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Analysis of Plate Vibration under Exponentially Varying Thermal Condition

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Abstract

A mathematical model is developed to assist the design engineers by analyzing the vibration response of non-homogeneous orthotropic rectangular plate under exponentially varying thermal condition. Plate thickness varies parabolically in both directions. Using Rayleigh Ritz approach, frequency parameter and two term deflection function is calculated for diverse values of taper constants. For the best comprehension of the vibration analysis, results are depicted graphically.

Keywords: Vibration, Non-Homogeneous, Plate, Thickness, Deflection Function, Taper Constants

1. Introduction

Study of vibration responses of plate has always been a principal concern for design engineers. These plates are used in numerous industries for the construction of innumerable vital structures and devices, such as space shuttles, rockets, air craft's, shafts, plate heat exchanger and many more. Satellite antenna booms are used in space as electric field or gravity-gradient probes. Heated by sunlight, a temperature gradient is built up across the cross-section. The thermal strain produces bending and torsion of the boom. Due to the twist the direction of the sunlight changes with respect to the cross-section and another temperature distribution is obtained which again causes another deformation. Similarly, a serious problem in mechanical design of heat exchanger is flow induced vibration.

Certain structures are less vulnerable against vibration impacts whereas certain other are more vulnerable. As we all know that vibration effects are now cannot be neglected, as our day to day life is affected by them; From kitchen to exercise centre, vibration effect are experienced. The only thing is we do not put an eye on them deliberately. Juicer, Mixer, massager, WholeBody *Vibration Plate* (*for fitness*), *all encompasses vibration effect.* Those were few inevitable and positive aspects of vibration. Controlled vibrations are utilized in health industry, paper industry, structural engineering, and aeronautical engineering and in many more industries.

But uncontrolled vibration causes devastation. Occurrences of Tsunami, earthquake, collapse of structures are few such most common devastating effects of vibration. Thus the study of vibration responses in advance is of immense importance for sustainable and positive effects of vibrations for the well being of humans.

Monograph written by Leissa [1] is ample source of knowledge in the field of plate vibration. Leissa [2] provides abundant knowledge about the numerous complicating effects that can be introduced in the analysis of plate vibration. Tomar and Gupta [3] evaluated the exponential thermal gradient effect on the vibration of orthotropic rectangular plate with variation in thickness. Rahimi & Davoodinik [4] studied the thermal behavior of functionally graded plates under the exponential and hyperbolic temperature conditions. They concluded that temperature distribution profile plays vital role in thermal resultant distribution of stresses and strains for FGP.

Shang, Wang and Li [5] analyzed the deformation of laminated plates under exponential distributions of temperature through the thickness. The plate under consideration is simply supported. Javaheri & Eslami [6] used classical plate theory for the buckling analysis of functionally graded plates under four different types of thermal load.Gupta, Johri & Vats [7] studied thermally induced vibrations of an orthotropic rectangular plate using Rayleigh Ritz approach. Gupta, Johri & Vats [8] calculated deflection function and frequency parameter for a rectangular plate under the effect of linear temperature distribution where thickness of plate was varied parabolically in both directions.

In the present scenario design engineers are indulge in making more efficient, vibration deficient and light weight structures.

Present study is truly devoted for design engineers utilizing rectangular plates for construction of devices or structures. The effect of exponentially varying temperature distribution is analyzed for a non- homogeneous orthotropic rectangular plate whose thickness varies bi-directionally in parabolic manner. The non-homogeneity is assumed to arise due to the variation in the density of the plate material in linear manner along the length of the plate. The frequencies and deflection function for first mode of vibration are calculated using Rayleigh Ritz technique, for clamped plate, for diverse values of non-homogeneity constant, taper constants and temperature gradient. Results are demonstrated graphically.

Authentication of work is done by comparing the results for a uniform unheated homogeneous orthotropic clamped rectangular plate with the results published by the authors [3]. Results are found to be in good agreement with those of published by Tomar & Gupta [3].

2. Methodology

Consider an orthotropic rectangular plate. Let us assume that complicating effects are introduced in the plate by density, thickness and thermal conditions.

Let the plate be subjected to an exponential thermal variation along X-axis only, *i.e.*

$$T = T_0\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right) \tag{1}$$

where T is the temperature excess above the reference temperature at a distance x/a and T_0 is the temperature excess above the reference temperature at the end of the plate *i.e.* at $x = a$, where a is length of plate.

Thickness h of the plate is assumed to be varying parabolically in both directions, *i.e.*

$$h = h_0\left(1+\beta_1\frac{x^2}{a^2}\right)\left(1+\beta_2\frac{y^2}{b^2}\right) \tag{2}$$

where, $h_0 = h|_{x=0,\,y=0}$ and β_1 & β_2 are two taper constants.

Non- homogeneity or variation in density ρ is assumed to varying linearly along X-axis, *i.e.*,

$$\rho = \rho_0\left(1+\alpha_1 x/a\right) \tag{3}$$

where, $\rho_0 = \rho|_{x=0}$ & α_1 is the non-homogeneity parameter.

For most orthotropic materials, moduli of elasticity (as a function of temperature) are defined as [8],

$$\begin{aligned} E_x(T) &= E_1(1-\gamma T)\\ E_y(T) &= E_2(1-\gamma T)\\ G_{xy}(T) &= G_0(1-\gamma T)\end{aligned} \tag{4}$$

where, E_x and E_y are Young's moduli in x- and y-directions respectively and G_{xy} is shear modulus, γ is Slope of variation of moduli with temperature. Using Equation (1) in Equation (4), one has,

$$\begin{aligned} E_x(T) &= E_1\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right]\\ E_y(T) &= E_2\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right]\\ G_{xy}(T) &= G_0\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right]\end{aligned} \tag{5}$$

The governing differential equation of transverse motion of an orthotropic rectangular plate of variable thickness in Cartesian coordinate [3], is;

$$\begin{aligned} &D_x\frac{\partial^4 w}{\partial x^4}+D_y\frac{\partial^4 w}{\partial y^4}+2H\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+2\frac{\partial H_x}{\partial x}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2}\\ &+2\frac{\partial H_y}{\partial y}\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial y}+2\frac{\partial D_x}{\partial x}\frac{\partial^3 w}{\partial x^3}+2\frac{\partial D_y}{\partial y}\frac{\partial^3 w}{\partial y^3}+\frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 w}{\partial x^2}\\ &+\frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+4\frac{\partial^2 D_{xy}}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\\ &+\rho h\frac{\partial^2 w}{\partial t^2}=0\end{aligned} \tag{6}$$

where w is transverse deflection of plate, at the point (x, y), ρ is mass density per unit volume of the plate material, t is time, h is thickness of the plate at the point (x, y), D_x & D_y are flexural rigidities in x- and y-directions respectively and D_{xy} is torsional rigidity [3],

$$D_x = \frac{E_x h^3}{12\left(1-\nu_x\nu_y\right)},\quad D_y = \frac{E_y h^3}{12\left(1-\nu_x\nu_y\right)},\quad D_{xy} = \frac{G_{xy}h^3}{12} \tag{7}$$

$D_1 = \nu_x D_y (=\nu_y D_x)$ and $H = D_1 + 2D_{xy}$, where ν_x & ν_y are Poisson's ratio.

Assuming time harmonic motion, solution of Equation (6), may be written as,

$$w(x,y,t) = W(x,y)e^{i\omega t} \tag{8}$$

where, w is frequency in radian and $W(x, y)$ is a two term deflection function.

For Clamped rectangular plate two term deflection function is expressed as,

$$W(x,y)=\begin{bmatrix} c_1\left(\frac{x}{a}\right)^2\left(\frac{y}{b}\right)^2\left(1-\frac{x}{a}\right)^2\left(1-\frac{y}{b}\right)^2 \\ +c_2\left(\frac{x}{a}\right)^3\left(\frac{y}{b}\right)^3\left(1-\frac{x}{a}\right)^3\left(1-\frac{y}{b}\right)^3 \end{bmatrix} \quad (9)$$

where, c_1 and c_2 are constants to be evaluated.

For a clamped plate, boundary conditions are,

$$W=\frac{\partial W}{\partial x}=0 \quad \text{at} \quad x=0,a$$

$$W=\frac{\partial W}{\partial y}=0 \quad \text{at} \quad y=0,b$$

In order to calculate the frequency ω, Rayleigh Ritz Technique is employed which states that maximum strain energy must be equal to maximum kinetic energy, *i.e.*

$$\delta(U-T)=0 \quad (10)$$

where, U is strain energy and T is kinetic energy for a plate executing transverse vibrations of mode shape $W(x, y)$, and are written as [3], respectively,

$$U=\frac{1}{2}\int_0^a\int_0^b \begin{bmatrix} D_x\left(\frac{\partial^2 W}{\partial x^2}\right)^2+D_y\left(\frac{\partial^2 W}{\partial y^2}\right)^2 \\ +2D_1\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right)+4D_{xy}\left(\frac{\partial^2 W}{\partial x\partial y}\right)^2 \end{bmatrix} dydx \quad (11)$$

$$T=\frac{1}{2}p^2\int_0^q\int_0^b \rho h W^2 \mathrm{d}y\mathrm{d}x \quad (12)$$

Using Equations (2), (3), (6), (7) and (9) in Equations (11) and (12) and then putting these values of U & T in Equation (10), one has,

$$\delta\left(U_1-\Omega^2 T_1\right)=0 \quad (13)$$

where, $\Omega^2=\frac{12a^4\rho_0\omega^2\left(1-v_x v_y\right)}{E_1 h_0^2}$ is the frequency parameter. Equation (13) contains two unknown constants c_1 and c_2 to be evaluated. Employing the following method, these constants may be evaluated:

$$\frac{\partial}{\partial c_k}\left(U_1-\Omega^2 T_1\right)=0 \quad (14)$$

where $k = 1,2$

On simplifying Equation (14), we get following form,

$$r_{k1}c_1+r_{k2}c_2=0 \quad (15)$$

where, r_{k1} & r_{k2} involves the parametric constants and the frequency parameter.

For a non-zero solution, determinant of coefficients of Equation (15) must vanish. In this way frequency equation comes out to be, as below,

$$\begin{vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{vmatrix}=0 \quad (16)$$

3. Result

Frequency Equation (16) provides the value of frequency parameter and deflection function for the first two modes of vibration for different values of taper constants, thermal gradient parameter and non-homogeneity constant, for a clamped plate with linear variation in thickness in both directions. Limitation of method used lies in the consideration of only first mode of vibration [3].

The parameter for orthotropic material has been taken as [8],

$$\frac{E_2}{E_1}=0.32\,,\quad v_x\frac{E_2}{E_1}=0.04\,,\quad \frac{G_0}{E_1}\left(1-v_x v_y\right)=0.09$$

Results are displayed graphically. **Figure 1** depicts the variation of frequency parameter Ω with the thermal gradient parameter 'α' for the following two cases: $\alpha_1 = 0.0, \beta_1 = 0.0, \beta_2 = 0.0$ and $\alpha_1 = 0.0, \beta_1 = 0.2, \beta_2 = 0.6$.

In **Figure 2**, Variation in frequency parameter with non-homogeneity of the plate material is taken into consideration for the following two cases:
$\alpha = 0.0, \beta_1 = 0.0, \beta_2 = 0.0$ and $\alpha = 0.0, \beta_1 = 0.2, \beta_2 = 0.6$

Figures 3 and **4** display the variation of taper constant 'β_1' and 'β_2' with frequency parameter 'Ω', respectively, for the following cases:

$\alpha_1 = 0.0, \alpha = 0.0, \beta_2$ or $\beta_1 = 0.0$
$\alpha_1 = 0.0, \alpha = 0.0, \beta_2$ or $\beta_1 = 0.6$
$\alpha_1 = 0.0, \alpha = 0.4, \beta_2$ or $\beta_1 = 0.0$
$\alpha_1 = 0.0, \alpha = 0.4, \beta_2$ or $\beta_1 = 0.6$
$\alpha_1 = 0.8, \alpha = 0.0, \beta_2$ or $\beta_1 = 0.0$
$\alpha_1 = 0.8, \alpha = 0.0, \beta_2$ or $\beta_1 = 0.6$
$\alpha_1 = 0.8, \alpha = 0.4, \beta_2$ or $\beta_1 = 0.0$
$\alpha_1 = 0.8, \alpha = 0.4, \beta_2$ or $\beta_1 = 0.6$

Figure 5 displays the variation of deflection function W with X for the following cases:

$\alpha_1 = 0.0, \alpha = 0.0, \beta_1 = 0.0, \beta_2=0.0, a/b = 1.5$ for $Y = 0.2$ and 0.4

$\alpha_1 = 0.8, \alpha = 0.4, \beta_1 = 0.2, \beta_2 = 0.6, a/b = 1.5$ for $Y = 0.2$ and 0.4

4. Conclusions

From the above results it is seen that the frequency of vibration reduces on increasing thermal gradient and non-homogeneity, whereas increase in taper constants increases the frequency of vibration. A comparative study was carried out for the plates regarding variation in thickness under exponential temperature gradient *i.e.*

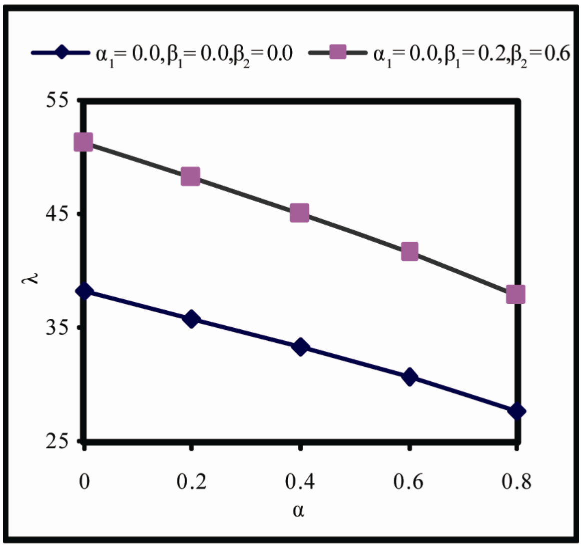

Figure 1. Frequency parameter ‘λ’ Vs. ‘α’.

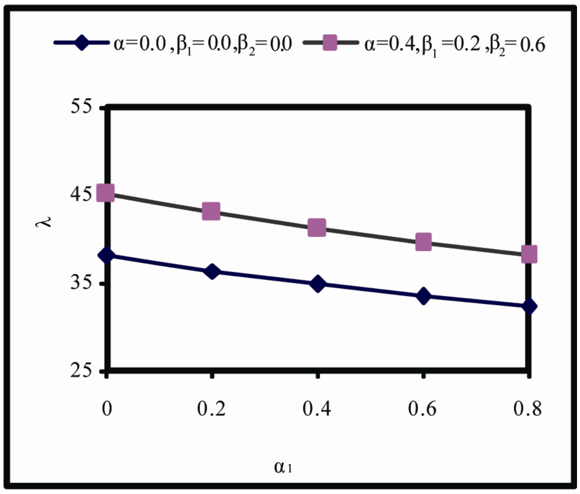

Figure 2. Frequency parameter ‘λ’ Vs. ‘α1’.

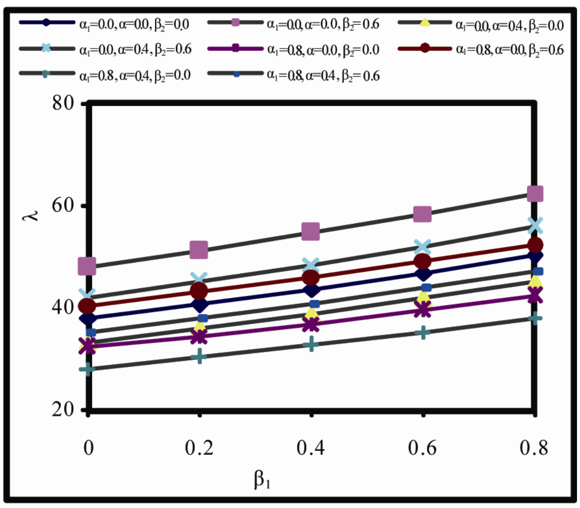

Figure 3. ‘λ’ Vs. taper constant ‘β_1’.

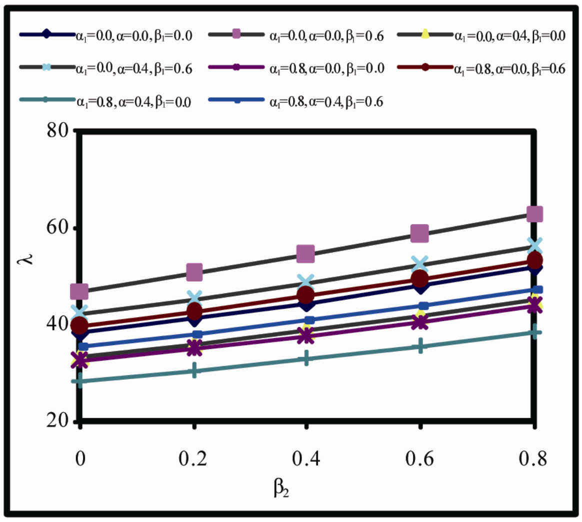

Figure 4. ‘λ’ Vs. taper constant ‘β_2’.

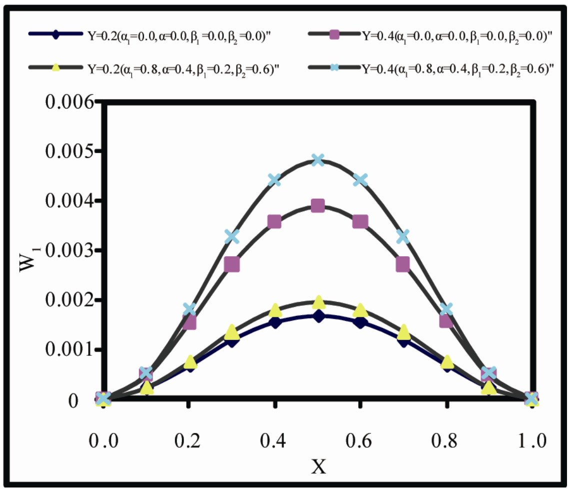

Figure 5. Deflection Vs. X (= *x*/*a*).

plates with linear and parabolic variations in thickness were compared. It was found that vibration effects were significantly less pronounced (lesser values of frequency parameter) for plates having parabolic bi-directional variation in thickness as compared to that of linear bi-directional variation in thickness. Hence it is concluded that plates with parabolic variation in thickness are more stable as compared to those of linearly varying thickness, for bearing up of exponential thermal gradient effects. Yet it was well thought-out that as compared to exponential variation in thermal gradient, linear variation in temperature is better. Hence parabolic bi-directional variation in thickness under linear temperature distribution is a nice combination of conditions for the bearing up of vibration effects, till the further considerations.

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