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Interacting generalized chaplygin gas model in bianchi type-I universe

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ABSTRACT

In this paper, we have studied the generalized chaplygin gas of interacting dark energy to obtain the equation of state for the generalized chaplygin gas energy density in anisotropic Bianchi type-I cosmological model. For negative value of *B* in equation of state of generalized chaplygin gas, we see that $\gamma_\Lambda^{eff} < -1$, that corresponds to a universe dominated by phantom dark energy.

Keywords: Cosmological Models; Chaplygin Gas; Cosmological Parameters

1. INTRODUCTION

One of the most important problems of cosmology, is the problem of so-called dark energy (DE). The type Ia supernova observations suggests that the universe is dominated by dark energy with negative pressure which provides the dynamical mechanism of the accelerating expansion of the universe [1-3]. The strength of this acceleration is presently matter of debate, mainly because it depends on the theoretical model implied when interpreting the data. Most of these models are based on dynamics of a scalar or multi-scalar fields. Primary scalar field candidate for dark energy was quintessence scenario [4,5], a fluid with the parameter of the equation of state lying in the range, $-1 < \gamma < -1/3$.

In a very interesting paper Kamenshchik, Moschella and Pasquier [6] have studied a homogeneous model based on a single fluid obeying the Chaplygin gas equation of state

$$p = -\frac{A}{\rho} \tag{1.1}$$

where p and ρ are respectively pressure and energy density in comoving reference frame, with $\rho > 0$; A is a positive constant. This equation of state has raised a certain interest [7] because of its many interesting and, in some sense, intriguingly unique features. Some possible motivations for this model from the field theory points of view are investigated in [8]. The Chaplygin gas emerges as an effective fluid associated with d-branes [9] and can also be obtained from the Born-infield action [10].

Inserting the equation of state (1.1) into the relativistic energy conservation equation, leads to a density evolving as

$$\rho_\Lambda = \sqrt{A + B/V^2} \tag{1.2}$$

where B is an integration constant.

There exist a wide class of anisotropic cosmological models, which also often studying in cosmology [11]. There are theoretical arguments that sustain the existence of an anisotropic phase that approaches an isotropic case [12]. Also, anisotropic cosmological models are found a suitable candidate to avoid the assumption of specific initial conditions in FRW models. The early universe could also characterized by irregular expansion mechanism. Therefore, it would be useful to explore cosmological models in which anisotropic, existing at early stage of expansion, are damped out in the course of evolution. Interest in such models have been received much attention since 1978 [13].

Setare [14] has obtained the equation of state for the generalized Chaplygin gas energy density in non-flat universe. Chaubey [15] has obtained the role of modified chaplygin gas in Bianchi type - I universe. In the present paper, using the generalized Chaplygin gas model of dark energy, we obtain equation of state for interacting Chaplygin gas energy density in anisotropic Bianchi type-I cosmological model. For negative value of B in equation of state of generalized chaplygin gas, we see that $\gamma_\Lambda^{eff} < -1$, that corresponds to a universe dominated by phantom dark energy.

2. INTERACTING GENERALIZED CHAPLYGIN GAS

In this section we obtain the equation of state for the generalized Chapligin gas when there is an interaction between generalized Chaplygin gas energy density ρ_Λ and a Cold Dark Matter (CDM) with $\gamma_m = 0$.

The continuity equations for dark energy and CDM are

$$\dot{\rho}_\Lambda + 3H(1+\gamma_\Lambda)\rho_\Lambda = -Q \tag{2.1}$$

$$\dot{\rho}_m + 3H\rho_m = Q. \tag{2.2}$$

The interaction is given by the quality $Q = \Gamma\rho_\Lambda$. This is a decaying of the generalized Chaplygin gas component into CDM with the decay rate Γ. Taking a ratio of two energy densities as $r = \rho_m/\rho_\Lambda$, the above equations lead to

$$\dot{r} = 3Hr\left[\gamma_\Lambda + \frac{1+r}{r}\frac{\Gamma}{3H}\right] \tag{2.3}$$

Following [3], if we define

$$\gamma_\Lambda^{eff} = \gamma_\Lambda + \frac{\Gamma}{3H}, \quad \gamma_m^{eff} = -\frac{1}{r}\frac{\Gamma}{3H} \tag{2.4}$$

Then, the continuity equations can be written in their standard form

$$\dot{\rho}_\Lambda + 3H\left(1+\gamma_\Lambda^{eff}\right)\rho_\Lambda = 0 \tag{2.5}$$

$$\dot{\rho}_m + 3H\left(1+\gamma_m^{eff}\right)\rho_m = 0 \tag{2.6}$$

We consider the homogeneous anisotropic Bianchi type-I cosmological model with line element

$$ds^2 = dt^2 - a_1{}^2dx^2 - a_2{}^2dy^2 - a_3{}^2dz^2 \tag{2.7}$$

where a_1, a_2 and a_3 are function of t only.

The Einstein field equations for the metric (2.1) are written in the form

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = \kappa[\rho_\Lambda + \rho_m]. \tag{2.8}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} = -\kappa[\rho_\Lambda]. \tag{2.9}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} = -\kappa[\rho_\Lambda]. \tag{2.10}$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} = -\kappa[\rho_\Lambda] \tag{2.11}$$

where $\kappa \equiv 8\pi G/c^4$ is constant.

We define

$$V = a_1a_2a_3 \tag{2.12}$$

By using the method of Singh *et al.* [16-19], we obtain

$$a_1(t) = D_1V^{1/3}\exp\left(X_1\int\frac{dt}{V(t)}\right) \tag{2.13}$$

$$a_2(t) = D_2V^{1/3}\exp\left(X_2\int\frac{dt}{V(t)}\right) \tag{2.14}$$

$$a_3(t) = D_3V^{1/3}\exp\left(X_3\int\frac{dt}{V(t)}\right) \tag{2.15}$$

where D_i $(i = 1, 2, 3)$ and X_i $(i = 1, 2, 3)$ satisfy the relation $D_1D_2D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Now, adding **Eqs.2.9**, **2.10** and **2.11** and three times **Eq.2.8**, we get

$$\left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) + 2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right) = \frac{3}{2}\kappa\left[(\rho_\Lambda + \rho_m) - p_\Lambda\right]. \tag{2.16}$$

From **Eqs.2.12** and **2.16**, we have

$$\frac{\ddot{V}}{V} = \frac{3}{2}\kappa\left[(\rho_\Lambda + \rho_m) - p_\Lambda\right]. \tag{2.17}$$

Define as usual

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{3\kappa\rho_m V^2}{\dot{V}^2}; \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{3\kappa\rho_\Lambda V^2}{\dot{V}^2} \tag{2.18}$$

From above, we obtain following relation for ratio of energy densities r as

$$r = \frac{\Omega_m}{\Omega_\Lambda} \tag{2.19}$$

In the generalized Chaplygin gas approach [10], the equation of state to (1.1) is generalized to

$$p_\Lambda = -\frac{A}{\rho_\Lambda^\alpha} \tag{2.20}$$

The above equation of state leads to a density evolution as

$$\rho_\Lambda = \left[A + \frac{B}{V^{(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{2.21}$$

Taking derivatives in both sides of above equation with respect to cosmic time, we obtain

$$\dot{\rho}_\Lambda = -B\left(\frac{\dot{V}}{V}\right)V^{-(1+\alpha)}\left[A + BV^{-(1+\alpha)}\right]^{-\frac{\alpha}{1+\alpha}} \tag{2.22}$$

Substituting this relation into **Eq.2.1** and using definition $Q = \Gamma\rho_\Lambda$, we obtain

$$\gamma_\Lambda = \frac{B}{V^{(1+\alpha)}[A + BV^{-(1+\alpha)}]} - \frac{\Gamma}{(\dot{V}/V)} - 1 \tag{2.23}$$

Here as in Ref. [20], we choose the following relation for decay rate

$$\Gamma = b^2(1+r)\frac{\dot{V}}{V} \tag{2.24}$$

with the coupling constant b^2. Using **Eq.2.14**, the above decay rate take following form

$$\Gamma = b^2\left(\frac{\dot{V}}{V}\right)\left(\frac{\Omega_\Lambda+\Omega_m}{\Omega_\Lambda}\right) \quad (2.25)$$

Substituting this relation into **Eq.2.23**, one finds the generalized Chaplygin gas energy equation of state

$$\gamma_\Lambda = \frac{B}{V^{(1+\alpha)}\left[A+BV^{-(1+\alpha)}\right]} - b^2\left(\frac{\Omega_\Lambda+\Omega_m}{\Omega_\Lambda}\right) - 1. \quad (2.26)$$

Now using the definition generalized Chaplygin gas energy ρ_Λ, and using Ω_Λ, we can rewrite the above equation as

$$\gamma_\Lambda = \frac{3B}{\left\{\frac{1}{\kappa}\left(\frac{\dot{V}^2}{V}\right)\Omega_\Lambda\right\}^{(1+\alpha)}} - b^2\left(\frac{\Omega_\Lambda+\Omega_m}{\Omega_\Lambda}\right) - 1 \quad (2.27)$$

From **Eqs.2.4**, **2.25** and **2.27**, we have the effective equation of state as

$$\gamma_\Lambda^{eff} = \frac{3B}{\left[\frac{1}{\kappa}\left(\frac{\dot{V}^2}{V}\right)\Omega_\Lambda\right]^{(1+\alpha)}} - 1 \quad (2.28)$$

By choosing a negative value for B we see that $\gamma_\Lambda^{eff} < -1$, that corresponds to a universe dominated by phantom dark energy, **Eq.2.28**, for $\alpha = 1$, is the effective parameter of state for Chaplygin gas. In this case, in the expression for energy density (1.2), term under square root should be positive, *i.e.* $V^2 > -B/A$, then the minimal value of the volume factor is given by

$$V_{min} = \left(-\frac{B}{A}\right)^{\frac{1}{2}} \quad (2.29)$$

Now, from **Eqs.2.13-2.15** and **2.29**, we have find the minimal value of the scale factors are given by

$$a_{1min} = D_1\left(-\frac{B}{A}\right)^{\frac{1}{6}} \exp\left[X_1\left(-\frac{B}{A}\right)^{-\frac{1}{2}} t\right] \quad (2.30)$$

$$a_{2min} = D_2\left(-\frac{B}{A}\right)^{\frac{1}{6}} \exp\left[X_2\left(-\frac{B}{A}\right)^{-\frac{1}{2}} t\right] \quad (2.31)$$

$$a_{3min} = D_3\left(-\frac{B}{A}\right)^{\frac{1}{6}} \exp\left[X_3\left(-\frac{B}{A}\right)^{-\frac{1}{2}} t\right] \quad (2.32)$$

According to this model we have a bouncing universe. Generally for this model $A > 0, B < 0$ and $1+\alpha > 0$. From **Eq.2.21**, we can realize that the cosmic scalar factors take values in the interval $a_{i\ min} < a_i < \infty$ (for i = 1, 2, 3) which corresponds to $0 < \rho < (2A)^{\frac{1}{1+\alpha}}$, where

$$V_{min} = \left(-\frac{B}{A}\right)^{\frac{1}{1+\alpha}} \quad (2.33)$$

and

$$a_{i\ min} = D_i\left(-\frac{B}{A}\right)^{\frac{1}{3(1+\alpha)}} \exp\left[X_i\left(-\frac{B}{A}\right)^{-\frac{1}{(1+\alpha)}} t\right], i = 1,2,3. \quad (2.34)$$

Using **Eq.1.2**, one can see that the Chaplygin gas interpolates between dust at small a_i and a cosmological constant at large a_i, but choosing a negative value of B, this quartessence idea lose. Following [6] if we consider a homogeneous scalar field $\phi(t)$ and a potential $V(\phi)$ to describe the Chaplygin cosmology, we find

$$\dot{\phi}^2 = \frac{B}{V^2\sqrt{A+B/V^2}} \quad (2.35)$$

Now, by choosing a negative value for B we see that $\dot{\phi}^2 < 0$, then we can write

$$\phi = i\psi \quad (2.36)$$

In this case the lagrangian of scalar field $\phi(t)$ can rewritten as

$$L = -\frac{1}{2}\dot{\phi}^2 - V(\phi) = -\frac{1}{2}\dot{\psi}^2 - V(i\psi) \quad (2.37)$$

The energy density and the pressure corresponding to the scalar field ψ are as respectively

$$\rho_\psi = -\frac{1}{2}\dot{\psi}^2 + V(i\psi) \quad (2.38)$$

$$p_\psi = -\frac{1}{2}\dot{\psi}^2 - V(i\psi) \quad (2.38)$$

Therefore, the scalar field ψ is a phantom field. This implies that one can generate phantom-like equation of state from an interacting generalized Chaplygin gas dark energy model in anisotropic universe.

3. CONCLUSIONS

We have studied the generalized chaplygin gas of interacting dark energy to obtain the equation of state for the generalized chaplygin gas energy density in anisotropic Bianchi type-I cosmological model. By choosing a negative value for B we see that $\gamma_\Lambda^{eff} < -1$, that corresponds to a universe dominated by phantom dark energy.

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