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Does Immigration Promote the Investment of the Monopolistic Firm?

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Abstract

In the present paper, we examine the effect of increasing uncertainty of immigrants' growth on the optimal timing of investment of a firm that has a monopolistic power over the labor market. It is revealed that when the uncertainty of immigrants' growth is more than a threshold level, increasing uncertainty of immigrants' growth accelerates the optimal timing of firms' investment and enhances the economic growth, even if the uncertainty of immigrants' growth is formulated by the geometric Brownian motion, which is in sharp contrast to the standard result that an increase in the uncertainty postpones the optimal timing.

Keywords

Optimal Stopping Theory, Geometric Brownian Motion, Value of Waiting, Immigrants, Monopolistic Power over Labor Market

1. Introduction

With an increase in the immigrants over the past ten years, workforces in the host countries have been growing significantly to the extent that the immigrants represent 70% of the increase in the workforce in Europe, and 47% in the United States as OECD [1] indicates. Although such increasing immigrants have been spurring interest in the relationship between the immigrants and the economic growth ([2] [3] and so on), few studies have tried to estimate the impact of migration on the economic growth, nor to conduct theoretical analyses by formulating economic models. The present study is an attempt to handle this situation by combining the optimal stopping theory with the market theory as in Fujita [4] [5] [6] and so on, to investigate the impact of the increase in the immigrants on investment of a monopolistic firm. More precisely, we construct a stochastic intertemporal model where the firm faces market uncertainty expressed by the

geometric Brownian motion, to examine the effect of increasing uncertainty of immigrants' growth on the optimal timing of the investment, and hence the economic growth.

It is revealed that when the uncertainty of immigrants' growth is more than a threshold level, increasing uncertainty of immigrants' growth accelerates the optimal timing of the firm's investment, even if the uncertainty of immigrants' growth is formulated by the geometric Brownian motion, which is in sharp contrast to the standard result of McDonald and Siegel [7], Dixit and Pindyck [8] and so on, which shows an increase in the uncertainty postpones the optimal timing. Nishimura and Ozaki [9] and Trojanowska and Kort [10] also reversed the standard result, but by introducing the Knightian uncertainty, which is different from the present paper that assumes the geometric Brownian motion.

Structure of this paper is as follows. Section 2 formulates the objective function of the firm after constructing a basic model. Based on these analyses, in section 3 we reveal the effect of increasing uncertainty of immigrants' growth on the optimal timing of the investment, and hence the economic growth. Concluding remarks are made in section 4.

2. Assumptions and Formulation of the Objective Function

In order to investigate the effect of growing number of immigrants on the economic growth, let us consider a stochastic intertemporal economy with one firm that has monopolistic power over the labor market. To contemplate when to conduct investment by incurring the cost of K to create a new product. For the simplicity of the analysis, we assume that the product is made only by the labor and the product price is constant at p. We also assume that the time passes continuously with importance of the future diminishing with the discount rate of ρ.

Letting $x(t)$ denote the firm's output in period t and specifying one unit of production requires one unit of labor, we assume the wage in period t, $w(t)$, is expressed by the following linear inverse labor supply function, $w(t) = c(t) x(t)$, where $c(t)$ is a positive variable that represents the size of the labor market, which is positively related to the amount of the immigrants, in period t, and fluctuates stochastically according as the following geometric Brownian motion as in Fujita (2016),

$$\frac{dc}{c(t)} = s dz \tag{1}$$

with initial value c_0, where s is a positive constant that expresses the labor market volatility in a sense that larger s means more uncertainty of the labor market, while dz is Wiener process that expresses the random movement. Then, the firm's profit in period t, $\pi(t)$, is described as $\pi(t) = \left(p - c(t) x(t) \right) x(t)$, so that, we have the firm's first order condition for the profit maximization as $\frac{\partial \pi}{\partial x} = p - 2c(t) x(t) = 0$, to yield the firm's output, and hence the profit, in the equilibrium in period t as $x(t) = \dfrac{p}{2c(t)}$ and $\pi(t) = \dfrac{p^2}{4c(t)}$, respectively.

Since we have the first and the second derivatives of the profit as

$\dfrac{d\pi}{dc} = -\dfrac{p^2}{4c(t)^2}$ and $\dfrac{d^2\pi}{dc^2} = \dfrac{p^2}{8c(t)^3}$ respectively, we can express the stochastic

process of the firm's profit, by making use of Equation (1) and Ito's lemma, as

$$\frac{d\pi}{\pi} = \mu dt + \sigma dz \qquad (2)$$

with initial value $\pi_0 = \dfrac{p^2}{4c_0}$, where $\mu = \dfrac{s^2}{4}$ and $\sigma = -s$.

Now, by making use of this stochastic process of the firm's profit, we are ready to express the firm's objective function to maximize in period 0,

$V = E\left[\int_{t^*}^{\infty} e^{-\rho t}\pi(t)\,dt - e^{-\rho t^*}K\right]$, as a function of c^*, the market size c in period t^*

when the firm's investment is carried out. For this purpose, if we let $G(\pi_0)$ denote the expected value of one unit of the firm's profit in period t^* (i.e., the expected value of $-e^{-\rho t^*}$) as a function of the initial profit π_0, the general solution to $G(\pi_0)$ is expressed as

$$G(\pi_0) = \alpha(\pi_0)^{\gamma_1} + \beta(\pi_0)^{\gamma_2} \qquad (3)$$

where $\gamma_1 < 0$ and $\gamma_2 > 0$ are solutions to the characteristic Equation

$\dfrac{\sigma^2}{2}x(x-1) - \mu x - \rho = 0$, which is rewritten as $x(2x-3) - \dfrac{4\rho}{s^2} = 0$ by substi-

tuting $\mu = \dfrac{s^2}{4}$ and $\sigma = -s$ into it. If we let π^* denote the firm's profit in period

t^*, it follows that $\alpha = 0$ and $\beta = \left(\dfrac{1}{\pi^*}\right)^{\gamma_2}$ since $G(\pi_0)$ satisfies $G(\infty) = 0$ and

$G(\pi^*) = 1$.

Therefore, we obtain $G(\pi_0) = \left(\dfrac{\pi_0}{\pi^*}\right)^{\gamma}$, where

$$\gamma = \frac{3 + \sqrt{9 + \dfrac{32\rho}{s^2}}}{4} \qquad (4)$$

by substituting $\alpha = 0$ and $\beta = \left(\dfrac{1}{\pi^*}\right)^{\gamma_2}$ into Equation (3) and letting γ denote γ_2.

Thus, we can derive the firm's objective function to maximize in period 0, V,

as $V = \left(\dfrac{\pi_0}{\pi^*}\right)^{\gamma}\left(\dfrac{\pi^*}{\rho - s^2} - K\right)$, which is rewritten as

$$V = \left(\frac{b^*}{b_0}\right)^{\frac{3 + \sqrt{9 + \frac{32\rho}{s^2}}}{4}}\left(\frac{1}{\rho - s^2}\frac{p^2}{4b^*} - K\right) \qquad (5)$$

by substituting $\pi_0 = \dfrac{p^2}{4c_0}$, $\pi^* = \dfrac{p^2}{4c^*}$ and Equation (4) into

$V = \left(\dfrac{\pi_0}{\pi^*}\right)^{\gamma}\left(\dfrac{\pi^*}{\rho - s^2} - K\right)$.

3. Optimal Timing of the Investment

Now, we can determine the firm's optimal timing of the investment, to examine

the effect of increased uncertainty caused by the growing number of the immigrants on the economic growth. Standard theory of the optimal timing of investment generally tells us that an increase in the uncertainty pulls up the value of waiting to postpone the optimal timing, which leads to the slowdown of the economic growth. The purpose of the paper is to investigate if this standard result holds if we construct a specific model to formulate the stochastic fluctuation of the wage rate.

Since the model of the present paper is stochastic, the optimal timing is expressed by the cut off level of the market size c^*. Therefore, by differentiating Equation (5) with respect to c^* and setting it to zero, we have the optimal cut off level of the market size c^* for the firm as

$$c* = \frac{\left(-3s + \sqrt{9s^2 + 32\rho}\right)^2 p^2}{32^2 \rho^2 \left(\rho - s^2\right) K} \tag{6}$$

Graph of c^* is depicted as a U-shaped curve with its minimum point at $\hat{s}$ on the s-c^* space as in **Figure 1**, which is in sharp contrast with the standard result of the value of waiting where c^* decreases monotonically as s increases. To be exact, by differentiating Equation (6) with respect to s and setting it to zero, we have the threshold level of the market uncertainty $\hat{s}$, so as to satisfy

$$3 = \frac{s\sqrt{9s^2 + 32\rho}}{\rho - s^2} \tag{7}$$

The graph of the left hand side of (7) is drawn as a horizontal line, while the graph of the right hand side of (7) is drawn as a monotonically upward curve that starts from the origin and goes infinity when $s = \sqrt{\rho}$. Therefore, $\hat{s}$ is determined uniquely as in **Figure 2**.
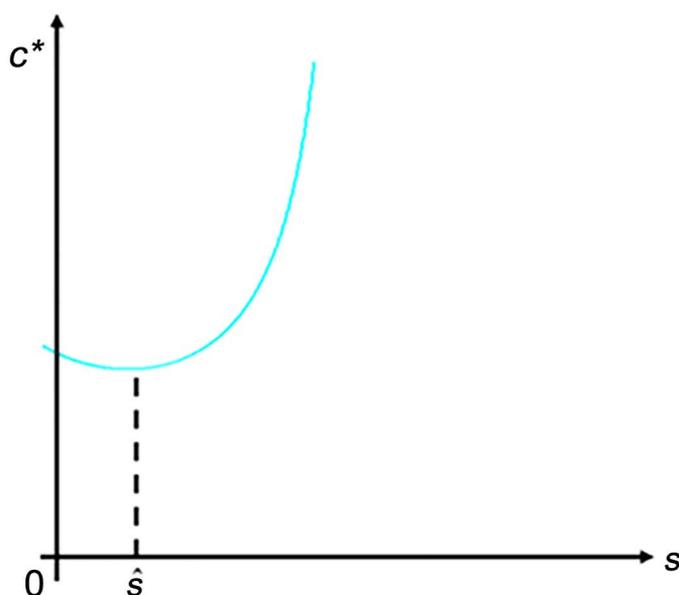
Figure 1. Increase in the number of immigrants and the timing of investment.
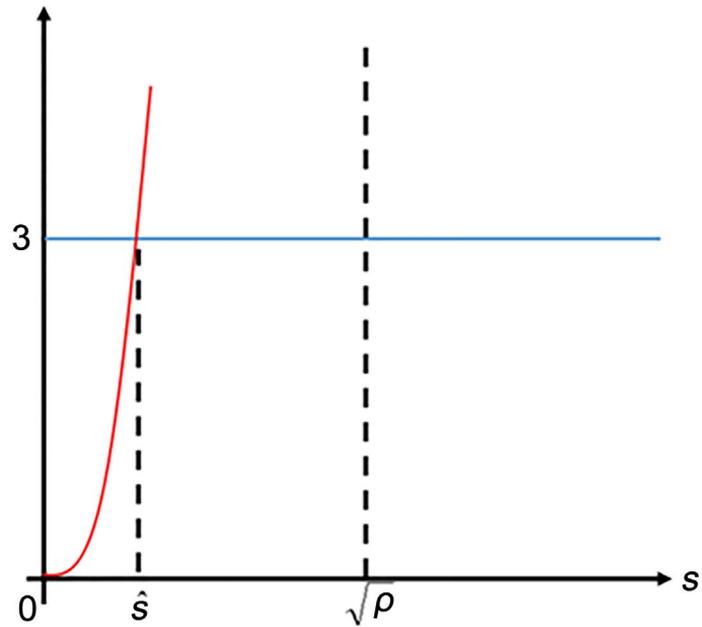
Figure 2. Determination of the threshold level of s.

Since large s means increasing uncertainty of immigrants' growth and small (large) c^* means postponement (acceleration) of the investment, we have the following proposition.

Proposition: (1) If uncertainty of immigrants' growth is less than a threshold level, increasing uncertainty of immigrants' growth postpones the optimal timing of the firm's investment; (2) if uncertainty of immigrants' growth is more than a threshold level, however, increasing uncertainty of immigrants' growth accelerates the optimal timing of the firm's investment.

We can conclude from Proposition (2) that when the uncertainty caused by the growing number of immigrants is more than the threshold level $\hat{s}$, an increase in the uncertainty of immigrants' growth accelerates the optimal timing and enhances the economic growth, which is in sharp contrast to the standard result of the theory of the optimal timing of investment.

In the present model, as Equation (2) shows, increasing uncertainty of immigrants' growth pulls up the growth rate of the firm's profit, which results in accelerating the timing of the investment. If this accelerating effect surpasses the standard postponing effect, overall effect accelerates the timing of the investment. Proposition (2) demonstrates that such a case prevails if uncertainty of immigrants' growth is more than the threshold level $\hat{s}$.

4. Conclusions

In the present paper, we attempted to investigate the effect of increased uncertainty caused by the growing number of immigrants on the economic growth by combining the optimal stopping theory with the standard market theory. Our result is: When the uncertainty of immigrants' growth is more than a threshold level, increasing uncertainty of immigrants' growth accelerates the optimal tim-

ing of firms' investment and enhances the economic growth, even if we formulate the immigrants' uncertainty by the geometric Brownian motion, not by the Knightian uncertainty.

In order to derive explicit results, we made several simplifying assumptions. Thus, it is necessary to lay out a general framework by relaxing those assumptions to examine the robustness of the results of the present paper. We also need to give an empirical testing of this model, in order to contribute to the advancement of the monopolistic theory as well as the optimal stopping theory. These are issues which are left for our future research.

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