

Crack-Tip Stress Analysis at a Bi-Material Interface by Photoelastic, Isopachic and FEA

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ABSTRACT

The paper investigates the stress state at the bi-material interface crack-tip by the Photoelastic and Isopachic methods and the Finite Element Analysis (FEA). The principal stresses at the bi-material interface crack-tip are theoretically determined using the combination photoelastic and isopachic fringes. The size and the shape of crack-tip isochromatic and isopachic fringes, at a bi-material interface under static load, are studied. When the crack-tip, which is perpendicular to interface, is placed at the interface of the bi-material, the isochromatic and the isopachic fringes depend on the properties of the two materials. Thus, the isochromatic and the isopachic fringes are divided into two branches, which present a jump of values at the interface. The size of the two branches mainly depends on the elastic modulus and the Poisson's ratio of the two materials. From the combination of the isochromatic and the isopachic fringes, the principal stresses σ_1 and σ_2 can be estimated and the contour curves around the crack-tip can be plotted. For the FEA analysis, the program ANSYS 11.0 was used. The bi-material cracked plates were made from Lexan (BCBA) and Plexiglas (PMMA).

Keywords: Crack, Bi-Material, Photoelasticity, Isochromatic, Isopachic, FEA

1. Introduction

The study of the behaviour of a transverse crack propagating through the mesophase of composites has become a subject of great interest. The problem of crack propagation in a duplex plate was studied by Williams *et al.* [1-5] and later was approached by Dally and Kobayashi [6] by means of dynamic photoelasticity. Theocaris *et al.* [7-9] have studied the influence of both the mesophase and the material characteristics of either phase, in biphasic plates consisting of different materials, on the stress distribution around the crack tip. They have extended their study to the magnitude and the variation of the crack propagation velocities during fracture in duplex plates under dynamic loading [10]. Also, Theocaris *et al.* [11,12] have studied the influence of the hard or soft fiber and the mesophase layers in a soft-hard-soft or hard-soft-hard combination of biphasic plate subjected to a dynamic tensile load, on the fracture mode and bifurcation process in both phases. Also, theoretical studies on this subject were carried out by Gdoutos *et al.* [13-15] and Theotokoglou *et al.* [16,17]. The study of the size and the shape of crack-tip caustics

at a bi-material interface was carried out by Papadopoulos [18]. Sadowski *et al.* [19,20] have studied the bi-material interface problem for ceramic materials by FEA. In the present paper, we estimate the principal stresses and its contour curves at the bi-material interface crack-tip from the combination of the isochromatic and the isopachic fringes and the FEA.

2. Crack-Tip Stresses at the Interface

Two plates of module E_1 and E_2 and Poisson's ratio ν_1 and ν_2 are perfectly bonded along their common interface (**Figure 1**). In plate (1) there is a crack perpendicular to the interface. The crack-tip is placed exactly at the interface of the two plates. The Airy stress function $X(r,\theta)$ for this problem, used by Zak and Williams [5], is:

$$X(r,\theta) = r^{\lambda+1}F(\theta) \quad (1)$$

with:

$$F(\theta) = \alpha \sin(\lambda+1)\theta + b \cos(\lambda+1)\theta + c \sin(\lambda-1)\theta + d \cos(\lambda-1)\theta \quad (2)$$

where α, b, c, d are constants and λ takes values between 0 and 1, which is depended on the ratio $E_{12} = E_1/E_2$ of the two plates moduli [18].

From the stress function (1) the polar stresses at the crack-tip are taken:

$$\sigma_r = -\lambda r^{\lambda-1} \begin{bmatrix} \alpha(\lambda+1)|\sin(\lambda+1)\theta| + \\ b(\lambda+1)|\cos(\lambda+1)\theta| + \\ +c(\lambda-3)|\sin(\lambda-1)\theta| + \\ d(\lambda-3)|\cos(\lambda-1)\theta| \end{bmatrix} \quad (3)$$

$$\sigma_\theta = \lambda(\lambda+1)r^{\lambda-1} \begin{bmatrix} \alpha|\sin(\lambda+1)\theta| + \\ b|\cos(\lambda+1)\theta| + \\ c|\sin(\lambda-1)\theta| + \\ d|\cos(\lambda-1)\theta| \end{bmatrix} \quad (4)$$

$$\tau_{r\theta} = -\lambda r^{\lambda-1} \begin{bmatrix} \alpha(\lambda+1)|\cos(\lambda+1)\theta| - \\ b(\lambda+1)|\sin(\lambda+1)\theta| + \\ c(\lambda-1)|\cos(\lambda-1)\theta| - \\ d(\lambda-1)|\sin(\lambda-1)\theta| \end{bmatrix} \quad (5)$$

3. Specimens

Specimens were made from Plexiglas (PMMA) with Poisson ratio $\nu = 0.34$ and Elasticity modulus $E = 3.4$ GPa and Lexan (BCBA) with $\nu = 0.36$ and $E = 2.8$ GPa, subjected to static uniaxial tension. The specimens thickness was $t = 0.003$ m. The crack width was 0.3mm with a sharp tip at the interface (Figure 1).

4. Theory of Photoelasticity

Isochromatic fringes are loci of points with the same

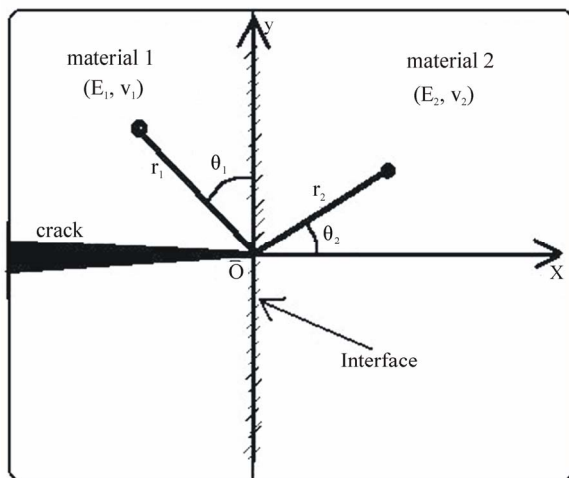


Figure 1. Geometry of bi-material plate

value for the difference of the principal stresses or the maximum shear stress. According to the stress optical law, the difference in the principal stresses is given by Frocht [21]:

$$\sigma_1 - \sigma_2 = 2\tau^{\max} = \frac{N_c f_c}{t} \quad (6)$$

where N_c is the isochromatic fringe order, t is the thickness of the plate and f_c is the material fringe value or stress-optical constant.

From equations (3)-(6) is obtained:

$$r = \left\{ \frac{N_c (f_c)_{1,2}}{\left[\left[(1-\lambda^2)F(\theta) + F''(\theta) \right]^2 + 4\lambda^2 (F'(\theta))^2 \right]^{1/2}} t \right\}^{\frac{1}{\lambda-1}} \quad (7)$$

The relation between the stress-optical constants of the materials 1 and 2 is:

$$(f_c)_2 = \frac{1}{E_{12}} \frac{\nu_1}{\nu_2} (f_c)_1 \quad (8)$$

where $E_{12} = E_1/E_2$ is the ratio of the elastic moduli and ν_1, ν_2 are the Poisson's ratio of the two materials.

5. Theory of Isopachic Fringes

Isopachics fringes are loci of points with the same value for the sum of the principal stresses. The fringe order N_p of isopachic is related to the sum of the principal stresses by:

$$\sigma_1 + \sigma_2 = \sigma_r + \sigma_\theta = \frac{N_p f_p}{t} \quad (9)$$

where N_p is the order of isopachics, t is the thickness of the plate and f_p is the isopachic fringe constant.

The sum of the stresses (equations (3) and (4)) is:

$$\sigma_1 + \sigma_2 = \sigma_r + \sigma_\theta = r^{\lambda-1} \left[(\lambda+1)^2 F(\theta) + F''(\theta) \right] \quad (10)$$

From equations (9) and (10) is obtained:

$$r = \left\{ \frac{1}{(\lambda+1)^2 F(\theta) + F''(\theta)} \frac{N_p (f_p)_{1,2}}{t} \right\}^{\frac{1}{\lambda-1}} \quad (11)$$

with:

$$(f_p)_2 = \frac{1}{E_{12}} \frac{\nu_1}{\nu_2} (f_p)_1 \quad (12)$$

where $(f_p)_{1,2}$ are the isopachic fringe constants of the materials 1 and 2 of the bi-material plate, respectively.

6. Principal Stresses Estimation from the Isochromatic and Isopachic Fringes

The principal stresses can be estimated from the system

of isochromatic and isopachic fringes (Eqs (6), (9)). The solution of the system is valid at the cross points of the isochromatic and isopachic fringes (**Figure 2**). **Figure 2** presents the overlapping of isochromatic and isopachic fringes by Eqs (7) and (11), for Lexan 1-Plexiglas 2 bi-material plate with $E_{12} = 0.82353$, $\nu_1 = 0.36$, $\nu_2 = 0.34$, $\lambda = 0.5192$, $t = 0.003$ and $d_2 = 1$ (d_2 is an arbitrary constant which represents the tensile load of the plate). At the cross points of the fringes the principal stresses can be calculated by the Eqs (6) and (9).

From the solution of the equations (6) and (9) system, the principal stresses are obtained:

$$\sigma_1 = \frac{\sigma_r + \sigma_\theta}{2} + \frac{1}{2} \sqrt{(\sigma_r - \sigma_\theta)^2 + 4\tau_{r\theta}^2} = \frac{N_p(f_p)_{1,2} + N_c(f_c)_{1,2}}{2t} \tag{13}$$

$$\sigma_2 = \frac{\sigma_r + \sigma_\theta}{2} - \frac{1}{2} \sqrt{(\sigma_r - \sigma_\theta)^2 + 4\tau_{r\theta}^2} = \frac{N_p(f_p)_{1,2} - N_c(f_c)_{1,2}}{2t} \tag{14}$$

By substituting the stresses from the Eqs (3)-(5) into Eqs (13), (14) the contour curves of the principal stresses, around the crack-tip, are obtained:

$$r_{\sigma_1} = \left\{ \frac{N_p(f_p)_{1,2} + N_c(f_c)_{1,2}}{t} \left[\frac{1}{(1+\lambda)^2 F(\theta) + F''(\theta)} + \frac{1}{\left[(1-\lambda^2)F(\theta) + F''(\theta) \right]^2 + 4\lambda^2(F'(\theta))^2} \right]^{1/2} \right\}^{\frac{1}{\lambda-1}} \tag{15}$$

$$r_{\sigma_2} = \left\{ \frac{N_p(f_p)_{1,2} - N_c(f_c)_{1,2}}{t} \left[\frac{1}{(1+\lambda)^2 F(\theta) + F''(\theta)} - \frac{1}{\left[(1-\lambda^2)F(\theta) + F''(\theta) \right]^2 + 4\lambda^2(F'(\theta))^2} \right]^{1/2} \right\}^{\frac{1}{\lambda-1}} \tag{16}$$

Figure 3 presents the contour curves of the principal stresses σ_1 and σ_2 around the crack-tip for Lexan 1-Plexiglas 2 bi-material plate with $E_{12}=0.82353$, $\nu_1 = 0.36$, $\nu_2 = 0.34$, $\lambda = 0.5192$, $d_2 = 1$ and $t = 0.003$ for (a) isopachic fringe order $N_p=1$ and isochromatic fringe orders $N_c = 3,4$ and (b) $N_p=5$ and $N_c = 3,4$.

7. Finite Element Analysis (FEA)

The program ANSYS 11 is used in the finite element analysis (FEA) modeling. The mesh is generated using triangular elements 8-node 82 for the crack and the crack-tip of the model (**Figure 4(a)**).

The elements have the material properties for Lexan

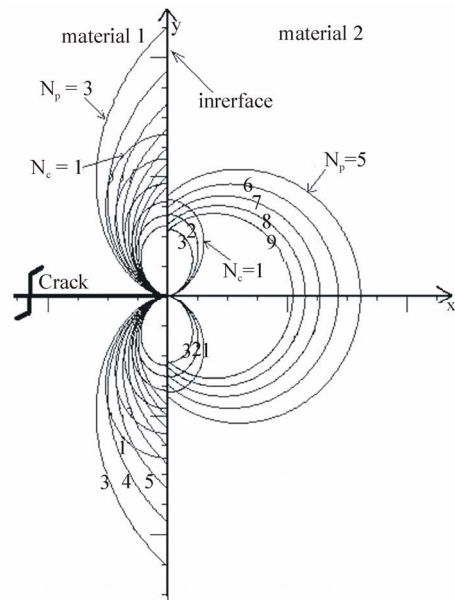


Figure 2. Theoretically overlapping of Isochromatic (N_c) and Isopachic (N_p) fringes.

$E = 2.8$ GPa and $\nu = 0.36$ and for Plexiglas $E=3.4$ GPa ($E_{12} = 0.82353$) and $\nu = 0.34$. **Figure 4(b)** shows the mesh at the crack-tip that had been used in the analysis. **Figure 4(c)** shows the FEA model mesh for the crack and the interface of bi-material plate. For the calculation of the stresses and its contour curves at the bi-material interface crack-tip, with Plexiglas as material 1 and Lexan as material 2 or with Lexan as material 1 and Plexiglas as material 2 (**Figure 1**), a tensile load of 100 N/m was applied.

Figure 5 shows the contour curves of principal stress σ_1 (Pa) at the bi-material interface crack-tip for Plexiglas-Lexan bi-material plate.

Figure 6 shows the contour curves of principal stress σ_2 (Pa) at the bi-material interface crack-tip for Plexiglas-Lexan bi-material plate.

Figure 7 shows the contour curves of principal stress σ_1 (Pa) at the bi-material interface crack-tip for Lexan-Plexiglas bi-material plate.

Figure 8 shows the contour curves of principal stress σ_2 (Pa) at the bi-material interface crack-tip for Lexan-Plexiglas bi-material plate.

The stresses variation in the direction of 45° relative to crack direction are presented in next figures. **Figure 9** shows the variation of principal stress σ_1 (Pa) and **Figure 10** shows the variation of principal stress σ_2 (Pa) at the bi-material crack-tip.

8. Results and Discussion

From **Figure 3** it is observed that the principal stresses σ_1 and σ_2 are rapidly increased approaching to the

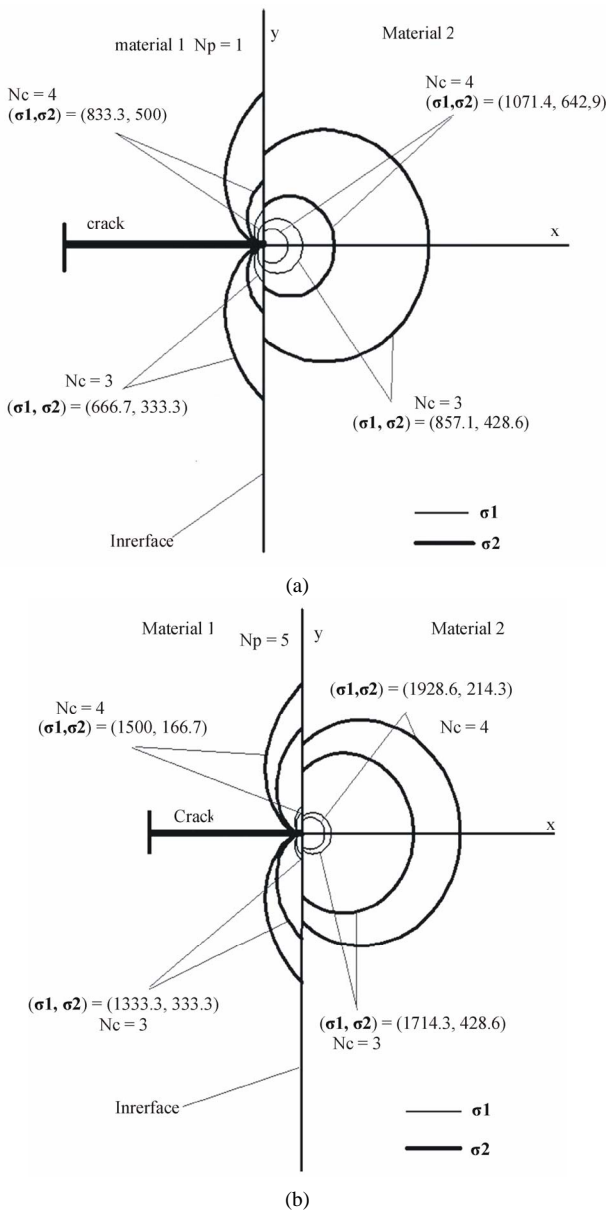


Figure 3. Contour curves of principal stresses around the crack-tip for (a) $N_p = 1$ and $N_c = 3,4$ and (b) $N_p = 5$ and $N_c = 3,4$.

crack-tip.

For isopachic fringe order $N_p = 1$ (Figure 3(a)) and for isochromatic fringe orders $N_c = 3$ and 4 the principal stresses in material 2 (Plexiglas) are $\sigma_1 = 857.1\text{Pa}$ and 1071.4Pa and $\sigma_2 = 428.6\text{Pa}$ and 642.9Pa , respectively. While, in material 1 (Lexan) the principal stresses are $\sigma_1 = 666.7\text{Pa}$ and 833.3Pa and $\sigma_2 = 333.3\text{Pa}$ and 500Pa , respectively. For isopachic fringe order $N_p = 5$ (Figure 3(b)), closer to the crack-tip than in state of Figure 3(a), and for isochromatic fringe orders $N_c = 3$ and 4 the principal stresses in material 2 (Plexiglas) are

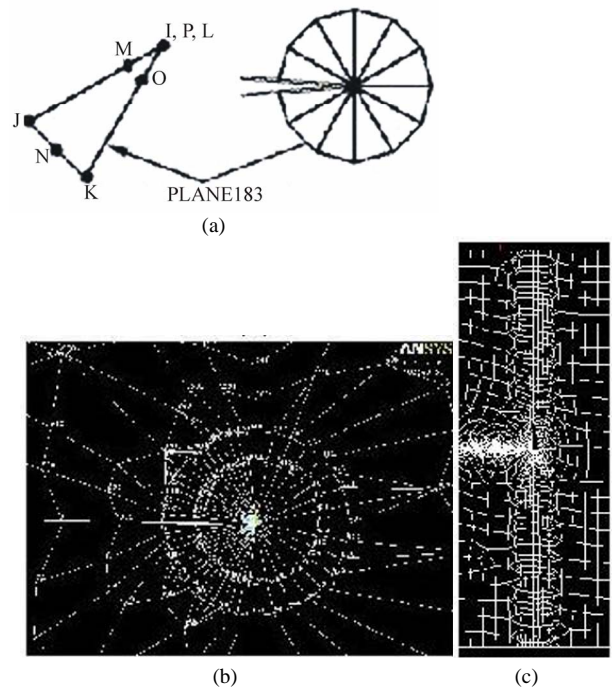


Figure 4. (a) Element 8-node 82, (b) FEA mesh at the crack tip, (c) Model FEA mesh.

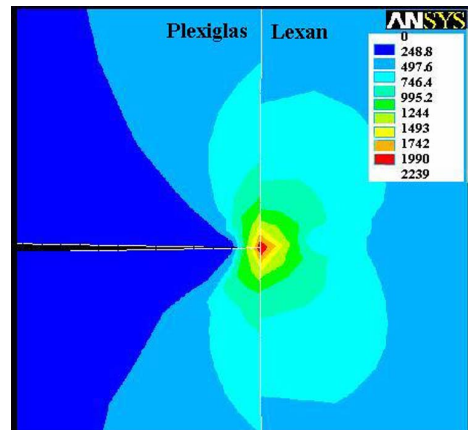


Figure 5. Contour of principal stress σ_1 (Pa) at the bi-material interface crack-tip tip for Plexiglas-Lexan bi-material plate.

$\sigma_1 = 1714.3\text{Pa}$ and 1928.6Pa and $\sigma_2 = 428.6\text{Pa}$ and 214.3Pa , respectively. While, in material 1 (Lexan) the principal stresses are $\sigma_1 = 1333.3\text{Pa}$ and 1500Pa and $\sigma_2 = 333.3\text{Pa}$ and 166.7Pa , respectively. At the interface a jump of principal stresses values is observed because of the difference constants of two materials. So, from the experimental combination of the isochromatic and isopachic fringes the contour curves of principal stresses around the crack-tip can taken.

Applying the FEA, the contour curves of principal stresses around the crack-tip can theoretically taken.

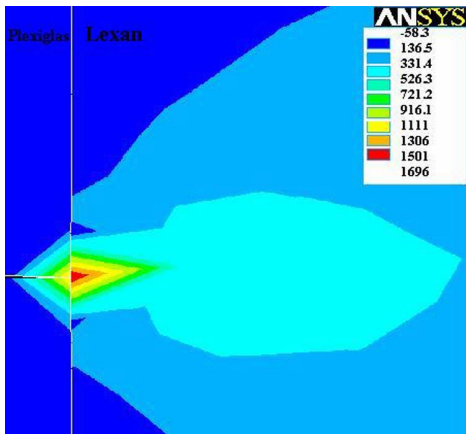


Figure 6. Contour of principal stress σ_2 (Pa) at the bi-material interface crack-tip tip for Plexiglas-Lexan bi-material plate.

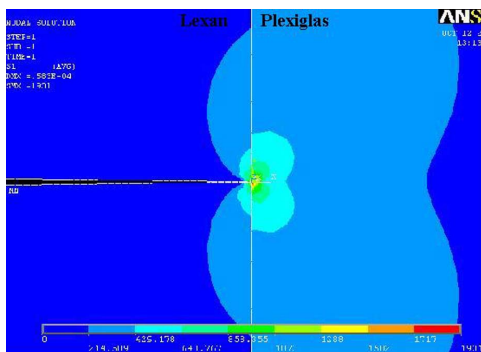


Figure 7. Contour of principal stress σ_1 (Pa) at the bi-material interface crack-tip tip for Lexan-Plexiglas bi-material plate.

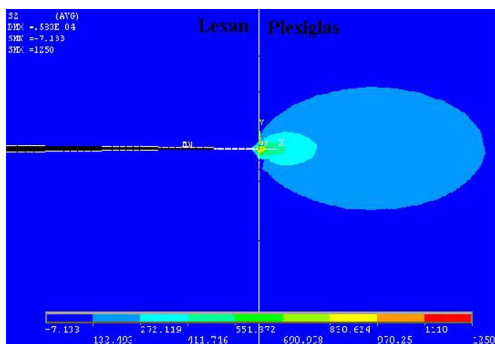


Figure 8. Contour of principal stress σ_2 (Pa) at the bi-material interface crack-tip for Lexan-Plexiglas bi-material plate.

From Figures 5-8 it is observed that the contour curves of principal stresses are zones with a average values of principal stresses. Approaching the crack-tip, the principal stresses σ_1 and σ_2 are rapidly increased. The variation of the average values of principal stresses σ_1 and σ_2 in front of the crack-tip in the direction of 45° relative to the

crack-axis are presented in Figures 9 and 10, respectively. Figure 11 presents the experimentally overlapping of isochromatic and isopachic fringes for $E_{12} = 1$ (one material, Lexan). At the cross points of the fringes the principal stresses can be calculated by the Eqs (6) and (9). Also, at crack tip the caustic was taken from which the stress intensity factor K_I can be calculated [22].

9. Conclusions

According to the above study is concluded that the distribution of the principal stresses close and far from the crack-tip can be experimentally considered by the methods of photoelasticity and isopachics. Also, the contour curves of principal stresses can be experimentally plotted

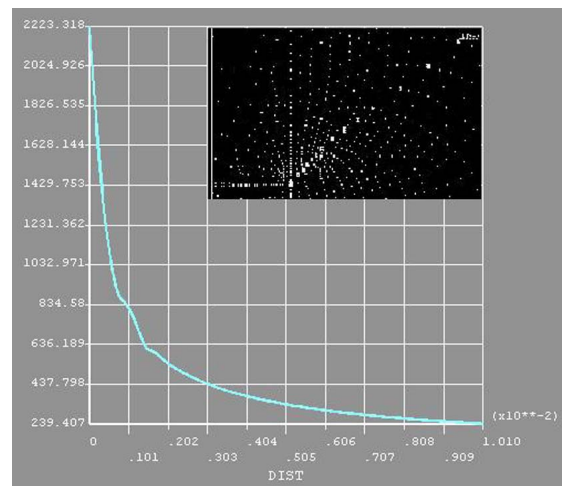


Figure 9. The variation of principal stress σ_1 (Pa) in the direction of 45° relative to crack direction (inset photo) at the bi-material interface crack-tip for Plexiglas-Lexan bi-material plate.

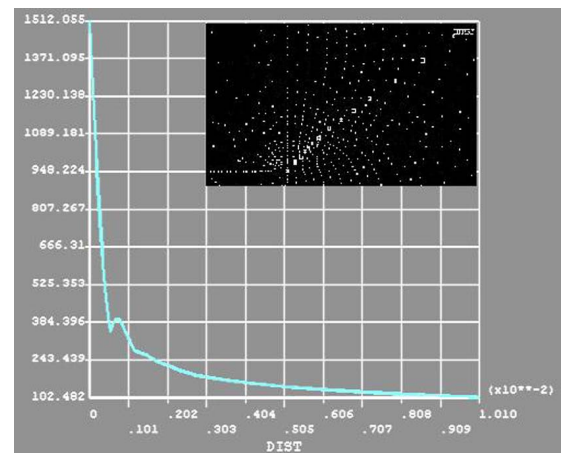


Figure 10. The variation of principal stress σ_2 (Pa) in the direction of 45° relative to crack direction (inset photo) at the bi-material interface crack-tip for Plexiglas-Lexan bi-material plate.

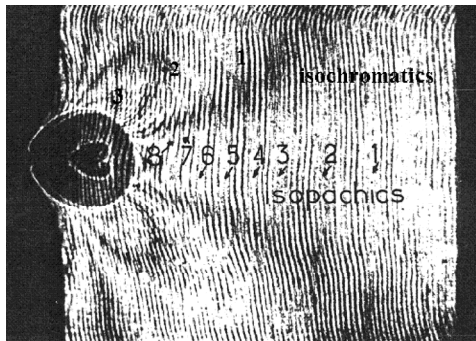


Figure 11. Experimentally overlapping Isochromatic and Isopachic fringes at the crack tip for one material (Lexan).

around the crack-tip at the interface of the bi-material from the overlapping of the isochromatic and isopachic fringes.

From the FEA analysis is concluded that: (a) The exactly contour curves of principal stresses can be theoretically estimated. (b) From the contour curves of stresses the variation of the principal stresses can be calculated in difference directions in front of bi-material crack-tip.

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