

# Partial Privatization in Price - Setting Mixed Duopolies with Complementary Goods

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## Abstract

We consider a domestic (resp. international) mixed duopoly model in which a domestic public firm and a domestic (resp. foreign) private firm produce complementary goods. First, the domestic government chooses the level of privatization to maximize domestic social welfare. Second, observing the level of privatization, the firms simultaneously and independently choose prices. We present the equilibrium outcomes of the two mixed duopoly models and shows that our result is in marked contrast to that of the price-setting mixed duopoly model with substitute goods.

**Keywords:** Partial Privatization, Price - Setting Model, Domestic Mixed Duopoly, International Mixed Duopoly, Complementary Goods

## 1. Introduction

The theoretical analysis of partial privatization of state-owned public firms has received significant attention in recent years and has been extensively studied by many economists, such as [1-17].

However, these studies analyze partial privatization in mixed duopoly competition in which public and private firms produce substitutable goods. To the best of my knowledge, the analysis of partial privatization in Bertrand mixed markets with public and private firms producing complementary goods has been ignored.

Therefore, we study partial privatization in price-setting mixed duopoly competition with complementary goods. We extend the analysis of Ohnishi [17], which investigates a price-setting mixed duopoly model involving a domestic public firm and a domestic private firm to reassess the welfare effect of partial privatization. Ohnishi demonstrates that partial privatization is not a reasonable choice for the government that wishes to maximize social welfare.

<sup>1</sup>As is well known, international mixed oligopolies are common in developed and developing countries as well as in former communist countries. Public firms compete against foreign private firms in many industries, such as banking, life insurance, automobiles, airlines, steel, shipbuilding and tobacco. For example, in the tobacco industries of France, Italy, Russia, Spain, Austria, Turkey, China, Japan, etc, we can find real-world examples in which public firms compete or competed against foreign private firms such as Philip Morris and R. J. Reynolds.

We consider both domestic and international mixed duopoly models with complementary goods.<sup>1</sup> We consider the following situation. In the first stage, the domestic government chooses the degree of privatization to maximize domestic social welfare. In the second stage, observing the degree of privatization, the firms simultaneously and independently choose prices.

The main purpose of this paper is to present the equilibrium outcomes of the two mixed duopoly models and to show that this result is in marked contrast to that of the price - setting mixed duopoly model with substitute goods.

The remainder of the paper is organized as follows. In Section 2, we examine a domestic mixed duopoly model. Section 3 examines an international mixed duopoly model. Section 4 concludes the paper.

## 2. Domestic Mixed Duopoly

In this section, we consider a domestic mixed model with two firms (firm P and firm D) and the government. These firms produce complementary goods. There is no possibility of entry or exit. On the consumption side, there is a continuum of consumers of the same type whose utility function is linear. Subscripts P and D denote firm P and firm D, respectively. Following Bárcena-Ruiz [18], we assume that the representative consumer maximizes  $U(q_P, q_D) - p_P q_P - p_D q_D$ , where  $q_i$  is the amount of

good  $i$  and  $p_i$  is its price ( $i = P, D$ ). The function  $U(q_P, q_D)$  is quadratic, strictly concave and symmetric in  $q_P$  and  $q_D$ :

$$U(q_P, q_D) = a(q_P + q_D) - \frac{1}{2}(q_P^2 + 2bq_Pq_D + q_D^2) \quad (1)$$

where  $a$  is a constant and  $b \in (-1, 0)$  is a measure of the degree of complementarity among products. The demand function is given by

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2} \quad (i, j = P, D; i \neq j) \quad (2)$$

For simplicity, we assume that  $a = 1$  and  $b = -0.5$ . Each firm's profit is

$$\pi_i = (p_i - c_i)q_i \quad (i = P, D), \quad (3)$$

where  $c_i$  is the marginal cost of firm  $i$ . Since the result of this paper is not affected by  $c_i$ , we normalize it to zero. Firm D aims to maximize its profit. Furthermore, domestic social welfare, which is the sum of consumer surplus and profits, is given by

$$W = CS + \pi_P + \pi_D. \quad (4)$$

The objective function of firm P is given by

$$V = \lambda\pi_P + (1-\lambda)W, \quad (5)$$

where  $\lambda \in [0, 1]$  is the level of privatization. That is, if  $\lambda = 0$  firm P is purely public, whereas if  $\lambda = 1$  it is purely private.

The game is constructed by the following two-stage decision-making. In the first stage, the government chooses the level of privatization,  $\lambda$ , to maximize domestic social welfare. Observing  $\lambda$ , the firms non-cooperatively choose prices in the second stage. Throughout this paper, the subgame perfect Nash equilibrium of the price-setting game is examined.

We now examine the welfare effect of partial privatization in the domestic mixed duopoly game. We obtain the reaction functions in prices of the two firms:

$$R_P = \frac{3 - (3 - 2\lambda)p_D}{2(3 - \lambda)}, \quad (6)$$

$$R_D = \frac{3 - p_P}{4} \quad (7)$$

From (6) and (7), the equilibrium can be derived as follows:

$$\begin{aligned} p_P &= \frac{1 + 2\lambda}{7 - 2\lambda}, & p_D &= \frac{5 - 2\lambda}{7 - 2\lambda} \\ q_P &= \frac{4(7 - 4\lambda)}{3(7 - 2\lambda)}, & q_D &= \frac{4(5 - 2\lambda)}{3(7 - 2\lambda)} \end{aligned}$$

Comparative static results yield  $dp_P/d\lambda > 0$ ,

$dp_D/d\lambda < 0$ ,  $dq_P/d\lambda < 0$ , and  $dq_D/d\lambda < 0$ . Thus, the privatization decreases each firm's output and firm D's price, and increases firm P's price.

Furthermore, the profits, consumer surplus, and social welfare can be expressed as follows:

$$\pi_P = \frac{4(1 + 2\lambda)(7 - 4\lambda)}{3(7 - 2\lambda)^2}, \quad (8)$$

$$\pi_D = \frac{4(5 - 2\lambda)^2}{3(7 - 2\lambda)^2}, \quad (9)$$

$$CS = \frac{8(29 + 19\lambda + 2\lambda^2)}{3(7 - 2\lambda)^2}, \quad (10)$$

In the first stage, the government chooses the level of privatization. Substituting (8), (9) and (10) into (4), the objective function of the government is obtained as

$$W = \frac{8(15 - 8\lambda)}{(7 - 2\lambda)^2}. \quad (11)$$

Social welfare  $W$  is illustrated as a function of  $\lambda$ . When  $\lambda = 0$ ,  $W = 2(22/49) \approx 2.449$ , and when  $\lambda = 1$ ,  $W = 2(6/25) = 2.24$ . In addition, when  $\lambda = 0.1$ ,  $W = 2(132/289) \approx 2.457$ . This can be stated in the following proposition.

**Proposition 1.** In the domestic mixed market with complementary goods, neither full nationalization nor full privatization is a reasonable choice for the government that wishes to maximize domestic social welfare; that is, the optimal solution is partial privatization.

### 3. International Mixed Duopoly

In this section, we consider an international mixed duopoly model in which a state-owned public firm (firm P) competes against a foreign private firm (firm F). Subscripts P and F denote firm P and firm F, respectively. In addition, an asterisk denotes the international mixed duopoly model.

Firm F aims to maximize its own profit. Furthermore, domestic social welfare is given by

$$W^* = CS^* + \pi_P^*. \quad (12)$$

The objective function of firm P is given by

$$V^* = \theta\pi_P^* + (1-\theta)W^*, \quad (13)$$

where  $\theta \in [0, 1]$  is the level of privatization. That is, if  $\theta = 0$  firm P is purely public, whereas if  $\theta = 1$  it is purely private. The timing of this model is identical to the domestic mixed duopoly model.

We examine the welfare effect of partial privatization in the international mixed duopoly model. We obtain the

reaction functions in prices of the two firms:

$$R_P^* = \frac{3-(2-\theta)P_F^*}{2(3-\theta)}, \quad (14)$$

$$R_F^* = \frac{3-P_P^*}{4}. \quad (15)$$

From (14) and (15), the equilibrium can be derived as follows:

$$P_P^* = \frac{3(2+\theta)}{22-7\theta}, \quad P_F^* = \frac{3(5-2\theta)}{22-7\theta},$$

$$q_P^* = \frac{2(13-7\theta)}{22-7\theta}, \quad q_F^* = \frac{4(5-2\theta)}{22-7\theta}.$$

Comparative static results yield  $dp_P^*/d\theta > 0$ ,  $dp_F^*/d\theta < 0$ ,  $dq_P^*/d\theta < 0$ , and  $dq_F^*/d\theta < 0$ . Thus, the privatization decreases each firm's output and firm F's price, and increases firm P's output.

Furthermore, the profits and consumer surplus can be expressed as follows:

$$\pi_P^* = \frac{6(2+\theta)(13-7\theta)}{(22-7\theta)^2}, \quad (16)$$

$$\pi_F^* = \frac{12(5-2\theta)^2}{(22-7\theta)^2}, \quad (17)$$

$$CS^* = \frac{2(367-263\theta+40\theta^2)}{(22-7\theta)^2}. \quad (18)$$

Substituting (16) and (18) into (12), the objective function of the government is obtained as

$$W^* = \frac{18(55-34\theta+3\theta^2)}{(22-7\theta)^2}. \quad (19)$$

Social welfare  $W^*$  is illustrated as a function of  $\theta$ . When  $\theta=0$ ,  $W^* = 2(11/242) \approx 2.045$ , and when  $\theta=1$ ,  $W^* = 1(23/25) = 1.92$ . In addition, when  $\theta=0.2$ ,  $W^* = 2(526/10609) \approx 2.050$ . This is stated in the following proposition.

**Proposition 2.** In the international mixed market with complementary goods, neither full nationalization nor full privatization is a reasonable choice for the government that wishes to maximize domestic social welfare; that is, the optimal solution is partial privatization.

## 4. Conclusions

We have studied partial privatization in price-setting mixed duopolies with complementary goods. First, the government chooses the level of privatization to maximize social welfare. Second, observing the level of privatization, the firms simultaneously and independently

choose prices. We have then presented the equilibrium outcomes of the two mixed duopoly markets. We have demonstrated that in each market, neither full nationalization nor full privatization is a reasonable choice for the government that wishes to maximize domestic social welfare; that is, the optimal solution is partial privatization. We have found that this result is in marked contrast to that of the price-setting mixed duopoly model with substitute goods.

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