

Using of the generalized special relativity (GSR) in estimating the neutrino masses to explain the conversion of electron neutrinos

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ABSTRACT

In this work the Generalized Special Relativity (GSR) is utilized to estimate masses of some elementary particles such as, neutrinos. These results are found to be in conformity with experimental and theoretical data. The results obtained may explain some physical phenomena, such as, conversion of neutrinos from type to type when solar neutrino reaches the Earth.

Keywords: Generalized; Neutrino Masses; Conversion; Phenomena

1. INTRODUCTION

The concept of mass plays an important role in physics. The mass is very important property of any substance and element. The mass of any element or elementary particle is utilized to characterize this element or particle. The masses of the elementary particles emitted from the stars can be utilized to understand the nuclear reactions and the mass conversions processes [1,2]. They can also utilized to study the gravitational waves [3] as well as supernova [4,5].

There are many problems associated with mass, especially in the world of the microscopic particles such as photons, electrons, neutrinos and protons. These problems appeared in the middle of the 20th century. Many physicists tried to solve the problems of mass [6,7]. The most famous one is the neutrino mass problem. Neutrino mass problem arises from the fact that the neutrino mass is strongly dependent on the elementary particle associated with it. It was discovered that electron, muon, and tau neutrino masses are different from each other [8,9]. This discovery is confirmed theoretically and experimentally by using very sensitive detectors [10].

Neutrinos have electric charge, interact very rarely with matter, and according to the text book version of

the Standard Model (SM) of particle physics – are mass less. For every hundred billion solar neutrinos that pass through the Earth, only about one interacts at all with stuff of which Earth is made. There are three known types of neutrinos. Nuclear fusion in the Sun produces only neutrinos that are associated with electrons, the so-called electron neutrinos (ν_e). The two other types of neutrinos, muon neutrinos (ν_μ) and tau neutrinos (ν_τ), are produced, for example, in laboratory accelerators or in exploding stars, together with heavier versions of the electron, the particles muon (μ) and tau (τ) [11].

Evidence obtained indicated that something must happen to the neutrinos on their way to detectors on Earth from the interior of the Sun. In 1990, Hans Bethe and John N. Bahcall pointed that new neutrino physics, beyond what was contained in the Standard Model particle physics textbook, was required to reconcile the results of the Davis Chlorine experiments and the Japanese -American water experiments. This lead directly to the relative sensitivity of the Chlorine and water experiments to neutrino number and energy. The newer Solar neutrino experiments in Italy and in Russia increased the difficulty of explaining the neutrino data without involving new physics [12]. On June 2001 the mystery of Solar neutrino had solved by a collaboration of Canadian, American, and British scientists. They reported the first Solar neutrino results obtained with a detector of 1000 tons of heavy water (D₂O). The new detector was able to study in different way the same higher-energy Solar neutrinos that had been investigated previously in Japan with the Kamiokande and Super – Kamiokande ordinary – water detectors. The Canadian detector is called SNO for Solar neutrino observatory [13].

The SNO collaboration made unique new measurements in which the total number of high energy neutrinos of all types was observed in the heavy water detector. These results from the SNO measurements alone show that most of the neutrinos produced in the interior of the Sun, all of which are electron neutrinos by the time they

reach the Earth. The solution of the mystery of missing Solar neutrinos is that neutrinos are not, in fact, missing. The previously uncounted neutrinos are changed from electron neutrinos (ν_e) into muon and tau neutrinos that are more difficult to detect [14].

The Standard Model of particle physics assumes that neutrinos are mass less. In order for neutrino oscillations to occur, some neutrinos must have masses. Therefore, the Standard Model of particle physics must be revised. The simplest model that fits all the neutrino data implies that the mass of the electron neutrino is about 100 million time smaller than the mass of the electron.

$$m_{\nu_e} = \frac{9.11 \times 10^{-31}}{10^8} = 9.11 \times 10^{-39} \quad (1)$$

But, the available data are not yet sufficiently definitive to rule out all but one possible solution. When we finally have a unique solution – as we will later, the values of the different neutrino masses may be clues that lead to understanding physics beyond the Standard Model of particle physics [15]. Neutrino mass given in many previous studies are as follow, all neutrino masses are given in the unit of eV. Experimental data shows that the neutrino mass is given as follows (see **Table 1**):

The best neutrino mass limits from direct measurements come from the tritium endpoint experiments Mainz and Troitsk [16,17], both of which place $m_\nu < 2.2$ eV. Measurements of the cosmic microwave background, coupled with cosmological models, have led to somewhat better (but model-dependent) constraints of $m_\nu < 1$ eV [18]. The next generation of tritium endpoint measurement is now being pursued by the KATRIN experiment [19]. They expect to push the limit on the neutrino mass as low as $m_\nu < 0.2$ eV. An independent avenue of research is neutrinoless double-decay, which could test the Majorana nature of the neutrino and possibly determine its mass [20].

Also theoretical works presented different values of the neutrino mass as follows:

The result $m_{\nu_e} < 4.5$ eV obtained by I. Hassan [21] and the correspondent muon and tau neutrino masses presented are:

$$m_{\nu_\mu} = 63.4 \text{ eV}, m_{\nu_\tau} = 225 \text{ eV} \quad (3)$$

Then Reference [17] presents $m_{\nu_e} < 10$ eV and the correspondent muon and tau neutrino masses presented are:

$$m_{\nu_\mu} = 31.88 \text{ eV}, m_{\nu_\tau} = 110 \text{ eV} \quad (4)$$

2. AIMS OF THE WORK

The aims of this work is to use the Generalized Special Relativity (GSR) to estimate the neutrino masses so as

Table 1. Experimental data of neutrinos mass values obtained in works [16-20].

work	Electron neutrino	Mass (eV)
[16,17]	m_{ν_e}	<2.2 eV
[18]	m_{ν_e}	<1 eV
[19]	m_{ν_e}	<0.2 eV
[20]	m_{ν_e}	<5.6 eV

to explain the problems associated with their masses, and to explain the conversion of neutrinos from type to type. Then to compare the result obtained using the (GSR) theory with other theoretical and experimental works.

3. GENERALIZED SPECIAL RELATIVITY (GSR) THEORY

The Generalized Special Relativity theory is a new form of the special relativity theory that adopts the gravitational potential, and it gives the formula of relative mass to be as follows [23]:

$$m = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \quad (5)$$

where $g_{00} = 1 + \frac{2\varphi}{c^2}$, and φ denotes the gravitational potential, or the field in which the mass is measured .

The derivation of the mass **Eq.5** using the generalized special relativity (GSR) can be found as follows:

In the special relativity (SR), the time, length, and mass can be obtained in any moving frame by either multiplying or dividing their values in the rest frame by a factor γ .

$$\gamma = \sqrt{1 - \frac{v^2}{c^2}} \quad (6)$$

where v is the velocity of the particle, and c is the speed of light.

It is convenient to re-express γ in terms of the proper time, associated with the impact of gravity on the previous physical quantities, (time, length, and mass) [23].

$$c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (7)$$

where $g_{\mu\nu}$ is the metric tensor, and, μ and ν denotes the contra variant (covariant) vectors.

Which is a common language to both special relativity SR, and general relativity (GR). We know that in special relativity (SR) **Eq.7** reduces to: [23].

$$c^2 d\tau^2 = c^2 dt^2 - dx^i dx^i, x^0 = ct \quad (8)$$

where i denotes the particle position (covariant) vector according to Lorentz covariance.

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{1}{c^2} \cdot \frac{dx^i}{dt} \cdot \frac{dx^i}{dt}} = \sqrt{1 - \frac{v^2}{c^2}} = \gamma \quad (9)$$

Thus we can easily generalize γ to include the effect of gravitation by using **Eq.7** and by adopting the weak field approximation where [23].

$$g_{11} = g_{22} = g_{33} = -1, g_{00} = 1 + \frac{2\varphi}{c^2} \quad (10)$$

$$\gamma = \frac{d\tau}{dt} = \sqrt{g_{00} - \frac{1}{c^2} \cdot \frac{dx^i}{dt} \cdot \frac{dx^i}{dt}} = \sqrt{g_{00} - \frac{v^2}{c^2}} \quad (11)$$

When the effect of motion only is considered, the expression of time in the special relativity (SR) is found to be [23].

$$dt = \frac{dt_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (12)$$

where the subscript 0 stands for the quantity measured in the rest frame. While if gravity only affect time, its expression is given by [23].

$$dt = \frac{dt_0}{\sqrt{g_{00}}} \quad (13)$$

In view of **Eqs.12, 13** and **11** the expression

$$dt = \frac{dt_0}{\gamma} \quad (14)$$

can be generalized to recognize the effect of motion as well as gravity on time, to get

$$dt = \frac{dt_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \quad (15)$$

The same result can be obtained for the volume where the effect of motion and gravity respectively gives [23].

$$V = V_0 \sqrt{1 - \frac{v^2}{c^2}} \quad (16)$$

$$V = \sqrt{g} V_0 = \sqrt{g_{00}} V_0 \quad (17)$$

The generalization can be done by utilizing **Eq.11** to find that

$$V = \gamma V_0 = \sqrt{g_{00} - \frac{v^2}{c^2}} \cdot V_0 \quad (18)$$

To generalize the concept of mass to include the effect of gravitation we use the expression for the Hamiltonian in general relativity, *i.e.* [23].

$$H = \rho c^2 = g_{00} T^{00} = g_{00} \rho_0 \left(\frac{dx^0}{d\tau} \right)^2 \quad (19)$$

$$= g_{00} \frac{\rho_0 c^2}{\gamma^2} = g_{00} \frac{m_0 c^2}{V_0 \gamma^2}$$

where H is Hamiltonian, ρ is the density, and T^{00} is energy tensor.

Using **Eqs.18,19**, yields:

$$\rho c^2 = \frac{m c^2}{V} = \frac{g_{00} m_0 c^2}{\gamma V} \quad (20)$$

Therefore

$$m = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \quad (21)$$

Which is the expression of mass in the presence of gravitational potential and it named the generalized special relativity (GSR) theory.

In view of **Eq.5**, and when we substitute the value of g_{00} , then the relative mass according to (GSR) is found to be

$$m = \frac{m_0 \left(1 + \frac{2\varphi}{c^2} \right)}{\sqrt{1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}}} \quad (22)$$

When the field is weak in the sense that

$$\frac{2\varphi}{c^2} \ll 1 \quad (23)$$

And when the speed v is very low such that

$$\frac{v^2}{c^2} \ll 1 \quad (24)$$

Eq.22 reduces to:

$$m = \frac{m_0 \left(1 + \frac{2\varphi}{c^2} \right)}{\sqrt{1 + \frac{2\varphi}{c^2}}} \rightarrow m = m_0 \sqrt{1 + \frac{2\varphi}{c^2}} \quad (25)$$

Using the identity $(1+x)^n \approx 1+n.x$ for $x \ll 1$ one can also gets:

$$m = m_0 \left(1 + \frac{\varphi}{c^2} \right) \quad (26)$$

And, when the field is so strong such that

$$\frac{2\varphi}{c^2} \gg 1 \text{ and } \frac{v^2}{c^2} \ll 1 \quad (27)$$

Then **Eq.22** reduces to

$$m = m_0 \sqrt{\frac{2\varphi}{c^2}} \quad (28)$$

Some gravitational models [21] propose the existence of short range gravitational field. The expression of the field potential is assumed to be:

$$\varphi = \frac{c_0}{r} e^{-c_1 r} \quad (29)$$

For very small distance, *i.e.* when $r \rightarrow 0$, **Eq.29** reduces to:

$$\varphi = \frac{c_0}{r} = \frac{G_s m}{r} \quad (30)$$

where G_s stands for the strong gravitational constant, to find G_s one can assume that the strong gravity field is the strong nuclear force itself. In this case we can use the nuclear proton potential φ_p to find G_s . Where:

$$\varphi = \frac{G_s m_p}{r_p} \quad (31)$$

4. RESULTS AND DISCUSSION

Let us recall **Eq.28** and estimate the results of neutrino masses. First of all one needs to find the strong gravity constant G_s by using **Eq.31** and substituting by the following values:

$$m_p = 1.67 \times 10^{-27} \text{ kg}, r_p = 1.32 \times 10^{-15} \text{ m}, \\ \varphi_p = 1.67 \times 10^{13} \text{ Nm/kg}$$

to get:

$$G_s = 1.23 \times 10^{25} \text{ Nm}^2/\text{kg}^2.$$

but since the electron radius and mass are given by:

$$r_e = 10^{-33} \text{ m}, m_e = 9.11 \times 10^{-31} \text{ kg},$$

Then from **Eq.31** we get:

$$\varphi_e = 11.07 \times 10^{27} \text{ Nm/kg}$$

Using **Eq.28** the electron neutrino zero mass can be adjusted to obey the relation.

$$m_{\nu_e} = 6.24 \text{ eV} = 1.11 \times 10^{-37} \text{ kg} \quad (32)$$

That means the mass of the electron neutrino (m_{ν_e}) is about million times smaller than the mass of the electron.

To calculate the mass of the muon neutrino (m_{ν_μ}), while the muon mass m_μ compared to the electron mass is given by:

$$m_\mu = \frac{105.7}{0.511} = 207 m_e$$

So that the mass of muon neutrino is given by:

$$m_{\nu_\mu} = m_e \sqrt{\frac{2\varphi_\mu}{c^2}} = \sqrt{207} m_{\nu_e} \quad (33)$$

$$m_{\nu_\mu} = 89.78 \text{ eV} = 1.59 \times 10^{-36} \text{ kg}$$

To calculate the mass of the tau neutrino (m_{ν_τ}), while the tau mass m_τ compared to the electron mass is given by:

$$m_\tau = \frac{1784}{0.511} = 3491 m_e$$

So that the mass of muon neutrino is given by:

$$m_{\nu_\tau} = m_e \sqrt{\frac{2\varphi_\tau}{c^2}} = \sqrt{3491} m_{\nu_e} \quad (34)$$

$$m_{\nu_\tau} = 368.7 \text{ eV} = 6.55 \times 10^{-34} \text{ kg}$$

5. CONCLUSIONS

The mass values found by **Eqs.32-34** using the (GSR) theory, for the three kinds of neutrinos compared to the values found by the different works as in **Eqs.1-3**, shows that the change in the neutrino mass is attributed in this model as resulted from the effect of the electron, muon, and tau strong gravitational field on the neutrino mass. The neutrino masses obtained due to the effect of e , μ and τ gravitational field are given in **Table 2**. The arrangements of these values are in agreement with that obtained in different experimental works in the ranges that given in Reference [17-20], and theoretical works [21]. A direct comparison between the values obtained using the generalized special relativity (GSR) model and the experimental and theoretical values shows that they are in conformity with each other. Experimental data found in different works agree that the neutrino mass estimated is the mass of electron neutrinos, which was able to detect in many detectors such as SNO collaborations [14]. The mass value of correspondent neutrinos (muon and tau), was calculated as a value that depend on the relation between the masses of the particles (electron, muon, and tau) themselves, as shown in the Results section. So the values found in this work using the genera-

Table 2. Neutrinos mass values obtained using the GSR model.

particle	Mass symbol	Mass (Mev)	Mass (kg)
Electron	m_e	0.511	9.11×10^{-31}
Electron neutrino	m_{ν_e}	6.24 eV	1.11×10^{-37}
Muon	m_μ	$207 m_e$	1.89×10^{-28}
Muon neutrino	m_{ν_μ}	89.78 eV	1.59×10^{-34}
Tau	m_τ	$3491 m_e$	4.57×10^{-27}
Tau neutrino	m_{ν_τ}	368.7 eV	6.55×10^{-34}

lized special relativity (GSR) model, affirm that the missing neutrinos are not, in fact, missing, but, actually are not able to detect. And that explains the conversion of a part of the electron neutrinos into muon and tau neutrinos, by the time that Solar neutrinos reach the earth, which explain why the number of detected neutrinos is less than that predicted by theoretical model of the sun and by textbook description of neutrinos.

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