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Binary Relations between Magnitudes of Different Dimensions Used in Material Science Optimization Problems Pseudo-State Equation of Soft Magnetic Composites

Krzysztof Z. Sokalski[1*], Bartosz Jankowski[2], Barbara Ślusarek[2]

[1]Institute of Computer Sciences, Częstochowa University of Technology, Częstochowa, Poland
[2]Tele and Radio Research Institute, Warszawa, Poland
Email: [*]sokalski_krzysztof@o2.pl

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Abstract

New algorithm for optimizing technological parameters of soft magnetic composites has been derived on the base of topological structure of the power loss characteristics. In optimization magnitudes obeying scaling, it happens that one has to consider binary relations between the magnitudes having different dimensions. From mathematical point of view, in general case such a procedure is not permissible. However, in a case of the system obeying the scaling law it is so. It has been shown that in such systems, the binary relations of magnitudes of different dimensions is correct and has mathematical meaning which is important for practical use of scaling in optimization processes. The derived structure of the set of all power loss characteristics in soft magnetic composite enables us to derive a formal pseudo-state equation of Soft Magnetic Composites. This equation constitutes a relation of the hardening temperature, the compaction pressure and a parameter characterizing the power loss characteristic. Finally, the pseudo-state equation improves the algorithm for designing the best values of technological parameters.

Keywords

Soft Magnetic Composites, Scaling, Binary Relations, Pseudo-State Equation

[*]Corresponding author.

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1. Introduction

Recently novel concept of technological parameters' optimization has been applied in Soft Magnetic Composites (SMC) by Ślusarek *et al.*, [1]. This concept is based on assumption that SMC is a self-similar system where function of loss of power obeys the scaling law [2]-[4]. The efficiency of scaling in solving problems concerning power losses in soft magnetic composites has already been confirmed in [1].
The scaling is very useful tool due to the three reasons:

- it reduces number of independent variables f and B_m to the effective one $f/(B_m)^\alpha$,
- and determines general form of loss of power characteristic in a form of homogenous function in general sense (h.f.g.s.),
- as well as enables us to use binary relations between magnitudes of different dimensions.

Reduction of independent variables is based on definition of the h.f.g.s., namely, $F(f, B_m)$ is the h.f.g.s. if:

$$\exists\{a,b,c\} \in \mathbb{R}\times\mathbb{R}\times\mathbb{R} : \forall\lambda \in \mathbb{R}_+ \Rightarrow F\left(\lambda^a f, \lambda^b B_m\right) = \lambda^c F\left(f, B_m\right). \tag{1}$$

According to the assumption concerning λ we are free to substitute any positive real number, for instance $\lambda = (B_m)^{-1/b}$ then we get:

$$\frac{F(f, B_m)}{B_m^\beta} = F\left(\frac{f}{B_m^\alpha}, 1\right) \tag{2}$$

where f and B_m are frequency and pik of magnetic inductance, respectively. $F(\cdot,1)$ is an arbitrary function, $\alpha = \frac{a}{b}$, $\beta = \frac{c}{b}$ are scaling exponents.

Choice for the $F(\cdot,1)$ depends on the power loss characteristics of investigated materials. In [1] we have modified the Bertotti decomposition rule [5] [6] which led to the following form for $P_{\text{tot}}(\cdot)/B_m^\beta = F(\cdot,1)$:

$$\frac{P_{\text{tot}}}{B_m^\beta} = \left(f/B_m^\alpha\right)\cdot\left(\Gamma_1 + f/B_m^\alpha\cdot\left(\Gamma_2 + f/B_m^\alpha\cdot\left(\Gamma_3 + f/B_m^\alpha\cdot\Gamma_4\right)\right)\right) \tag{3}$$

where Γ_n, α and β have been estimated for different values of the technological parameters [1] (pressure and temperature). For purpose of this paper we take into account only one family of power loss characteristics which are presented in **Figure 1** and **Figure 2**. The corresponding estimated values of the model parameters are presented in **Table 1**. For all other details concerning SMC material and measurement data we refer to [1]. Now we are ready to formulate the goals of this paper. Main goal is to minimize the power loss in SMC by using model density of power loss (3) and corresponding values of the model parameters. From the first row of **Table 1**, we can see that dimensions of the Γ_n coefficients depend on the values of the α and β exponents. Therefore, the

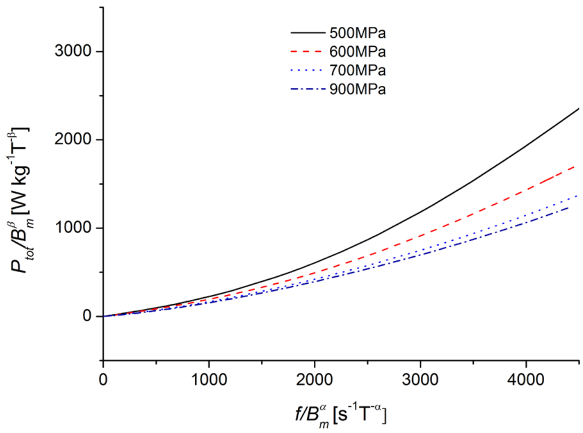

Figure 1. Selection of the power loss characteristics $P_{tot}/(B_m)^\alpha$ vs. $f/(B_m)^\alpha$ calculated according to (3) and **Table 1** for Somaloy 500 [1], $T = 500^\circ$C.

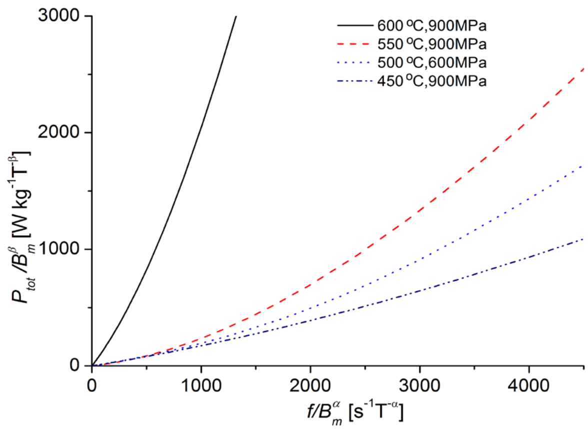

Figure 2. Selection of the power loss characteristics $P_{tot}/(B_m)^{\alpha}$ vs. $f/(B_m)^{\alpha}$ calculated according to (3) and **Table 1** for Somaloy 500 [1].

Table 1. Somaloy 500. Values of scaling exponents and coefficients of (3) vs. compaction pressure and hardening temperature, a selection from [1].

T	p	α	β	Γ_1	Γ_2	Γ_3	Γ_4
[˚C]	[MPa]	[−]	[−]	[m^2·s^{-2}T$^{\alpha-\beta}$]	[m^2·s^{-1}T$^{2\alpha-\beta}$]	[m^{2}T$^{3\alpha-\beta}$]	[m^2·sT$^{4\alpha-\beta}$]
500	500	−1.312	−0.011	0.171	3.606×10^{-5}	1.953×10^{-8}	-2.255×10^{-12}
500	600	−1.383	−0.125	0.153	3.328×10^{-5}	9.254×10^{-8}	-1.177×10^{-12}
500	700	−1.735	−0.517	0.156	2.393×10^{-5}	2.309×10^{-8}	-8.075×10^{-14}
500	900	−1.395	−0.082	0.101	6.065×10^{-5}	-8.031×10^{-8}	7.877×10^{-13}
400	800	−1.473	−0.28	0.183	1.347×10^{-5}	3.689×10^{-9}	1.185×10^{-13}
450	800	−1.596	−0.123	0.145	2.482×10^{-5}	-1.218×10^{-9}	6.120×10^{-14}
550	800	−2.034	−1.326	0.106	1.407×10^{-4}	-1.066×10^{-8}	4.541×10^{-13}
600	800	−1.608	−0.232	1.220	8.941×10^{-4}	-5.302×10^{-8}	1.664×10^{-11}

power loss characteristics presented in **Figure 1** and **Figure 2** are different dimensions. So, we have to answer the following question: are we able to relate them in the optimization process which has been described in [1]?

In this paper we will prove that if the considered characteristics obey the scaling, then the binary relation between them is invariant with respect to this transformation and comparison of two magnitudes of different dimensions has mathematical meaning. Reach measurement data of power losses in Somaloy 500 have been transformed into parameters of (3) vs. hardening temperature and compaction pressure **Table 1** in [1]. Information contained in this table enable us to infer about topological structure of set of the power loss characteristics and finally to construct pseudo-state equation for SMC, and derive new algorithm for the best values of technological parameters.

2. Scaling of Binary Relations

Let the power loss characteristic has the form determined by the scaling (2). It is important to remain that α and β are defined by initial exponents a, b and c (see after Formula (2)):

$$\alpha = \frac{a}{b}; \quad \beta = \frac{c}{b} \tag{4}$$

Let us concentrate our attention at the point on the $f/(B_m)^{\alpha}$ axis of **Figure 1** and **Figure 2**:

$$\frac{f}{B_m^{\alpha}} = \frac{f_1}{B_{m1}^{\alpha 1}} = \frac{f_2}{B_{m2}^{\alpha 2}} = \frac{f_3}{B_{m3}^{\alpha 3}} = \frac{f_4}{B_{m4}^{\alpha 4}} \tag{5}$$

Let us take into account the two characteristics and let us assume that

$$\frac{P_{\text{tot1}}}{B_{m1}^{\beta 1}} > \frac{P_{\text{tot2}}}{B_{m2}^{\beta 2}} \tag{6}$$

Therefore, the considered binary relation is the strong inequality and corresponds to natural order presented in **Figure 1** and **Figure 2**. The most important question of this research is whether (6) is invariant with respect to scaling:

$$\frac{P'_{\text{tot1}}}{B'^{\beta 1}_{m1}} > \frac{P'_{\text{tot2}}}{B'^{\beta 2}_{m2}} \tag{7}$$

Let $\lambda > 0$ be an arbitrary positive real number. Then, the scaling of (7) goes according to the following algorithm:

• Let us perform the scaling with respect to λ of all independent magnitudes and the dependent one:

$$f_i' = \lambda^{a_i} f_i; \quad B'_{mi} = \lambda^{b_i} B_{mi}; \quad P'_{\text{tot}} = \lambda^{c_i} P_{\text{tot}} \tag{8}$$

where $i = 1, 2, \cdots, 4$ labels the considered characteristics.

• Substituting appropriate relations of (8) to (7) we derive:

$$\frac{P_{\text{tot1}}}{B_{m1}^{\beta 1}} \lambda^{c_1 - b_1 \beta_1} > \frac{P_{\text{tot2}}}{B_{m2}^{\beta 2}} \lambda^{c_2 - b_2 \beta_2} \tag{9}$$

• Collecting all powers of λ on the left-hand side of (9) and taking into account (4) we derive the resulting power to be zero and

$$\lambda^{c_1 - b_1 \beta_1 - c_2 + b_2 \beta_2} = 1 \tag{10}$$

Therefore (6) is invariant with respect to scaling. This binary relation has mathematical meaning and constitutes the total order in the set of characteristics.

3. Binary Equivalence Relation

The result derived in Section 2 can be supplemented with the following binary equivalence relation. Let

$$X_{i,j} = \left(\frac{f_{i,j}}{B_{m\,i,j}^{\alpha_i}}, \frac{P_{\text{tot}\,i,j}}{B_{m\,i,j}^{\beta_i}} \right) \tag{11}$$

be the j-th point of the i-th characteristic. Two points j and k are related if they belong to the same i-th characteristic:

$$X_{i,j} \boldsymbol{R} X_{i,k} \tag{12}$$

Theorem: $\boldsymbol{R}$ is equivalence relation. (The proof is trivial and can be done by checking out that the considered relation is: reflexive, symmetric and transitive.) Therefore, $\boldsymbol{R}$ constitutes division of the positive-positive quarter of plane spanned by (11). The characteristics do not intersect each other except in the origin point which is excluded from the space. The result of this section implies that the power loss characteristics (2) and (3) are invariant with respect to scaling. Structure of derived here the set of all characteristics of which some examples are presented in **Figure 1** and **Figure 2** enables us to derive a formal pseudo-state equation of SMC. This equation constitutes a relation of the hardening temperature, the compaction pressure and a parameter characterizing the power loss characteristic corresponding to the values of these technological parameters. Finally, the pseudo-state equation will improve the algorithm for designing the best values of technological parameters.

4. Pseudo-State Equation of SMC

Let $\mathbb{C}$ be set of all possible power loss characteristics in considered SMC. Each characteristic is smooth curve in $\left[f / (B_m)^\alpha, P_{\text{tot}} / (B_m)^\beta \right]$ plane which corresponds to a point in $[T, p]$ plane. In order to derive the pseudo-state equation we transform each power loss characteristic into a number V corresponding to (T, p) point.

By this way we obtain a function of two variables:

$$(T, p) \to V \tag{13}$$

This function must satisfy the following condition. Let us concentrate our attention at the two following points:

$$\frac{f_1}{B_{m1}^{\alpha_1}} = \frac{f}{B_m^{\alpha}}; \quad \frac{f_2}{B_{m2}^{\alpha_2}} = \frac{f}{B_m^{\alpha}} \tag{14}$$

Let us consider the two characteristics $P_{\text{tot1}}/(B_{m1})^{\beta 1}$ and $P_{\text{tot2}}/(B_{m2})^{\beta 2}$ of the two samples composed under $T1$, $p1$ and $T2$, $p2$ values of temperature and pressure, respectively.

While, the other technological parameters powder compositions and volume fraction are constant. Let us assume that for (14) the following relation holds:

$$\frac{P_{\text{tot1}}}{B_{m1}^{\beta 1}} > \frac{P_{\text{tot2}}}{B_{m2}^{\beta 2}} \tag{15}$$

It results from the derived structure of $\mathbb{C}$ that (15) holds for each value of (14).Therefore we have to assume the following condition of sought $V \ (T, p)$: If the relation (15) holds for $T1$, $p1$, $T2$, $p2$ then the following relation has to be satisfied for $V \ (T, p)$:

$$V(T_1, p_1) > V(T_2, p_2). \tag{16}$$

Moreover, $V \ (T, p)$ has to indicate place of corresponding characteristic in the ordered $\mathbb{C}$. The simplest choice satisfying these requirements is the following average:

$$V(T, p) = \frac{1}{\varphi_{\max} - \varphi_{\min}} \int_{\varphi_{\min}}^{\varphi_{\max}} \frac{P_{\text{tot}}\left(\frac{f}{B_m^{\alpha}}\right)}{B_m^{\beta}} \mathrm{d}\left(\frac{f}{B_m^{\alpha}}\right) \tag{17}$$

where the integration domain is common for the all characteristics. We have selected the common domain of **Figure 1** and **Figure 2**: $\varphi_{\min} = 0$, $\varphi_{\max} = 4000$ [$s^{-1} \cdot T^{-\alpha}$]. Using (3) we transform (17) to the working formula for the measure V:

$$V(T, p) = \frac{1}{\varphi_{\max} - \varphi_{\min}} \int_{\varphi_{\min}}^{\varphi_{\max}} x\left(\Gamma_1 + x\left(\Gamma_2 + x\left(\Gamma_3 + x\Gamma_4\right)\right)\right) \mathrm{d}x \tag{18}$$

where $x = f/(B_m)^{\alpha}$, Γ_i are coefficients dependent on T and p, see **Table 1**. The values of $V \ (T, p)$ are tabulated in **Table 2**. **Table 2** enables us to draw pseudo-isotherm. It is presented in **Figure 3**. However, in order to derive the complete pseudo-state equation we must create a mathematical model. On basis of **Figure 3** we start from the classical gas state-equation as an initial approximation:

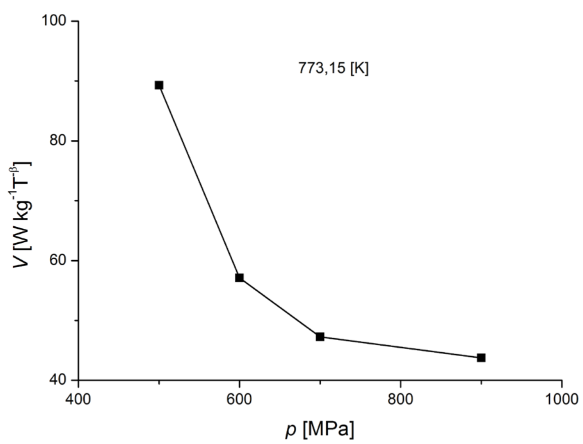

Figure 3. Pseudo-Isotherm T = 500˚C of the Low-losses phase, according to data of **Table 2** for Somaloy 500 [1].

Table 2. *V* measure vs. hardening temperature and compaction pressure.

T	*p*	*V*
[K]	[MPa]	[W·kg^{-1}T$^{-\beta}$]
723.15	800	40.60
773.15	900	43.75
773.15	700	47.25
673.15	800	50.30
773.15	600	57.12
823.15	800	81.50
773.15	500	89.28
742.15	764	492.3
753.15	780	509.2
804.15	764	528.5
711.15	764	547.0
873.15	800	720.0

$$\frac{p \cdot V}{k_B \cdot T} = 1 \tag{19}$$

where k_B is pseudo-Boltzmann constant.

In order to extent (19) to a realistic equation we apply again the scaling hypothesis (2) [2]-[4]:

$$V\left(\frac{T}{T_c}, \frac{p}{p_c}\right) = \left(\frac{p}{p_c}\right)^{\gamma} \Phi\left(\frac{\frac{T}{T_c}}{\left(\frac{p}{p_c}\right)^{\delta}}\right) \tag{20}$$

where $\Phi(\cdot)$ is an arbitrary function to be determined. γ, δ and T_c, p_c are scaling exponents and scaling parameters respectively, to be determined. For our conveniences we introduce the following variables:

$$\tau = \left(\frac{T}{T_c}\right); \quad \pi = \frac{p}{p_c}; \quad X = \frac{\frac{T}{T_c}}{\left(\frac{p}{p_c}\right)^{\delta}} = \frac{\tau}{\pi^{\delta}} \tag{21}$$

In order to extent (19) to a full state-equation we apply the Padé approximant by analogy to virial expansion derived by Ree and Hoover [7]:

$$V(\tau, \pi) = \pi^{\gamma} \frac{G_0 + X\left(G_1 + X\left(G_2 + X\left(G_3 + XG_4\right)\right)\right)}{1 + X\left(D_1 + X\left(D_2 + X\left(D_3 + XD_4\right)\right)\right)} \tag{22}$$

where $G_0, \cdots, G_4$, $D_1, \cdots, D_4$ are parameters of the Padé approximant. All parameters have to be determined from the data presented in **Table 2**.

5. Estimation of the Pseudo-State Equation's Parameters

At the beginning we have to notice that the data collected in **Table 2** reveal sudden change of V between two points: [773, 15; 500, 0] and [742, 15; 764, 0]. This suggests existence of a crossover between two phases: low-losses phase and high losses phase. We take this effect into account and we divide the data of **Table 2** into two subsets corresponding to these two phases, respectively. Since the cross over consists in changing of characteristic exponents for the given universality class it is necessary to perform estimations of the model parameters for each phase separately. Minimizations of χ^2 for both phases have been performed by using MICROSOFT EXCEL 2010, where

$$\chi^2 = \sum_{i=1}^{N}\left(V(\tau_i,\pi_i) - \pi_i^{\gamma}\frac{G_0 + X_i\left(G_1 + X_i\left(G_2 + X_i\left(G_3 + X_i G_4\right)\right)\right)}{1 + X_i\left(D_1 + X_i\left(D_2 + X_i\left(D_3 + X_i D_4\right)\right)\right)}\right)^2 \tag{23}$$

where $N = 7$ and $N = 5$ for the low-losses and high-losses phases, respectively. **Table 3** and **Table 4** present estimated values of the model parameters for the low-losses and for high-losses phases, respectively.

6. Optimization of Technological Parameters

Function $V\ (T,p)$ serves a power loss measure versus the hardening temperature and compaction pressure. In order to explain how to optimize the technological parameters with the pseudo-state Equation (22) we plot the phase diagram of considered SMC **Figure 4**. Note that all losses' characteristics collapsed to a one curve for the eachphase. Taking into account the Low-losses phase we determine the lowest losses at $\tau\cdot\pi^{-\delta} = 19.75$. This gives the following continuous subspace of the optimal points:

$$\frac{\frac{T}{T_c}}{\left(\frac{p}{p_c}\right)^{\delta}} = 19.75 \tag{24}$$

Formula (24) represents the minimal iso-power loss curve. All points satisfying (24) are solutions of the optimization problem for technical parameters of SMC.

7. Conclusion

By introducing the binary relations we have revealed twofold. The power loss characteristics do not cross each other which makes the topology's set of this curves very useful and effective that we can perform all calculations in the one-dimension space spanned by the scaled frequency or here in the case of pseudo-statee quation in the scaled temperature. For general knowledge concerning such a topology we refer to the papers by Egenhofer [8] and by Nedas *et al.* [9]. However, to our knowledge this paper is the first one about the binary relations be-

Table 3. Somaloy 500, low-losses phase. Values of pseudo-state equation's parameters and the Padé approximant's coefficients of (22).

		T_c	p_c	G_0	G_1	G_2
0.1715	1.2812	21.622	37.729	370,315,315	−47,752,251	1,734,952
G_3	G_4	D_1	D_2	D_3	D_4	-
−1.3764	−678.26	170.80	6243.8	386.96	−28.699	-

Table 4. Somaloy 500, high-losses phase. Values of pseudo-state equation's parameters and the Padé approximant's coefficients of (22).

		T_c	p_c	G_0	G_1	G_2
0.1810	1.5550	22.949	30.197	365,210,688	−47,714,207	1,762,773
G_3	G_4	D_1	D_2	D_3	D_4	-
−1.3763	−683.38	170.77	5739.9	387.81	−22.514	-

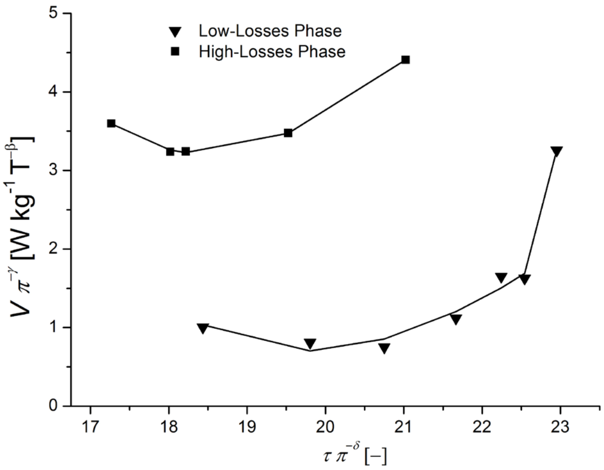

Figure 4. Phase diagram for Somaloy 500.

tween magnitudes of different dimensions in the sense of different physical magnitudes. Also, this paper is the first one which presents an application of scaling in designing the technological parameters' values by using the pseudo-state equation of SMC. The obtained result is the continuous set of points satisfying (24). All solutions of these equations are equivalent for the optimization of the power losses. Therefore, the remaining degree of freedom can be used for optimizing magnetic properties of the considered SMC. Ultimately, one must say that the degree of success achieved when applying the scaling depends on the property of the data. The data must obey the scaling.

Acknowledgements

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