time(NULL);
helloworld<<<grid,block>>>();
cudaDeviceSynchronize();
printf ("\n\n\nComplete CUDA Time: %i ",
int(time(NULL))-seconds);
return 0;
-------------------------------------------------------------
```

APPENDIX II

A maple script one could use to check the program output.

```
-------------------------------------------------------------
with(ScientificConstants);
c0 := evalf(Constant(c));
c0u := GetUnit(Constant(c));
h0 := evalf(Constant(Planck_constant));
h0u := GetUnit(Constant(Planck_constant));
G0 :=
evalf(Constant(Newtonian_constant_of_gravitation));
G0u := GetU-
nit(Constant(Newtonian_constant_of_gravitation));
w0 :=
evalf(Constant(Wien_displacement_law_constant));
w0u := GetU-
nit(Constant(Wien_displacement_law_constant));
R0 := evalf(Constant(R));
R0u := GetUnit(Constant(R));
N0 := evalf(Constant(Avogadro_constant));
N0u := GetUnit(Constant(Avogadro_constant));
A0 := 1;
A0u := Unit('A');
F0 := 96485.3399;
F0u := Unit('C')/Unit('mol');
```