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Analysis on Impact of Individual's Bid Propensity on Optimal Bidding of "Naive-Rational" Bidders

Yiqing Tang[1], Weijie Yuan[1*], Cuihong Luo[2]

[1]School of Economics and Management, Southwest Jiaotong University, Chengdu, China
[2]Foreign Language Department, Chengdu Textile College, Chengdu, China
Email: *jingzheywj@163.com

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Abstract

This paper considers the emotional factors of the naïve and rational bidder in the auctions, and introduces it in the bidder's utility function. We study the impact of individual's bid propensity on optimal bidding in the different case of "naive-rational" bidders' value judgments by establishing the naive and rational bidder's comprehensive utility function including monetary utility and emotional utility. Further, we analyze who is the last winner in the condition of difference of individual's bid propensity.

Keywords

Naive, Rational, Emotional Utility, Monetary Utility, Judge Value, Individual's Bid Propensity

1. Introduction

Auction, a special mode of commodity trading in human society, a mature market trading system under the market economy, is closely related to competitive markets. In the period of Chinese economy transition, it is of great practical significance to study the auction problem. There have been a lot of researches on the auction problem. The research of auction problem in western countries mainly includes these aspects: research on auction mechanism and auction principle design in auction market [1] [2] [3] [4] research on the discrimination and strategy of public information and private information of bidders, in the case of incomplete information in auction market [5] [6] [7] [8] research on the bidding behavior of bidders based on psychological factors. In Chinese, much of the earlier work mainly focused on the types of auction, the establishment of auction mechanism and its principles and so on. Few work focused on the bidders' bidding behavior through information recognition. George and Richard [1] studied the income of Naive-Rational bidders and the stability of the auction market. Ding and Jehoshua studied the optimal bidding strategy of bidders in reverse auction market, considering the emotional factors of the bidders. Based on these two literatures, this paper studies the impact of individual's bid propensity on optimal bidding of Naive-Rational bidder. Besides, this paper also shows the type of bidder who eventually wins the auction because of the difference in the bid propensity of the bidders.

2. Utility Analysis of Naive Bidder and Rational Bidder

2.1. The Characteristics of the Naive Bidder and the Rational Bidder

In the auction market,because of overconfidence [1], the naive bidder does not analyze the information of his opponent's bid and considers naively. He always thinks his judging in the value of the auction item is correct and other bidders judge the value of the item the same as he does. In this case, the naive bidder's bidding strategy relies entirely on his own judgment of the value of the item. The rational bidder, however, is rational and never overconfident. He can judge the naïve bidder's valuation of the auction, which is not necessarily equal to judgment value of naïve bidder himself, through the naïve bidder's bid. Then he integrate his own valuation and analysis of the opponent's judgment on the value of the auction to choose his own bidding strategy. In this case, the rational bidder's bidding strategy depends on his own and analysis of the opponent's valuation of the auction item.

2.2. Utility Analysis of Bidders

It has been shown that human emotion has a certain influence on human behavior. Therefore, in the auction market, bidders' emotions would have a certain influence on their bidding behaviors. In this paper, we consider two types of emotion, sense of excitement and sense of frustration. At auction, the excitement caused by the bidder's expectation of bidding success is generally positive, and the frustration caused by the bidder's expectation of bidding failure is generally negative. These two emotional effects play a common role in the bidding process instead of playing a single role. Their combination effect would affect the bidding level of bidders. In this paper, we establish a comprehensive utility model of bidders to analyze the impact of individual's bid propensity on optimal bidding of naïve bidders and rational bidders and what type of bidder is most likely to win the auction.

In an auction, the excitement or frustration of a bidder would lead to a certain utility. According to the prospect theory, there is a reference point for the evaluation of the utility. In this paper, we choose the expect price of the bidder as the reference point, which is set as the expect bid of the bidder according to the judgment value of the auction product before the bidding. Because the success or failure of the bidding would lead to different level of emotional utilities, the emotional utility functions of excitement and frustration of bidders are given by

$$u_{i,a}(p_i)=\begin{cases}\alpha\dfrac{s_i-p_i}{p_{i,0}}, & p_i>p_j\\ 0, & \text{otherwise},\end{cases}\quad u_{i,b}(p_i)=\begin{cases}\beta\dfrac{p_i}{p_{i,0}}, & p_i>p_j\\ 0, & \text{otherwise},\end{cases}$$

$i=n,r$ (n refers to naïve bidder, r refers to rational bidder), $i\neq j$, when i refers to the naïve bidder, j refers to the rational bidder. p_i is the bid of the bidder i. $p_{i,0}$ is the expect price of the bidder i. s_i is the judgment value of the bidder i on the auction. α is the parameter of excitement reaction, which shows the bidders' sensitivity to success. β is the parameter of frustration reaction, which shows the bidders' sensitivity to failure.

Besides, another part of the bidder's utility is monetary utility, which consists of the bidder's judgment on the value of the auction and the bid to win the auction. It's given by

$$u_{i,m}=\theta(s_i-p_i),\ p_i>p_j$$

θ is the monetary utility coefficient of bidders.

2.3. Utility Model of Bidders

Hypothesis:

i) Set the naïve bidder's judgment value of auction as s_n, and s_r for the rational bidder.

ii) According to the characteristics of bidders, set the bid of naïve bidder as $p_n\in[0,s_n]$. For rational bidder, set his judgment of the naïve bidder's valuation of the auction item as s_n', so his bid would be $p_r \in [0, f_r(s_r,s_n')$ [1]].

iii) The bid of both the naïve and the rational are uniformly distributed, and the density function is $f(p_i)$. Therefore, the probability density functions of bid for the naïve bidder and the rational bidder are given by

$$f(p_n)=\begin{cases}\dfrac{1}{s_n}, & 0\le p_n\le s_n\\ 0, & \text{otherwise}\end{cases},\quad f(p_r)=\begin{cases}\dfrac{1}{f_r(s_r,s_n')}, & 0\le p_n\le f_r(s_r,s_n')\\ 0, & \text{otherwise}\end{cases}$$

The utility function of the bidder i is $U_i=u_{i,m}+u_{i,a}-u_{i,b}$, that is

$$U_i=\int_0^{p_i}u_{i,m}(p_i)f(p_j)\mathrm{d}p_j+\int_0^{p_i}u_{i,a}(p_i)f(p_j)\mathrm{d}p_j-\int_{p_i}^{s_{j,\max}}u_{i,b}f(p_j)\mathrm{d}p_j \quad (1)$$

When i stands for the naïve bidder, $s_{j,\max}$ stands for $f_r(s_r,s_n')$, the superior limit of the rational bidder's bid price. When i stands for the rational bidder, $s_{j,\max}$ stands for s_n, the superior limit of the naive bidder's bid price.

The next is an analysis of the best bid for the naive and the rational. According to (1), the utility function of the naïve bidder:

$$U_n=\int_0^{p_n}\theta(s_n-p_n)f(p_r)\mathrm{d}p_r+\int_0^{p_n}\alpha\left(\frac{s_n-p_n}{p_{n,0}}\right)f(p_r)\mathrm{d}p_r-\int_{p_n}^{f_r(s_r,s_n')}\beta\frac{p_n}{p_{n,0}}f(p_r)\mathrm{d}p_r$$

Substituting the first-order condition of optimization $\dfrac{\mathrm{d}U_n}{\mathrm{d}p_n}=0$ into this, the optimal bidding strategy for the naïve bidder is

[1] $f_r(s_r,s_n')$ may be other expressions of s_r and s_n', such as weights or products.

$$p_n^* = \frac{s_n\left(\theta p_{n,0} + \alpha\right) - \beta f_r\left(s_r, s_n'\right)}{2\left(\theta p_{n,0} + \alpha - \beta\right)} \tag{2}$$

According to (1), the utility function of the rational bidder:

$$U_r = \int_0^{p_r} \theta\left(\frac{s_r}{2} - p_r\right) f\left(p_n\right) \mathrm{d}p_n + \int_0^{p_r} \alpha\left(\frac{s_r - p_r}{p_{r,0}}\right) f\left(p_n\right) \mathrm{d}p_n - \int_{p_r}^{s_n} \beta \frac{p_r}{p_{r,0}} f\left(p_n\right) \mathrm{d}p_n$$

Substituting the first-order condition of optimization $\frac{\mathrm{d}U_r}{\mathrm{d}p_r} = 0$ into this, the optimal bidding strategy for the rational bidder is

$$p_r^* = \frac{s_r\left(\theta p_{r,0} + \alpha\right) - \beta s_n}{2\left[\left(\theta p_{r,0} + \alpha\right) - \beta\right]} \tag{3}$$

3. Analysis on Impact of People's Bid Propensity on Optimal Bidding Strategy

According to (2) and (3), the factors influencing the optimal bidding strategy for both the naive and the rational include: the judgment value, reserve price, monetary utility coefficient of bidders and the sensitivity to the success and failure of the two bidders. And then we will find how these factors would influence the optimal bidding strategy comprehensively. First, we quote the bidding tendency of bidders as $x_i = \frac{\theta p_{i,0} + \alpha}{\beta}$ [2], which can comprehensively reflect how reserve price, monetary utility coefficient of bidders and the sensitivity to the success and failure of the two bidders influence bidding. Therefore, substituting x_i into (2) and (3), we can find the optimal bidding strategy of the naïve and rational as

$$p_n^* = \frac{s_n x_n - f_r\left(s_r, s_n'\right)}{2\left(x_n - 1\right)} \tag{4}$$

$$p_r^* = \frac{s_r x_r - s_n}{2\left(x_r - 1\right)} \tag{5}$$

(1) For the optimal strategy of the naïve, first-order conditions for bid propensity, respectively, are

$$\frac{\mathrm{d}p_n^*}{\mathrm{d}x_n} = \frac{f_r\left(s_r, s_n'\right) - s_n}{2\left(x_r - 1\right)^2} = \begin{cases} > 0, & f_r\left(s_r, s_n'\right) > s_n \\ = 0, & f_r\left(s_r, s_n'\right) = s_n \\ < 0, & f_r\left(s_r, s_n'\right) < s_n \end{cases}$$

and $\frac{\mathrm{d}^2 p_n^*}{\mathrm{d}x_n^2} = -\frac{f_r\left(s_r, s_n'\right) - s_n}{\left(x_n - 1\right)^3}$

We can derive the following conditions: when $f_r\left(s_r, s_n'\right) > s_n$, $\frac{\mathrm{d}^2 p_n^*}{\mathrm{d}x_n^2} < 0$. And p_n^*, the optimal bidding strategy, is a concave upward increasing function about x_n; when $f_r\left(s_r, s_n'\right) < s_n$, $\frac{\mathrm{d}^2 p_n^*}{\mathrm{d}x_n^2} < 0$, p_n^* is a concave upward decreasing function about x_n; when $f_r\left(s_r, s_n'\right) = s_n$, p_n^* is constant about x_n.

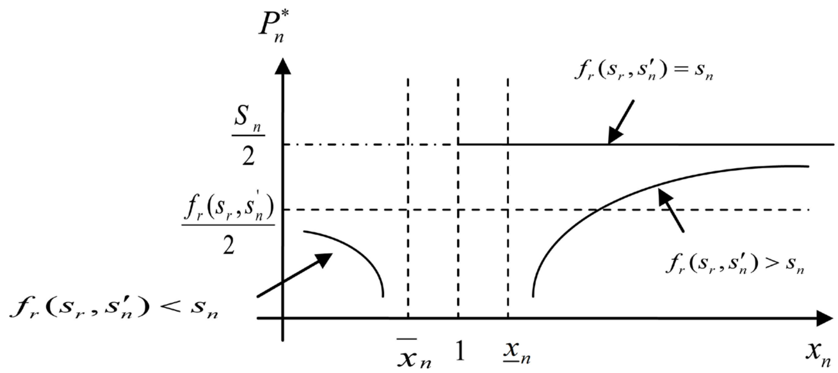

Figure 1. The relation (3 situations) between the optimal bidding strategy and the propensity to bid for the naive bidder.

In addition, we substitute (4) into the expected utility function of the naïve bidder, and let $U_n = 0$. In this case, $x_n = \frac{f_r(s_r, s_n')}{s_n}$, which is the superior or inferior limits of x_n (we name it $\overline{x}_n$ and $\underline{x}_n$), that is, when $f_r(s_r, s_n') < s_n$, $\overline{x}_n$ is the superior limit of x_n; when $f_r(s_r, s_n') > s_n$, $\underline{x}_n$ is the inferior limit of x_n; when $f_r(s_r, s_n') = s_n$, the inferior limit of x_n equal to 1 but can never reach it. Taking x_n as the abscissa axis and p_n^* as the ordinate axis, the relation (3 situations) between the optimal bidding strategy and the propensity to bid for the naive bidder can be expressed in Figure 1.

In Figure 1, the According to Figure 1, when $f_r(s_r, s_n') > s_n$, p_n^*, the optimal bid price of the naïve bidder increases as the bid propensity increases, and gradually tends to 1/2 of his judgment value of the auction, that is $\lim\limits_{x_n \to +\infty} p_n^* \to \frac{s_n}{2}$; when $f_r(s_r, s_n') < s_n$, p_n^* decreases as the bid propensity increases, and gradually goes to 0, that is $\lim\limits_{x_n \to \overline{x}_n} p_n^* \to 0$; when $f_r(s_r, s_n') = s_n$, there is always $p_n^* = \frac{s_n}{2} = \frac{f_r(s_r, s_n')}{2}$, in which case the two lines overlap.

(2) For the optimal strategy of the rational, first-order conditions for bid propensity, respectively, are

$$\frac{\mathrm{d}p_r^*}{\mathrm{d}x_r} = \frac{s_r - s_n}{2(x_r - 1)^2} = \begin{cases} > 0, & s_r > s_n \\ = 0, & s_r = s_n \\ < 0, & s_r < s_n \end{cases}$$

$$\frac{\mathrm{d}^2 p_r^*}{\mathrm{d}x_r^2} = -\frac{s_r - s_n}{(x_r - 1)^3}$$

Therefore, when $s_r > s_n$, $x_r > 1$, p_n^*, the optimal bidding strategy of the rational, is a concave downward increasing function about x_r; when $s_r > s_n$, $x_r < 1$, p_n^* is a convex upward increasing function about x_r; when $s_r < s_n$, $x_r > 1$, p_n^* is a concave downward decreasing function about x_r; when $s_r < s_n$, $x_r < 1$, p_n^* is a convex upward decreasing function about x_r; when $s_r = s_n$, p_n^* is constant about x_r.

In addition, we substitute (5) into the expected utility function of the rational

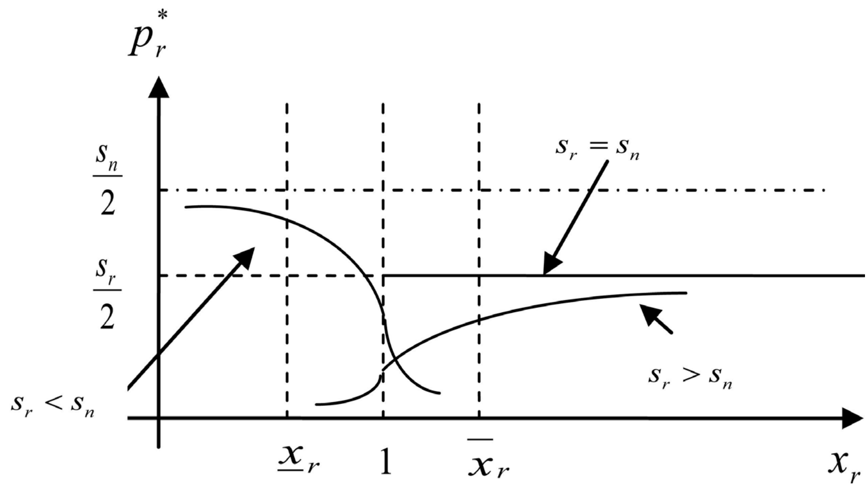

Figure 2. The relation (3 situations) between the optimal bidding strategy and the propensity to bid for the rational bidder.

bidder, and let $U_r = 0 : x_r = \frac{s_n}{s_r}$, which is the superior or inferior limits of x_r (we name it $\overline{x}_r$ and $\underline{x}_r$), that is, when $s_r < s_n$, $\overline{x}_r$ is the superior limit of x_n; when $s_r > s_n$, $\underline{x}_n$ is the inferior limit of x_n; when $s_r = s_n$, the inferior limit of x_r equal to 1 but can never reach it. The relation between the optimal bidding strategy and the propensity to bid for the rational bidder can be expressed in **Figure 2**.

According to **Figure 2**, when $s_r > s_n$, p_r^*, the optimal bid price of the rational bidder increases as the bid propensity increases, and gradually tends to 1/2 of his judgment value of the auction, that is $\lim_{x_r \to +\infty} p_r^* \to \frac{s_r}{2}$; when $s_r < s_n$, p_r^* decreases as the bid propensity increases, and gradually goes to 0, that is $\lim_{x_r \to \overline{x}_r} p_r^* \to 0$; when $s_r = s_n$, there is always $p_n^* = \frac{s_n}{2} = \frac{s_r}{2}$, in which case the two lines overlap.

4. Auction Result Analysis

From the above analysis, we can see that the difference between the judgment value of the naive and the rational leads to the optimal bidding of the bidder changes with its bid propensity. In this chapter, we try to find which type of bidder is likely to win this auction at last through analyzing this difference and the difference of bid propensity. According to the analysis above, we have the following conclusions.

(1) in the case of $f_r(s_r, s_n') > s_n, s_r > s_n$, the optimal bidding of the two types of bidders increases with the increase of bid propensity. In this case, the inferior limit of bid propensity of the rational bidder $\underline{x}_r = \frac{s_n}{s_r} < 1$, and for the naïve bidder, there is $\overline{x}_n = \frac{f_r(s_r, s_n')}{s_n} > 1$. The optimal bidding for the naive and the rational is shown in **Figure 3**.

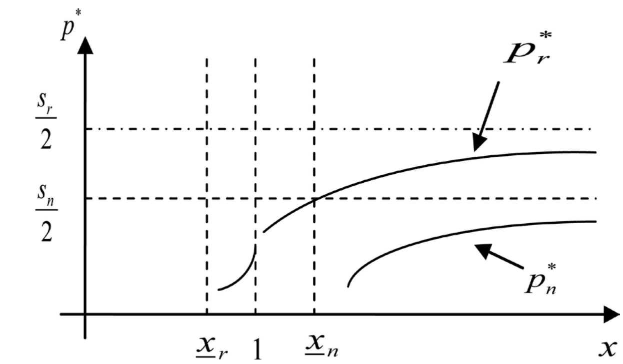

Figure 3. The optimal bidding for the naive and the rational when $f_r(s_r, s_n') > s_n$, $s_r > s_n$.

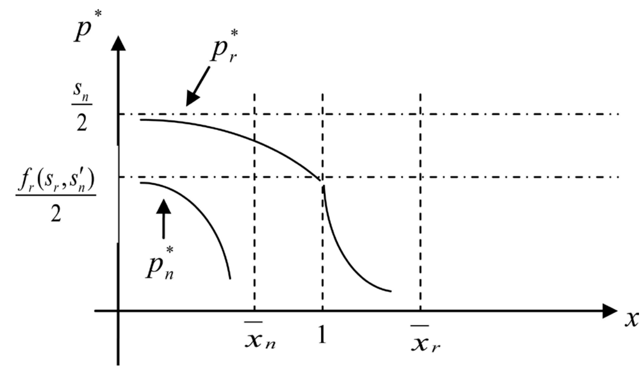

Figure 4. The optimal bidding for the naive and the rational when $f_r(s_r, s_n') < s_n$, $s_r < s_n$.

According to **Figure 3**, when $f_r(s_r, s_n') > s_n$, $s_r > s_n$, and the two types have the same bid propensity, because the rational bidder's optimal bidding is greater than the naïve bidder's, finally the rational bidder would win the auction.

(2) in the case of $f_r(s_r, s_n') < s_n$, $s_r < s_n$, the optimal bidding of the two types of bidders decreases with the increase of bid propensity. In this case, the superior limit of bid propensity of the rational bidder $\bar{x}_r = \frac{s_n}{s_r} > 1$, and for the naïve bidder, there is $\bar{x}_n = \frac{f_r(s_r, s_n')}{s_n} < 1$. The optimal bidding for the naive and the rational is shown in **Figure 4**.

According to **Figure 4**, when $f_r(s_r, s_n') < s_n$, $s_r < s_n$ and the two types have the same bid propensities, because the rational bidder's optimal bidding is greater than the naïve bidder's, finally the rational bidder would win the auction.

(3) in the case of $f_r(s_r, s_n') > s_n$, $s_r < s_n$, the optimal bidding of the naïve bidder p_n^* increases with the increase of bid propensity, but the optimal bidding of the rational bidder p_r^* decreases with the increase of bid propensity. In this case, the inferior limit of bid propensity of the naïve $\underline{x}_n = \frac{f_r(s_r, s_n')}{s_n} > 1$, the superior limit of bid propensity of the rational bidder $\bar{x}_r = \frac{s_n}{s_r} > 1$. When

$\underline{x}_n = \frac{f_r(s_r, s_n')}{s_n} \geq \frac{s_n}{s_r} = \overline{x}_r$, the optimal bidding of the two types of bidders is shown as Figure 5.

According to Figure 5, for any $x' \in (0, \overline{x}_r)$, when the bid propensity of the rational $x_r \in (0, x')$, and the bid propensity of the naïve $x_n \in (\underline{x}_n, x'')$, the rational bidder would finally win the auction; when the bid propensity of the rational $x_r \in (x', \overline{x}_r)$, and the bid propensity of the naïve $x_n \in (x'', +\infty)$, the naïve bidder would finally win the auction.

When $\underline{x}_n = \frac{f_r(s_r, s_n')}{s_n} < \frac{s_n}{s_r} = \overline{x}_r$, it's shown as Figure 6.

According to Figure 6, when the bid propensity of the rational $x_r \in (0, x')$ and the naïve $x_n \in (\underline{x}_n, x')$, the rational bidder would win the auction; when the bid propensity of the rational $x_r \in (x', \overline{x}_r)$ and the naïve $x_n \in (x', +\infty)$, the naïve bidder would win the auction.

(4) in the case of $f_r(s_r, s_n') < s_n$, $s_r > s_n$, the optimal bidding of the rational bidder p_r^* increases with the increase of bid propensity, and the optimal bidding of the naive bidder p_n^* decreases with the increase of bid propensity. In this case, the inferior limit of bid propensity of the rational $\underline{x}_r = \frac{s_n}{s_r} < 1$, the superior limit of bid propensity of the naive $\overline{x}_n = \frac{f_r(s_r, s_n')}{s_n} < 1$.

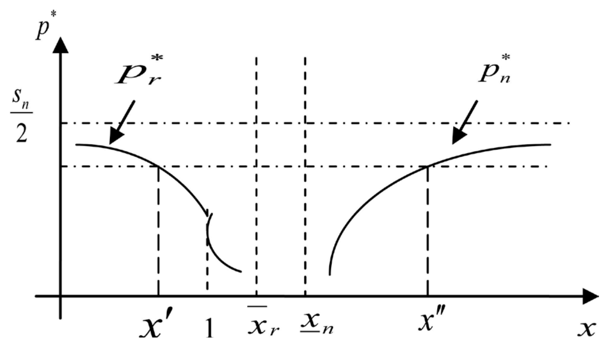

Figure 5. The optimal bidding for the naive and the rational when $f_r(s_r, s_n') > s_n$, $s_r < s_n$.

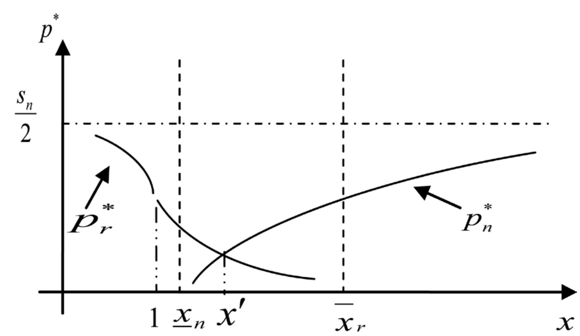

Figure 6. The optimal bidding for the naive and the rational when $f_r(s_r, s_n') > s_n$, $s_r < s_n$ and $\underline{x}_n = \frac{f_r(s_r, s')}{s_n} < \frac{s_n}{s_r} = \overline{x}_r$.

When $\overline{x}_n = \frac{f_r(s_r, s_n')}{s_n} < \frac{s_n}{s_r} = \underline{x}_r$,the optimal bidding of the two types of bidders is shown as Figure 7

According to Figure 7.

(a) when the bid propensity of the rational $x_r \in (x''', +\infty)$, the rational bidder would always win the auction.

(b) for any $x_r \in (0, \overline{x}_n)$, when the bid propensity of the naïve $x_n \in (0, x')$ and the rational $x_r \in (\underline{x}_r, x'')$, the naïve bidder would win the auction; when the bid propensity of the naïve $x_n \in (x', \overline{x}_n)$ and the rational $x_r \in (x'', x''')$, the rational bidder would win the auction.

When $\overline{x}_n = \frac{f_r(s_r, s_n')}{s_n} > \frac{s_n}{s_r} = \underline{x}_r$, the optimal bidding of the two types of bidders is shown as Figure 8.

According to Figure 8,

(a) when the bid propensity $x_n \in (x'', +\infty)$, the rational bidder would win the auction.

(b) set the intersection of the optimal bidding curve between the innocent and

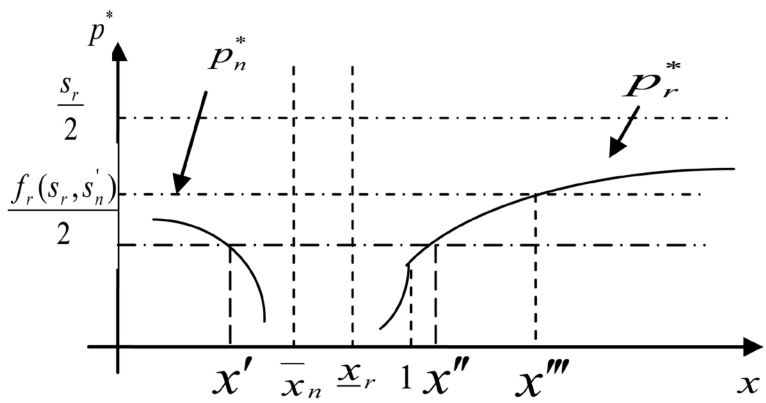

Figure 7. The optimal bidding for the naive and the rational when $f_r(s_r, s_n') < s_n$, $s_r > s_n$ and $\overline{x}_n = \frac{f_r(s_r, s_n')}{s_n} < \frac{s_n}{s_r} = \underline{x}_r$.

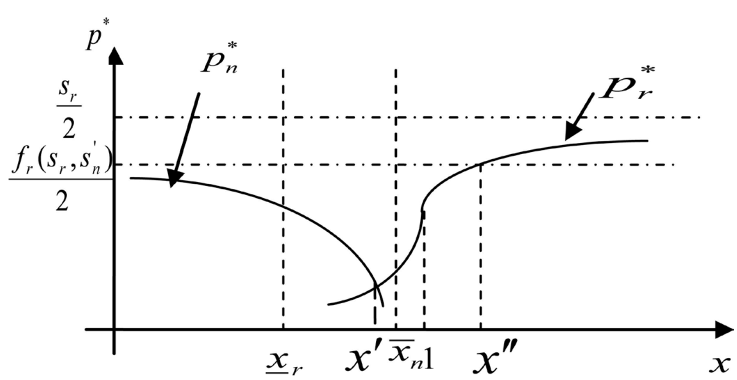

Figure 8. The optimal bidding for the naive and the rational when $f_r(s_r, s_n') < s_n$, $s_r > s_n$ and $\overline{x}_n = \frac{f_r(s_r, s_n')}{s_n} > \frac{s_n}{s_r} = \underline{x}_r$.

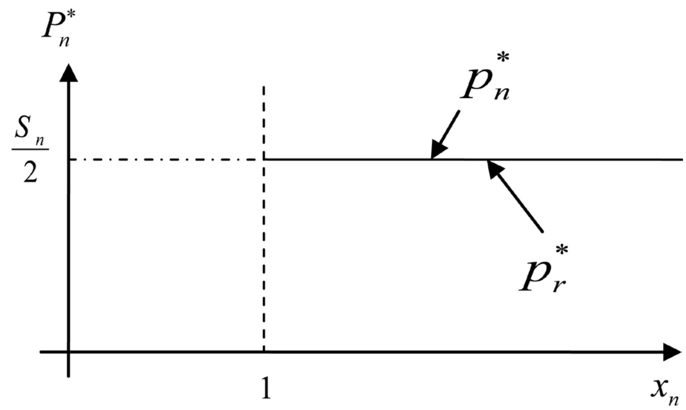

Figure 9. The optimal bidding for the naive and the rational when $f_r(s_r, s_n') = s_n$.

the rational as x'. When the bid propensity of the naïve $x_n \in (0, x')$ and the rational $x_r \in (\underline{x}_r, x')$, the naïve bidder would win the auction; when the bid propensity of the naïve $x_n \in (x', \overline{x}_n)$ and the rational $x_r \in (x', x'')$, the rational bidder would win the auction.

(5) in the case of $f_r(s_r, s_n') = s_n$, p_n^* and p_r^* are both constants about their own bid propensity, $p_n^* = p_r^* = \frac{s_n}{2} = \frac{f_r(s_r, s_n')}{2}$, and the inferior limits of both are 1, as shown in **Figure 9**.

In this case, according to the rule of auction, who calls the optimal bidding first is most likely to win the auction.

5. Conclusions

In this paper, we abstract the bidders in the auction market as two bidders, the naïve bidder and the rational bidder, to study bidding strategies of bidders with different experience types, which is the innovation of this paper. In the auction market, emotional factors have a certain influence on the bidder's bidding. Therefore, according to building a two bidder's model about the comprehensive effectiveness of monetary utility and emotional utility, we analyzed the influence of the optimal bidding strategy for the naive and rational and their bid propensity on optimal bidding. And then we studied that because of the difference of bidder's judgment value of the auction and bidder's bidding tendency, which type of bidder may eventually win the auction.

Of course, in the real auction market, there are more bidders than just a naïve bidder and a rational bidder. We can divide these bidders into two types, the naïve and the rational, in which case we can still use the idea of this paper to consider the problem. However, the proportion of the naïve and the rational in real auction market is not always the same. The population change of two types of bidders and equilibrium problem need to be discussed in the future study.

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