Vol.1, No.3, 204-209 (2009)
doi:10.4236/ns.2009.13027

Natural Science

Wavelet chaotic neural networks and their application to continuous function optimization

Jia-Hai Zhang[1], Yao-Qun Xu[2]

[1]College of Electrical and Automatic Engineering, Sanjing University, Nanjing, China; zhangjh6688@sohu.com
[2]Institute of Computer and Information Engineering Harbin Commercial University, Harbin, China; xuyaoq@hrbcu.edu.cn

Received 7 September 2009; revised 10 October 2009; accepted 12 October 2009.

ABSTRACT

Neural networks have been shown to be powerful tools for solving optimization problems. In this paper, we first retrospect Chen's chaotic neural network and then propose several novel chaotic neural networks. Second, we plot the figures of the state bifurcation and the time evolution of most positive Lyapunov exponent. Third, we apply all of them to search global minima of continuous functions, and respectively plot their time evolution figures of most positive Lyapunov exponent and energy function. At last, we make an analysis of the performance of these chaotic neural networks.

Keywords: Wavelet Chaotic Neural Networks; Wavelet; Optimization

1. INTRODUCTION

Hopfield and Tank first applied the continuous-time, continuous-output Hopfield neural network (HNN) to solve TSP [1], thereby initiating a new approach to optimization problems [2,3]. The Hopfield neural network, one of the well-known models of this type, converges to a stable equilibrium point due to its gradient decent dynamics; however, it causes sever local-minimum problems whenever it is applied to optimization problems. M-SCNN has been proved to be more power than Chen's chaotic neural network in solving optimization problems, especially in searching global minima of continuous function and traveling salesman problems [4].

In this paper, we first review the Chen's chaotic neural network. Second, we propose several novel chaotic neural networks. Third, we plot the figures of the state bifurcation and the time evolution of most positive Lyapunov exponent. Fourth, we apply all of them to search global minima of continuous functions, and respectively plot their time evolution figures of most positive Lyapunov exponent and energy function. At last, simulation results are summarized in a Table in order to make an analysis of their performance.

2. CHAOTIC NEURAL NETWORK MODELS

In this section, several chaotic neural networks are given. And the first is proposed by Chen, the rest proposed by ourselves.

2.1. Chen's Chaotic Neural Network

Chen and Aihara's transiently chaotic neural network [5] is described as follows:

$$x_i(t) = f(y_i(t)) = \frac{1}{1+e^{-y_i(t)/\varepsilon}} \tag{1}$$

$$y_i(t+1) = ky_i(t) + \alpha\left[\sum_j W_{ij}x_j + I_i\right] - z_i(t)(x_i(t) - I_0) \tag{2}$$

$$z_i(t+1) = (1-\beta)z_i(t) \tag{3}$$

where $x_i(t)$ is output of neuron i ; $y_i(t)$ denotes internal state of neuron i ; W_{ij} describes connection weight from neuron j to neuron i , $W_{ij} = W_{ji}$; I_i is input bias of neuron i , a a positive scaling parameter for neural inputs, k damping factor of nerve membrane, $0 \le k \le 1$, $z_i(t)$ self-feedback connection weight (refractory strength) ≥ 0, β damping factor of $z_i(t)$, $0 < \beta < 1$, I_0 a positive parameter, ε steepness parameter of the output function, $\varepsilon > 0$.

2.2. Morlet Wavelet Chaotic Neural Network (MWCNN)

Morlet wavelet chaotic neural network is described as follows:

$$x_i(t) = f(y_i(t)) = e^{-(uy_i(t))^2/2} cos(5uy_i(t)) \tag{4}$$

$$y_i(t+1) = ky_i(t) + \alpha\left[\sum_j W_{ij}x_j + I_i\right] - z_i(t)(x_i(t) - I_0) \quad (5)$$

$$z_i(t+1) = (1-\beta)z_i(t) \quad (6)$$

where $x_i(t)$, $y_i(t)$, W_{ij}, α, k, I_i, $z_i(t)$, I_0 are the same with the above. And the **Eq.4** is the Morlet wavelet function. u is a steepness parameter of the output function which is varied with different optimization problems.

2.3. Mexican Hat Wavelet Chaotic Neural Network (MHWCNN)

Mexican hat wavelet chaotic neural network is described as follows:

$$x_i(t) = f(y_i(t)) = \frac{2}{\sqrt{3\sqrt{\pi}}}(1-(uy_i(t))^2)e^{-(uy_i(t))^2/2} \quad (7)$$

$$y_i(t+1) = ky_i(t) + \alpha\left[\sum_j W_{ij}x_j + I_i\right] - z_i(t)(x_i(t) - I_0) \quad (8)$$

$$z_i(t+1) = (1-\beta)z_i(t) \quad (9)$$

where $x_i(t)$, $y_i(t)$, W_{ij}, α, k, I_i, $z_i(t)$, I_0, u are the same with the above. And the **Eq.7** is the Shannon wavelet function.

3. RESEARCH ON CONTINUOUS FUNCTION PROBLEMS

In this section, we apply all the above chaotic neural networks to search global minima of the following three continuous functions.

The three continuous functions are described as follows [6]:

$$f_1(x_1,x_2) = \frac{\sin^2\sqrt{x_1^2+x_2^2} - 0.5}{[1+0.001(x_1^2+x_2^2)]^2} - 0.5 \quad |x_i| \le 100 \quad (10)$$

$$f_2(x_1,x_2) = 4x_1^2 - 2.1x_1^4 + x_1^6/3 + x_1x_2 - 4x_2^2 + 4x_2^4 \quad |x_i| \le 5 \quad (11)$$

$$f_4(x_1,x_2) = (x - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1-\frac{1}{8\pi})\cos x_1 + 10$$
$$-5 \le x_1 \le 10, 0 \le x_2 \le 15 \quad (12)$$

The minimum value of **Eq.10**, **11**, **12** respectively are -1, -1.0316285, 0, 0.398 and its responding point are (0, 0), (0.08983, -0.7126) or (-0.08983, 0.7126), (-3.142, 2.275) or (3.142, 2.275) or (9.425, 2.425).

In order to make comparison conveniently, we set some parameters such as the annealing speed β, the self-feedback $z(0,0)$ and the initial value of internal state $y(0,0)$ as follows: β =0.002, $z(0,0)$ =[0.8, 0.8], $y(0,0)$ =[0.283, 0.283]. Meanwhile, we set the iteration as large as 5000 so as to get stable state of a global minimum.

3.1. Chen's Chaotic Neural Network

1) Simulation on the First Continuous Function

The rest parameters are set as follows:

$$k=1, \alpha=0.5, \varepsilon=1/10, I_0=0.85.$$

The time evolution figures of the biggest positive Lyapunov exponent and energy function of Chen's in solving the first continuous function are shown as **Figure 1, Figure 2.**

The global minimum and its responding point of the simulation are respectively -0.99989 and (0.0073653, 0.0073653).

2) Simulation on the Second Continuous Function

The rest parameters are set as follows:

$$k=1, \alpha=0.02, \varepsilon=1/20, I_0=0.85.$$

The time evolution figures of most positive Lyapunov exponent and energy function of Chen's in

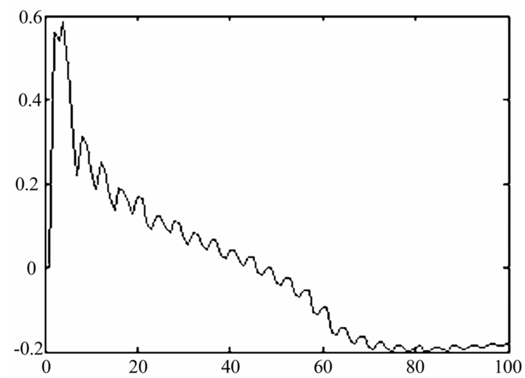

Figure 1. Time evolution figure of Lyapunov exponent.

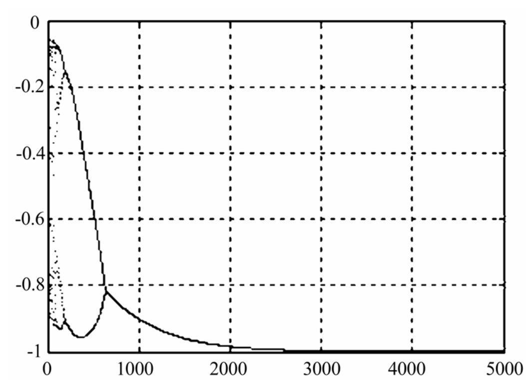

Figure 2. Time evolution figure of energy function.

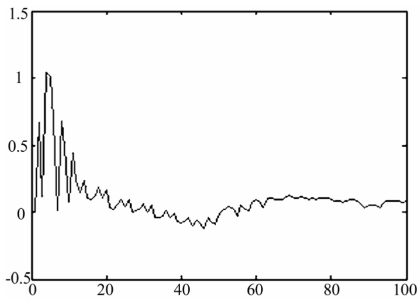

Figure 3. Time evolution figure of Lyapunov exponent.

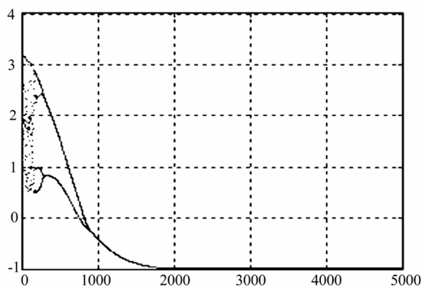

Figure 4. Time evolution figure of energy function.

solving the first continuous function are shown as **Figure 3**, **Figure 4**.

The global minimum and its responding point of the simulation are respectively -1 and (0, 0.70712).

3) Simulation on the Third Continuous Function

The rest parameters are set as follows:

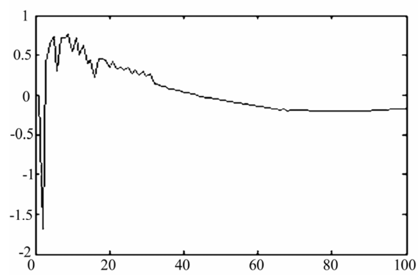

Figure 5. Time evolution figure of Lyapunov exponent.

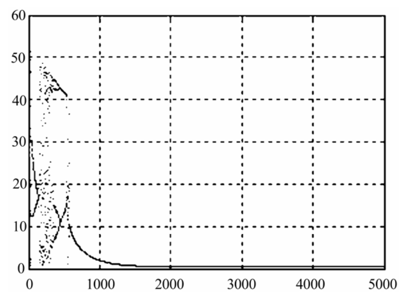

Figure 6. Time evolution figure of energy function.

$$k=1, \alpha=0.2, \varepsilon=1, I_0=0.5.$$

The time evolution figures of most positive Lyapunov exponent and energy function of Chen's in solving the first continuous function are shown as **Figure 5**, **Figure 6.**

The global minimum and its responding point of the simulation are respectively 0.39789 and (9.4246, 2.4747).

3.2. Morlet Wavelet Chaotic Neural Network (Mwcnn)

1) Simulation on the First Continuous Function

The rest parameters are set as follows:

$$k=1, \alpha=0.5, u=0.5, I_0=0.65.$$

The time evolution figures of most positive Lyapunov exponent and energy function of MWCNN in solving the first continuous function are shown as **Figure 7, Figure 8.**

The global minimum and its responding point of the simulation are respectively -0.99997 and (0.0038638, 0.0038638).

2) Simulation on the Second Continuous Function

The rest parameters are set as follows:

$$k=1, \alpha=0.05, u=0.7, I_0=0.2.$$

The time evolution figures of most positive Lyapunov exponent and energy function of MWCNN in solving the first continuous function are shown as **Figure 9**, **Figure 10.**

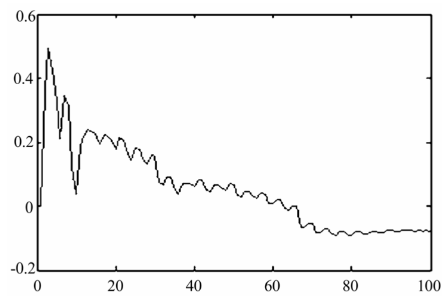

Figure 7. Time evolution figure of Lyapunov exponent.

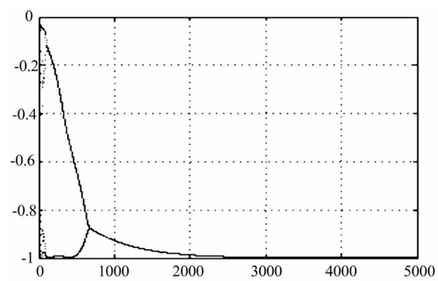

Figure 8. Time evolution figure of energy function.

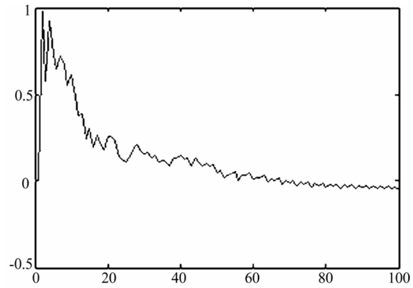

Figure 9. Time evolution figure of Lyapunov exponent.

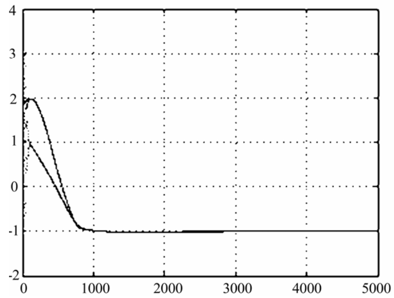

Figure 10. Time evolution figure of energy function.

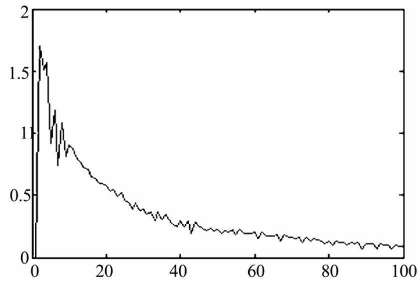

Figure 11. Time evolution figure of Lyapunov exponent.

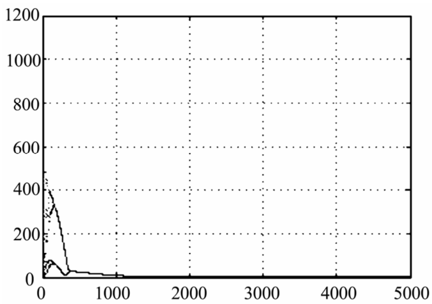

Figure 12. Time evolution figure of energy function.

The global minimum and its responding point of the simulation are respectively -1.0021 and (-0.074007, 0.76863).

3) Simulation on the Third Continuous Function

The rest parameters are set as follows:

$$k=1, \alpha=0.02, u=0.09, I_0=0.2.$$

The time evolution figures of most positive Lyapunov exponent and energy function of MWCNN in solving the first continuous function are shown as **Figure 11, Figure 12.**

The global minimum and its responding point of the simulation are respectively 0.39789 and (3.1413, 2.2733).

3.3. Mexican Hat Wavelet Chaotic Neural Network (MHWCNN)

1) Simulation on the First Continuous Function

The rest parameters are set as follows:

$$k=1, \alpha=0.5, u=0.2, I_0=0.8.$$

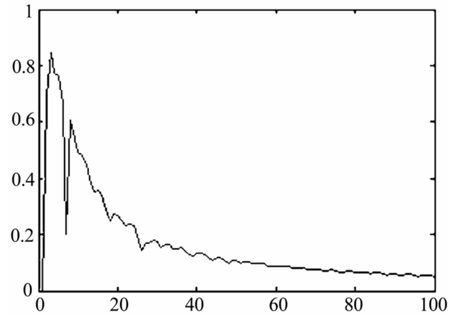

Figure 13. Time evolution figure of Lyapunov exponent.

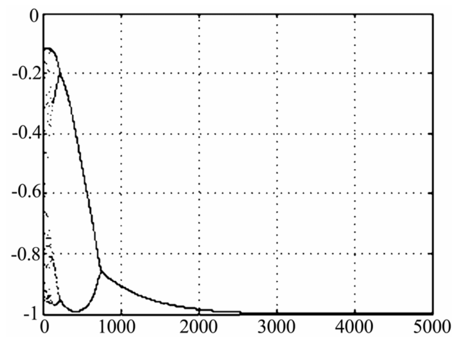

Figure 14. Time evolution figure of energy function.

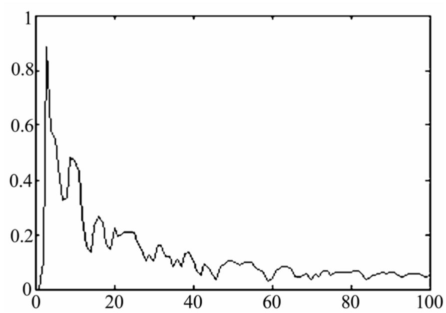

Figure 15. Time evolution figure of Lyapunov exponent.

The time evolution figures of most positive Lyapunov exponent and energy function of MHWCNN in solving the first continuous function are shown as **Figure 13**, **Figure 14.**

The global minimum and its responding point of the simulation are respectively -0.99996 and (0.0043259, 0.0043259).

2) Simulation on the Second Continuous Function

The rest parameters are set as follows:

$$k=1, \alpha=0.05, u=2.8, I_0=0.05.$$

The time evolution figures of most positive Lyapunov exponent and energy function of MHWCNN in solving the first continuous function are shown as **Figure 15**, **Figure 16.**

The global minimum and its responding point of the simulation are respectively -1.0316 and (-0.089825, 0.71263).

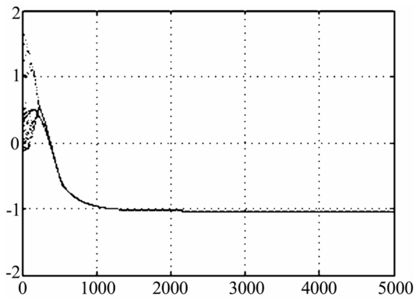

Figure 16. Time evolution figure of energy function.

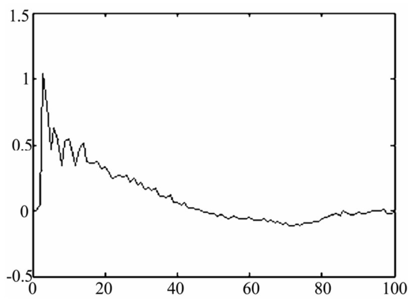

Figure 17. Time evolution figure of Lyapunov exponent.

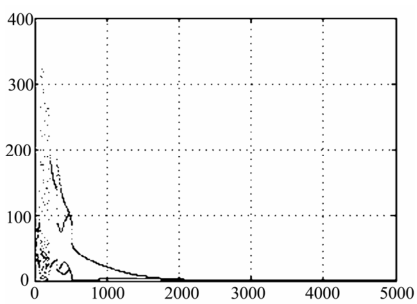

Figure 18. Time evolution figure of energy function.

Table 1. the simulation results of the chaotic neural networks.

Fun \ Model GM/ER		Chen's	MWCNN	MHWCNN
f_1	TGM	-1	-1	-1
	PGM	-0.99989	*-0.99997*	-0.99996
	ER	0.00011	0.00003	0.00004
f_2	TGM	-1.0316285	-1.0316285	-1.0316285
	PGM	-1	-1.00021	-1.0316
	ER	0.0316285	0.0314185	0.0000285
f_4	TGM	0.398	0.398	0.398
	PGM	0.3789	0.3789	0.3789
	ER	0.0191	0.0191	0.0191
AVE	AVER	0.01270962	0.01263712	0.00479212

3) Simulation on the Third Continuous Function**.**

The rest parameters are set as follows:

$$k=1, \alpha=0.05, \quad u=0.3, I_0=0.2.$$

The time evolution figures of most positive Lyapunov exponent and energy function of MHWCNN in solving the first continuous function are shown as **Figure 17**, **Figure 18.**

The global minimum and its responding point of the simulation are respectively 0.39789 and (3.1415, 2.2743).

4. ANALYSIS OF THE SIMULATION RESULTS

Simulation results are summarized in **Table 1**. The columns "GM/ER", "TGM", "PGM" and "AVER" represent, respectively, global minimum/error rate; theoretical global minimum; practical global minimum; average error.

Seen from the **Table 1**, we can conclude that the wavelet chaotic neural networks are superior to Chen's in AVER

5. CONCLUSION

We have introduced Chen's and wavelet chaotic neural networks. We make an analysis of them in solving continuous function optimization problems, and find out that wavelet chaotic neural networks are superior to Chen's in general.

REFERENCES

[1] J. J. Hopfield and D. W. Tank, (1985) Neural computation of decision in optimization problems. Biol. Cybern., **52**, 141-152.

[2] J. Hopfield, (1982) Neural networks and physical systems with emergent collective computa-tional abilities. In: Proc Natl Acad Sci, **79**, 2554-2558.

[3] J. Hopfield, (1984) Neural networks and physical systems with emergent collective computational abilities. 81, Proc. Natl. Acad. Sci., 3088-3092.

[4] Y. Q. Xu, M. Sun, and G. R. Duan, (2006) Wavelet chaotic neural networks and their application to optimization problems. Lecture Notes in Computer Science.

[5] L. P. Wang and F. Y. Tian, (2000) Noisy chaotic neural networks for solving combinatorial optimization problems. International Joint Conference on Neural Networks. Italy, IEEE, 37-40.

[6] L. Chen and K. Aihara, (1995) Chaotic simulated annealing by a neural network model with transient chaos. Neural Networks., **8**, 915-930.

[7] L. Wang, (2001) Intelligence optimization algorithm and its application. Press of TUP.

[8] C. Q. Xie, C. He, and H. W. Zhu, (2002) Improved noisy chaotic neural networks for solving combinational optimization problems. Journal of Shanghai Jiaotong university, **36**, 351-354.

[9] Y. Q. Xu and M. Sun, (2006) Gauss-morlet-sigmoid chaotic neural networks. ICIC, LNCS.