

# Thermomechanical Stress in the Evolution of Shear of Fiber-Matrix Interface Composite Material

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## ABSTRACT

*This work aims to describe the behavior of the interface using the method of load transfer between fiber and matrix in a composite material. Our contribution is to track the Evolution of the thermomechanical behavior by establishing a new mathematical model that describes the variation of shear stress along the interface. This model has been implemented in code in C<sup>++</sup>. The results revealed that the shear of the interface increases with temperature. This increase is partly due to the difference in expansion coefficient between fiber and matrix. The composite studied is T300/914; Carbon-Epoxy.*

**Keywords:** Interface, Fiber, Matrix, Thermal Expansion, Shear, Stress

## 1. Introduction

Composite materials with fiber reinforcements are used in structural applications where mechanical properties are essential. The charge transfer fiber-matrix is largely conditioned by the mechanical response of the interface. Unlike the constituent fiber and matrix, which may be a specification and be subject to specific controls, the interface escapes in part to the efforts of analysis and forecasting and may be the spot of concentration of defects what Bikerman called weak boundary layers [1]. Thanks to finite element analysis, Broutman and Agarwal [2] have confirmed the role of the interface, this study has been illustrated by the work of Théocaris [3], and the model of Adams [4].

For a single fiber surrounded by matrix, many analytical solutions have been proposed to evaluate the shear stress along a fiber, the Cox model [5] in the elastic case and the model of Kelly [6] in the case plastic. These depend of course the mechanical characteristics of the reinforcing fiber and matrix, but also how the stress is transmitted from the matrix to the fiber.

The purpose is to illustrate on simple cases, the mechanisms of charge transfer at fiber-matrix interface and show their impact on macroscopic mechanical properties of the composite is seen clearly in the work of Pig-

gott [7] and [8] On the other hand, the technique is well explained by Favre [9] and Amestoy [10].

To better understand the mechanical behavior of the interface we may refer to works of Berthelot [11] and J.Garrigues [12].

Our contribution has been to follow the evolution of the thermomechanical behavior by establishing a new mathematical model that describes the variation of the shear stress along the interface and viewed on a microscopic scale, the distribution of shear stresses in the fiber and interface based on thermomechanical properties of each component, their respective volume fraction of the fiber length renfortet especially difference expansion coefficients of the fiber and matrix. We became interested in two materials: the Peek/ APC2 and T300/914.

## 2. Development Model

### 2.1. Hypotheses

Consider a representative volume element RVE consisting of  $a$  fiber radius and length  $2L$  surrounded by a matrix cylinder of radius  $R$ . The fiber gives a volume

fraction with:  $V_f = \frac{a^2}{R^2}$ .

**The resolution by transfer stress method is:**

- Enter the equilibrium equations.

- Proposition a solution through the law of thermo-linear elasticity.
- Check the boundary conditions in effort.

## 2.2. Setting Equations

The load transfer between fiber and matrix operates in the vicinity of a discontinuity in the fiber or the matrix. This results in a stress gradient in the fiber is balanced by an interfacial shear  $\tau_i$ :

$$\frac{d\sigma_f}{dx} = -\frac{2\tau_i}{a} \quad (1)$$

A first we assume the behavior of the elastic matrix:

$$\tau = G_m \gamma \quad (2)$$

where  $\gamma = \gamma_{12}$  is the shear deformation,  $G_m$  the shear modulus of the matrix and  $\tau$  is the shear matrix. Let  $W$  be the matrix displacement along the direction of  $x$ ; One compatibility condition follows:

$$\frac{dW}{dr} = \varepsilon_{ij} = \varepsilon_{12} = \gamma \quad (3)$$

The balance of shear forces is written as:

$$\tau = \frac{a\tau_i}{r} \quad (4)$$

After integration of (3) on  $[a, R]$ , using (4):

$$\int_a^R dW = \frac{\tau_i a}{G_m} \int_a^R \frac{dr}{r}; \text{ then: } W_R - W_a = \frac{\tau_i}{G_m} a \ln(R/a)$$

We found the expression of known shear interface:

$$\tau_i = \frac{G_m}{a \ln\left(\frac{R}{a}\right)} (W_R - W_a) \quad (5)$$

The linear thermo elasticity gives:

$$\begin{cases} \frac{dW_R}{dx} = \varepsilon_m = \varepsilon_1 + \frac{\sigma_m}{E_m} + \Delta\theta\alpha_m & \text{if } r = R \\ \frac{dW_a}{dx} = \varepsilon_f = \frac{\sigma_f}{E_f} + \Delta\theta\alpha_f & \text{if } r = a \end{cases} \quad (6)$$

where  $\varepsilon$ ,  $E$ ,  $\alpha$ ,  $\Delta\theta$  are respectively the strain, Young's modulus, the coefficient of thermal expansion and the temperature difference.

The indices "f" and "m" spot sizes on either the fiber or the matrix, which allows describing the equilibrium thermo elastic system by the following differential equation:

$$\frac{d^2\sigma_f}{dx^2} = -\beta^2 \left[ \varepsilon_1 + \frac{\sigma_m}{E_m} + \frac{\sigma_f}{E_f} + (\alpha_m - \alpha_f)\Delta\theta \right] \quad (7)$$

$$\text{With: } \beta^2 = \left[ \frac{2G_m}{a^2 \ln\left(\frac{R}{a}\right)} \right]$$

and given the following equilibrium conditions [7]:

$$V_f \Delta\sigma_f + (1 - V_f) \Delta\sigma_m = 0 \quad (8)$$

It comes:

$$\frac{d^3\sigma_f}{dx^3} = \beta^2 \left[ \frac{V_f}{(1 - V_f)E_m} + \frac{1}{E_f} \right] \frac{d\sigma_f}{dx} \quad (9)$$

We assume:

$$n^2 = \beta^2 \left[ \frac{V_f}{(1 - V_f)E_m} + \frac{1}{E_f} \right]$$

The general solution of Equation (9) is of the form:

$$\sigma_f(x) = A \cosh(nx) + B \sinh(nx) + C \quad (10)$$

$$\sigma_m(x) = \frac{a^2}{R^2 - a^2} [A \cosh(nx) + B \sinh(nx) + D] \quad (11)$$

By using the boundary conditions  $\sigma_f = 0$  at the extremities of the fiber  $x = -L$  and  $x = L$ , we find after integration of the Equation (9), the value of coefficients  $A$ ,  $B$ ,  $C$  and  $D$ :

$$\begin{cases} A = 0 \\ B = -\left(\frac{\beta}{n}\right)^2 \frac{(\alpha_m - \alpha_f)\Delta\theta}{\cosh(nL)} \\ C = \left(\frac{\beta}{n}\right)^2 (\alpha_m - \alpha_f)\Delta\theta \\ D = \frac{a^2}{R^2 - a^2} \left(\frac{\beta}{n}\right)^2 (\alpha_m - \alpha_f)\Delta\theta \end{cases} \quad (12)$$

The general shape of the resulting stress:

$$\sigma_f(x) = \left[ \frac{(\alpha_m - \alpha_f)\Delta\theta + \varepsilon_1}{\frac{V_f}{(1 - V_f)E_m} + \frac{1}{E_f}} \right] \left( 1 - \frac{\cosh(nx)}{\cosh(nL)} \right) \quad (13)$$

## 2.3. Model Interface

The interface shear model in terms of the various parameters can be expressed as:

$$\tau_i(x) = \frac{a\beta^2}{2n} \left[ (\alpha_m - \alpha_f)\Delta\theta + \varepsilon_1 \right] \left( \frac{\sinh(nx)}{\sinh(nL)} \right) \quad (14)$$

After variable change:  $\begin{cases} x = \left(\frac{l}{2} - X\right); \\ l = 2L \end{cases}$ ; We have:

$$\tau_i(X) = \frac{a\beta^2}{2n} [(\alpha_m - \alpha_f)\Delta\theta + \varepsilon_1] \left[ \frac{\sinh n\left(\frac{l}{2} - X\right)}{\sinh\left(\frac{nl}{2}\right)} \right] \quad (15)$$

For  $X = 0$ ; The shear is maximal:

$$\tau_{\max} = \frac{a\beta^2}{2n} [(\alpha_m - \alpha_f)\Delta\theta + \varepsilon_1] \tanh\left(\frac{nl}{2}\right) \quad (16)$$

#### 2.4. Isothermal Case and Comparison with the Cox Model

To understand the shear model of the interface expressed by (16) It would be interesting to see the isothermal case by asking:  $\Delta\theta = 0$ .

It comes:

$$\tau_{1\max} = \varepsilon_1 \frac{a\beta^2}{2n} \tanh\left(\frac{nl}{2}\right). \quad (17)$$

It is at near constant the Cox model [6].

#### 2.5. Development of the Cox Model

Consider a representative volume element RVE consisting of  $a$  fiber radius and length  $2L$  surrounded by a matrix cylinder of radius  $R$  [9].

We apply the direction parallel to fibers (longitudinal direction) uniaxial traction  $\sigma = \sigma_{11} \neq 0$ . Every direction normal to the fibers is called transverse direction.

The law of elasticity applied to isotropic elastic material is written:

$$\overline{\overline{\sigma}} = \lambda \left( \text{trace } \overline{\overline{\varepsilon}} \right) \overline{\overline{\mathbf{1}}} + 2\mu \overline{\overline{\varepsilon}} \quad (18)$$

With  $\lambda, \mu$  Lamé constants and  $\overline{\overline{\sigma}}, \overline{\overline{\varepsilon}}$  are respectively the tensors of stress and strain Inversely:

$$\overline{\overline{\varepsilon}} = \frac{1+\nu}{E} \overline{\overline{\sigma}} - \frac{\nu}{E} \left( \text{trace } \overline{\overline{\sigma}} \right) \overline{\overline{\mathbf{1}}} \quad (19)$$

then: 
$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \frac{\sigma}{E} & 0 & 0 \\ 0 & \frac{-\sigma\nu}{E} & 0 \\ 0 & 0 & \frac{-\sigma\nu}{E} \end{bmatrix}$$

where:  $\nu = \frac{\lambda}{2(\lambda + \mu)}$  and  $E = E_f$ .

This explains the existence of a transverse strains where the shear stresses and in the matrix and interface respectively.

The method of load transfer between fiber and matrix [8] and [9], gives:

$$\tau_1(x) = \varepsilon \frac{E_f a}{2} \beta_1 \frac{\text{sh}\left[\beta_1\left(\frac{l}{2} - x\right)\right]}{\text{ch}\left(\frac{\beta_1 l}{2}\right)} \quad (20)$$

with: 
$$\beta_1^2 = \left( \frac{2G_m}{E_f a^2 \ln\left(\frac{R}{a}\right)} \right)$$

$$\tau_1(0) = \varepsilon \frac{a}{2} \beta_1 \cdot \text{th}\left(\frac{\beta_1 l}{2}\right) \quad (21)$$

A near constant, the two expressions shear (17) and (21) are the same.

### 3. Results and Discussion

We were interested at T300/914 carbon epoxy composite with known mechanical properties and a well defined fiber length; the variable parameters are the temperature and the volume fiber fraction, taking into account the considerable difference of thermal expansion coefficients of the fiber and matrix.

**Figure 1** shows the shearing of the interface corresponding to the thermomechanical model which we have accomplished in Subsection 2.3, while **Figure 2** and **Figure 3** represent the Cox model we developed in Subsection 2.5.

The **Figure 1** allows us to conclude and compare; shear increases with temperature and our model is consistent with Cox model.

The **Figure 2** and **Figure 3** indeed, in the Cox model [7] the shear varies with the deformation applied, we have shown it for the two different materials Peek/APC2 (**Figure 2**) and Carbone-epoxy T300/914 (**Figure 3**). We note that the shear strength of fiber-matrix interface is 4000 MPa at the extremity of Peek, while of carbon epoxy is 3500 MPa for a strain and for the same length of fiber. We find that our model describes the behavior of the interface; the comparison with the Cox model is the proof.

The **Figure 4** shows the influence of temperature on the stress for a fixed fraction at 10% and the **Figure 5** shows the influence of fiber volume fraction on the stress

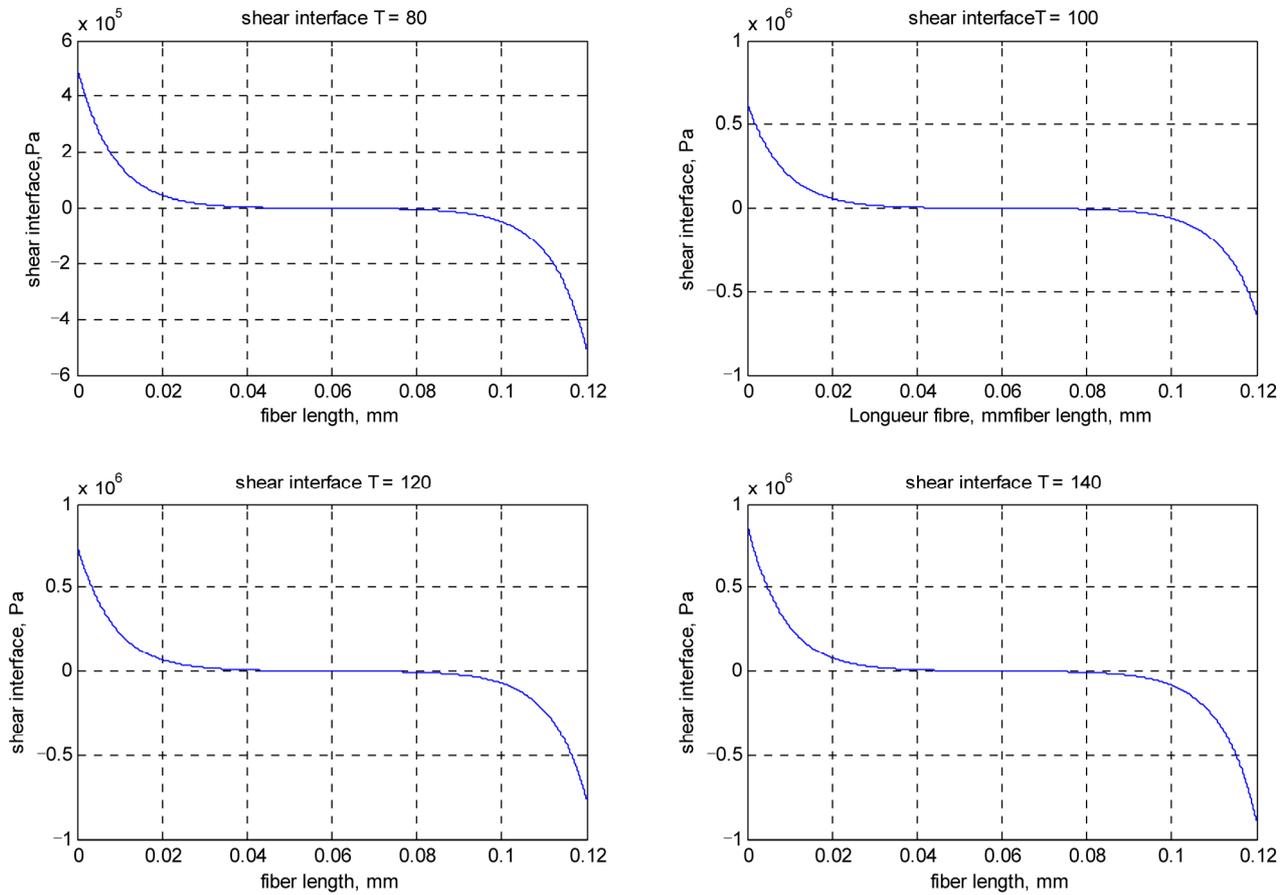


Figure 1. Influence of temperature on the shear interface.

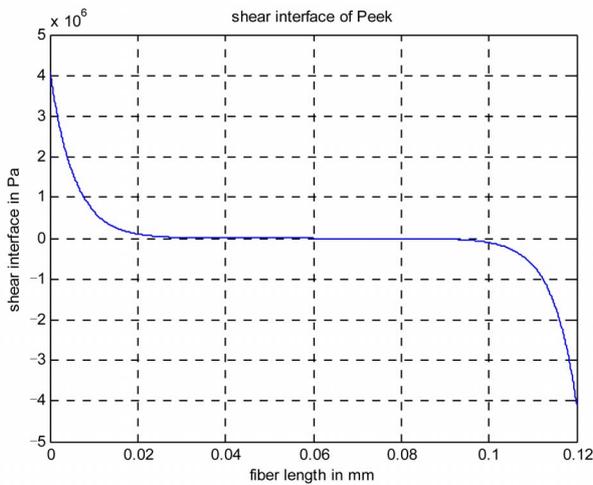


Figure 2. Shear interface of Peek.

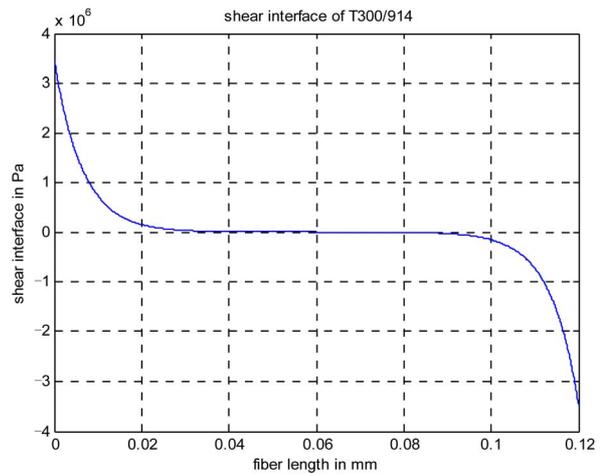


Figure 3. Shear interface of T300/914.

for a fixed temperature to 140°.

**4. Conclusions**

It is well known that the composite mechanical behavior

strongly depends on the fiber-matrix interface. This interface is accessible only indirectly through the behavior it engenders in the composite or those attributed to it.

The mechanical behavior of the interface depends on

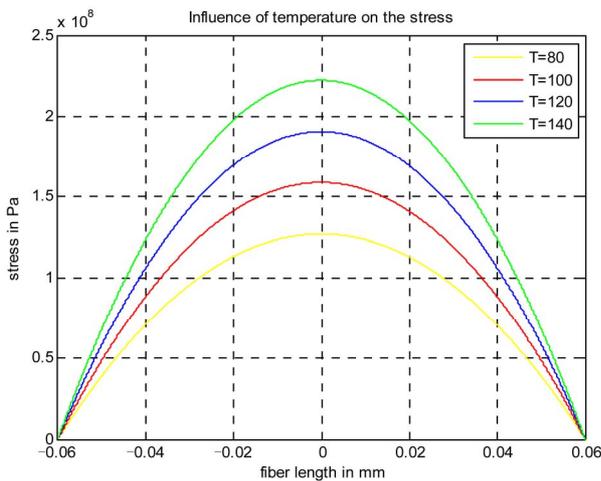


Figure 4. Influence of temperature on the stress.

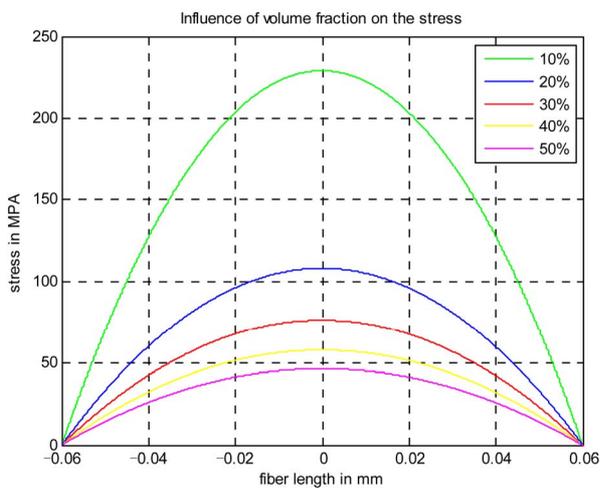


Figure 5. Influence of volume fraction on the stress.

several parameters of their components, fiber and matrix.

We found that there is a greater influence of the temperature on the fiber-matrix interface behavior. We believe that the volumic fraction of reinforcement has a greater contribution. This work shows that the percentage and the fiber type must be defined as they play a major role in the interface behavior of composite structures.

## REFERENCES

- [1] J. Bickerman, "The Science of Adhesive Joints," Academic Press, Inc, New York & London, 1968, p. 258.
- [2] L. J. Broutman and B. D. Agarwal, "Theoretical Study of the Effect of an Interfacial Layer on the Properties of Polymer," *Engineering & Science*, Vol. 14, No. 8, 1974, pp. 581-588.
- [3] P. S. Theocaris, "The Unfolding Model for the Representation of the Mesophase Layer in Composites," *Journal of Applied Polymer Science*, Vol. 30, No. 2, 1985, pp. 621-645. [doi:10.1002/app.1985.070300214](https://doi.org/10.1002/app.1985.070300214)
- [4] D. F. Adams, "Micromechanical Predictions/Experimental Correlations of the Influence of the Interface on the Mechanical and Physical Properties of a Unidirectional Composite," *Composite Interfaces*, Cleveland, 27-30 May 1986, pp. 351-365.
- [5] H. L. Cox, "The Elasticity and Strength of Paper and Other Fibrous Materials," *British Journal of Applied Physics*, Vol. 3, No. 3, 1952, pp. 72-79. [doi:10.1088/0508-3443/3/3/302](https://doi.org/10.1088/0508-3443/3/3/302)
- [6] A. Kelly and W. R. Tyson, "Fiber Strengthened Materials," In: V. F. Zackay, Ed., *High Strength Materials*, John Wiley & Sons, London, 1964, pp. 578-602.
- [7] M. R. Piggott, "How the Interface Controls the Properties of Fibre Composites," *Progress in Science and Engineering of Composites: Proceedings of the 4th International Conference on Composite Materials*, Tokyo, 25-28 October 1982, pp. 193-202.
- [8] M. R. Piggott, "Debonding and Friction at Fibre-Polymer Interfaces. I: Criteria for Failure and Sliding," *Composites Science and Technology*, Vol. 30, No. 4, 1987, pp. 295-306. [doi:10.1016/0266-3538\(87\)90017-0](https://doi.org/10.1016/0266-3538(87)90017-0)
- [9] J. P. Favre, "Interface dans les composites fibreux," *Technique de l'Ingenieur, Traité Plastiques et Composites*, Vol. AM6, No. A7765, Article ID A7765, pp. 1-21.
- [10] M. Amestoy, "Introduction à la Mécanique des Milieu Déformables," Ecole des mines de Paris, Paris, 1995.
- [11] J. M. Berthelot, "Matériaux Composites, Comportement Mécanique et Analyse des Structures," Hermès-Lavoisier, Paris, June 2005.
- [12] J. Garrigues, "Fondements de la Mécanique des Milieux Continus," Lavoisier, Paris, 2007.