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Hybrid Vehicle (City Bus) Optimal Power Management for Fuel Economy Benchmarking

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ABSTRACT

In this paper a global optimization method (dynamic programming) is used to find the optimal power management in hybrid electric city bus for the objective to reduce the fuel consumption. Knowing that when using a global optimization method the results cannot be used in real-time control; because we need to know the entire vehicle speed in advance to perform the optimization, but in spite of that this method is very useful to make a benchmark for hybrid electric city buses fuel economy and to judge the effectiveness and improve real-time control strategies. Finally results of optimal power management are shown and discussed.

Keywords: Hybrid Vehicle; Power Management; Fuel Economy; Optimization; Dynamic Programming

1. Introduction

The study of ground vehicles has taken a tremendous interest in recent years due to the increased price of fuel and emission stringent laws. In this way, Hybrid Electric Vehicles (HEV) seems to be the most promising short-term solution and is under enthusiastic development by many automotive companies. An HEV adds an electric motor to the conventional powertrain, which helps to improve fuel economy by engine downsizing, load leveling, and regenerative braking. A downsized engine has better fuel efficiency and smaller heat loss. The reduced engine power is compensated by the electric motor. Load leveling can be achieved by adding the electric motor, which enables the engine to operate more efficiently, independent from the road load. Regenerative braking allows the electric machine to capture part of the vehicle kinetic energy.

Power management strategies for parallel HEVs can be classified into three categories. The first type uses heuristic control techniques such as control rules [1], fuzzy logic [2,3] or neural networks [4] for estimation and control algorithm development. The second approach is based on static optimization methods [5-7]. Generally, electric power is translated into an equivalent amount of fuel rate in order to calculate the overall fuel cost. The optimization scheme and figures out the proper split between the two energy sources using steady-state efficiency maps. The third type of HEV control algrithms considers the dynamic nature of the system when performing the optimization [8,9]. Furthermore, the optimization is with respect to a time horizon, rather than for an instant in time. In general, power split algorithms resulting from dynamic optimization approaches are more accurate, but are computationally more intensive.

In this paper we use the dynamic programming method to solve the problem of optimal power management in a HEV, for that reason the reference speed should be known in advance to solve the problem; thus we use a simple reference speed (not a normalized drive cycle), this reference speed contains linear acceleration and deceleration and constant speed in order to facilitate the interpretation.

2. System Specification

2.1. System Structure and Modeling

The hybrid vehicle structure is a parallel single shaft topology, which utilizes a PMSM motor placed before the transmission and coupled with the diesel engine via clutch. The engine, motor and battery are modeled using experimental data (efficiency maps for engine and motor) and an equivalent electric circuit for the battery with experimental data.

2.2. Problem Formulation

In this paper we seek to find the optimal power split between engine and electric motor in HEV in order to achieve minimum fuel consumption, this is a problem of optimal control; for that reason we need to define the criterion of optimization the constraints and the state equation.

2.2.1. Criterion

The criterion of optimization also known as the cost function or the objective function is the function that we seek to minimize, which is the fuel consumption in this case

$$\text{Minimize } J = \sum_{i=0}^{N} C\left(Te(i), we(i)\right)Ts \quad (1)$$

2.2.2. Constraints

In a parallel single shaft hybrid powertrain topology, the sum of engine and motor torque must be instantaneously equal to the torque demand described in the engine shaft. and the engine and motor speeds are proportional to the wheel speed by the final drive and gearbox ratios. Also we must constrain the engine and motor torque to make sure that they do not exceed their maximum torques and finally constrain the battery state of charge to remain between two limits denoted as SOC_{max} and SOC_{min}. Constraining the battery SOC in this way helps to prolong its life, the constraints are described by the equations below:

$$Td(i) = Te(i) + Tm(i)$$

$$\left(if \cdot it(i)\right) \cdot w(i) = we(i) = wm(i)$$

$$we,\min \le we(i) \le we,\max$$

$$Te,\min \le Te(i) \le Te,\max$$

$$wm,\min \le wm(i) \le wm,\max$$

$$Tm,\min \le Tm(i) \le Tm,\max$$

$$\text{SOC1} \le \text{SOC}(i) \le \text{SOC2}$$

2.2.3. State Equation

The state equation gives the variation of the energy stored in the battery (*X*) as a function of the electric power furnished by this battery. In discrete time this variation is described by

$$X(i+1) = X(i) - Pe\left(wm(i), Tm(i)\right) \cdot Ts \quad (2)$$

2.2.4. Limit Condition

In order to be able to perform the optimization the zone of acceptable solution must be closed, which leads to constraining the battery SOC to converge to a known limit, this limit is described by SOCfinal, in our article a limit condition used is described by:

$$\text{SOCfinal} = \text{SOCinitial} = 80\% \quad (3)$$

3. Principal of the Method of Dynamic Programming

Dynamic Programming (DP) is a powerful mathematical technique developed to solve dynamic optimization problems. The advantage is that it can easily handle the constraints and nonlinearity of the problem while obtaining a globally optimal solution. The DP technique is based on Bellman's Principle of Optimality, which states that the optimal policy can be obtained if we first solve a one stage sub problem involving only the last stage and then gradually extend to sub-problems involving the last two stages, last three… etc. until the entire problem is solved (backward method). In this manner, the overall dynamic optimization problem can be decomposed into a sequence of simpler minimization problems [10].

In HEV the sequence of choices represents the power split between the internal combustion engine and the electric motor at successive time steps. The objective function can be fuel consumption, emissions, or any other design objective. The set of choices at each instant is determined by considering the state of each powertrain component and the total power requested by the driver. Given the current vehicle speed and the driver's demand (accelerator position); the controller determines the total power that should be delivered to the wheels. Then, using maps of the components and feedback on their present state, it also determines the maximum and minimum power that each energy source can deliver. If the power demand equals or exceeds the total available power from both sources, there is no choice to be made: each of them should be used at the maximum of its capabilities. Otherwise, there are infinite combinations such that the sum of the power from engine and motor equals the power demand. In most algorithms, including dynamic programming, instead of considering this continuum of solutions, a discrete number is selected and evaluated. The number of solution candidates that can be considered is a compromise between the computational capabilities and the accuracy of the result: in fact, the minimum cost may not exactly coincide with one of the selected points, but the closer these are to each other, the better the approximation of the optimal solution. Once the grid of possible power splits, or solution candidates, is created associating a cost to each of the solution candidates, the optimal cost is calculated for each grid point, and stored in a matrix of costs. When the entire cycle has been examined, the path with the lowest total cost represents the optimal solution (**Figure 1**).

3.1. Torque Demand Calculation

As we said before, this method requires the knowledge of the whole reference speed in advance to precede the optimization; thus; after knowing the speed, we can calculate the power demand and also the torque demand (since both engine and motor run at the same speed) in each sample time using the wheel speed and its derivatives as shown below:

Energy of the power source (Engine and Motor)

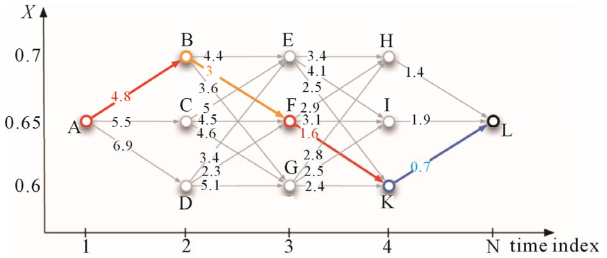

Figure 1. A grid with small number of discretization, the elementary costs and the optimal path are shown.

$$Es = \int Td \cdot we \cdot \mathrm{d}t$$

Energy of rotation of different inertia plus energy of translation of the vehicle (kinetic energy)

$$Ek1 + Ek2 = \frac{1}{2} J1 \cdot we^2 + \frac{1}{2} J2 \cdot wt^2 + \frac{1}{2} J3 \cdot w^2 + \frac{1}{2} M \cdot V^2$$

Energy to overcome resistant forces

$$Er = \int Faer \cdot V \cdot \mathrm{d}t + \int Froll \cdot V \cdot \mathrm{d}t$$

$$Faer = \frac{1}{2} \cdot Cd \cdot \rho \cdot A \cdot V^2$$

$$Froll = M \cdot (Cr1 + Cr2 \cdot V)$$

So after neglecting the energy lost by friction and damping we get:

$$Es = Ek1 + Ek2 + Er$$

That means:

$$\begin{aligned}\int Td \cdot we \cdot \mathrm{d}t &= \frac{1}{2} J1 \cdot we^2 + \frac{1}{2} J2 \cdot wt^2 + \frac{1}{2} J3 \cdot w^2 \\ &+ \int \left(\frac{1}{2} \mathrm{Cd} \cdot \rho \cdot A \cdot V^2 \right) \cdot V \cdot \mathrm{d}t \\ &+ \int M \left(Cr1 + Cr2 \cdot V \right) \cdot V \cdot \mathrm{d}t + \frac{1}{2} M \cdot V^2\end{aligned}$$

The derivation by time gives us the equation below

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t} \int Td \cdot we \cdot \mathrm{d}t &= \frac{\mathrm{d}}{\mathrm{d}t} \left[\frac{1}{2} J1 \cdot we^2 + \frac{1}{2} J2 \cdot wt^2 + \frac{1}{2} J3 \cdot w^2 \right. \\ &+ \frac{1}{2} M \cdot V^2 + \int \left(\frac{1}{2} \mathrm{Cd} \cdot \rho \cdot A \cdot V^2 \right) \cdot V \cdot \mathrm{d}t \\ &\left. + \int M \left(Cr1 + Cr2 \cdot V \right) \cdot V \cdot \mathrm{d}t \right]\end{aligned}$$

After derivation and arrangement we get this last differential equation

$$Td = \left\{ \frac{\left(J3 + M \cdot R^2 + if^2 \cdot J2 + it^2 \cdot if^2 \cdot J1 \right) \cdot \frac{\mathrm{d}w}{\mathrm{d}t}}{it \cdot if} + \frac{M \cdot Cr2 \cdot R^2 \cdot w + \frac{1}{2} Cd \cdot A \cdot \rho \cdot R^3 \cdot w^2 + R \cdot M \cdot Cr1}{it \cdot if} \right\}$$

This last equation gives a relation between wheel angular speed (w) and torque demand: $(Td = Te + Tm)$.

3.2. Creation of the Grid of Acceptable Solution

If we have the reference speed in advance, we can know the wheel speed $w(i)$ at any sample time; this means that we can know the torque demand $Td(i) = Te(i) + Tm(i)$ at any sample time, but the goal in this paper is to decide how much is $Te(i)$ and $Tm(i)$ in each sample time.

To make that choice we should define the grid of all possible solution that satisfy the constraints discussed before, for that we suppose that $\overline{X}$ and $\underline{X}$ are maximum and minimum battery charge that can possibly be achieved.

$$\overline{X}(i+1) = \overline{X}(i) - Pe\left(wm(i), Tm(i)\right) \cdot Ts$$

$$\underline{X}(i+1) = \underline{X}(i) - Pe\left(wm(i), Tm(i)\right) \cdot Ts$$

By the same logic and if we start from the last point (backward) we can find the maximum battery energy that make the system converge to SOC limit described in (3).

$$\overline{X}b(i) = \overline{X}b(i+1) + Pe\left(wm(i), Tm(i)\right) \cdot Ts$$

$$\underline{Xb}(i) = \underline{Xb}(i+1) + Pe\left(wm(i), Tm(i)\right) \cdot Ts$$

It is obvious that the grid of possible solution is limited by:

$$X\min = \max(\underline{X}, \underline{Xb}, X1)$$

$$X\max = \min(\overline{X}, \overline{Xb}, X2)$$

After defining the zone of possible solution (**Figure 2**) and using a sample time (Ts = 1 s) and a sample of battery energy (dx = 500 J) to make a mesh, knowing that the number of samples of battery energy $n(i)$ in a time $t(i) = i \cdot Ts$ is described by the integer value plus one

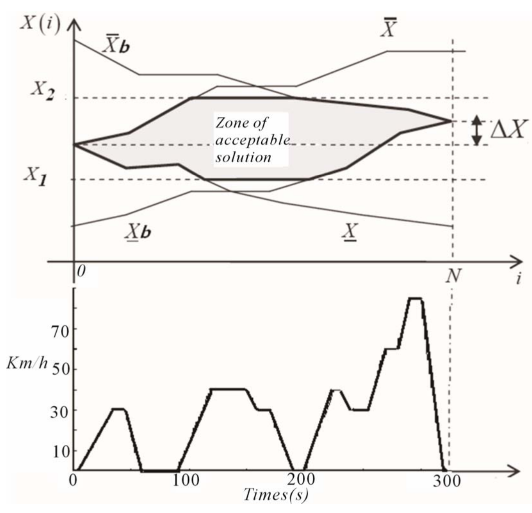

Figure 2. Zone of possible solution over the reference speed.

of $n(i)$:

$$n(i) = \frac{X\max(i) - X\min(i)}{dx}$$

In our case $\Delta X = 0$, in order to make the fuel economy easy to interpret.

3.3. Optimal Trajectory Calculation

Suppose that $(i \times Ts, X(j))$ is a point in the mesh and $Q_j(i)$ is the cost to bring the system from the point $(i \times Ts, X(j))$ to the final point $(N \times Ts, X(0) + \Delta X)$.

$$Q_j(i) = \min_j \sum_i^N C(i, j) \cdot Ts$$

where $C(i, j)$ is the specific fuel consumption at the sample time $t(i)$ and for different engine torque corresponding to different motor torque that makes the battery energy to vary from $X_{\min}$ to $X_{\max}$ by a step of dx (knowing that always we have $Td(i) = Te(i) + Tm(i)$.

In another hand let $\underset{j \to l}{\mathrm{Ce}}(i)$ be the elementary cost to bring the battery from the point $((i-1) \times Ts, X(l))$ to the point $(i \times Ts, X(j))$ and using Bellman's principle of optimality we can finally find that

$$Q_l(i-1) = \min_j \left(Q_j(i) + \underset{j \to l}{\mathrm{Ce}}(i) \right)$$

This means that if we start from the last point and by recurrence until the first point we can solve the problem backwardly.

In this paper a program is made using MATLAB (*M* file) to seek the optimal path based on the relation discussed in Equation (4).

4. Results and Discussion

After optimization; the optimal torque split between the engine and motor is finally found. **Figures 3** and **4** show the engine torque distribution and motor torque distribution all over the speed reference (**Figure 5**).

We notice that the motor torque is engaged at start of the vehicle which is very understandable since the engine efficiency is too poor at low power and we believe that the motor drag the engine with it until a better operating point and then only the engine torque is used.

Also we notice that at low speed coasting, only motor torque is used which is also understandable for the same reason as before, since we know that when coasting at low speed, the power demand is low; which makes the engine not efficient; thus the use of motor is more economic (in fuel consumption). But when coasting at higher speed we notice that the engine torque is used.

Another result we found is that when the bus is decalerating part of the power is stocked in the battery by regenerative braking when the motor become generator and acts as brake by using a negative torque; which makes the battery SOC to increase (another profit of hybrid vehicle over conventional vehicle) (**Figure 6**).

Finally the fuel consumption of the bus can be easily integrated from the engine Map, because in **Figure 3**, we have the engine torque and since we know the speed at the wheel which is proportional to engine speed by the transmission and final drive ratios, the fuel rate can be

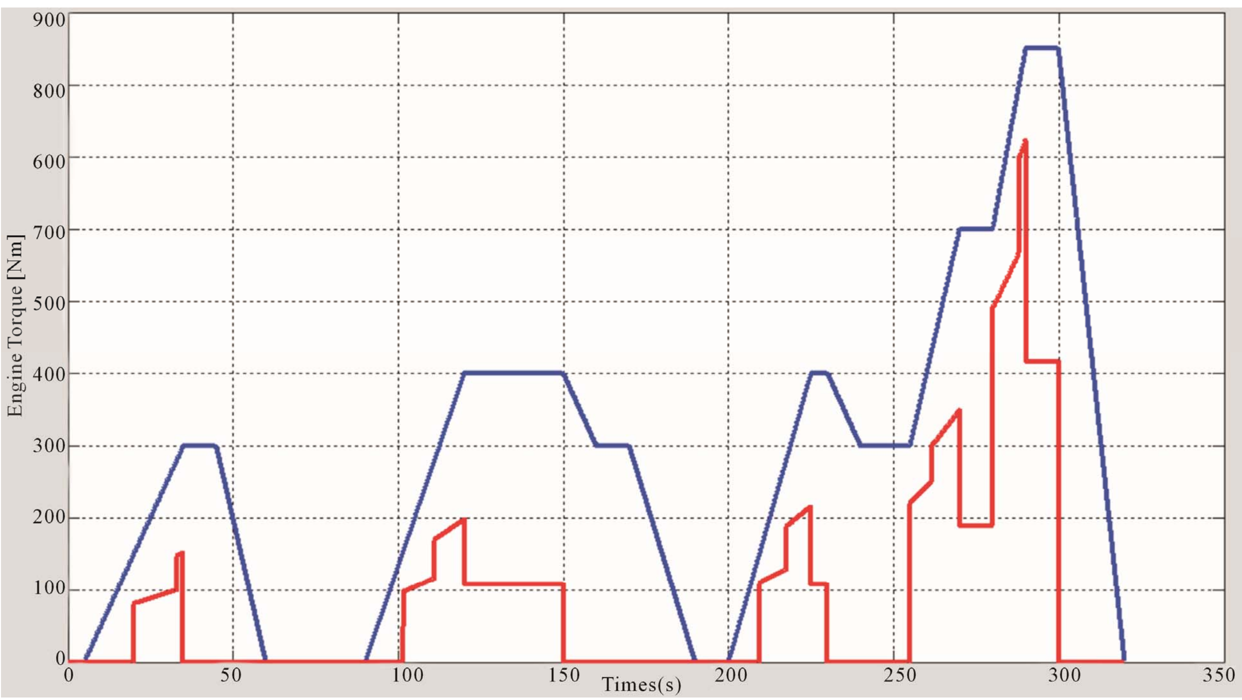

Figure 3. Engine torque distribution over the reference speed.

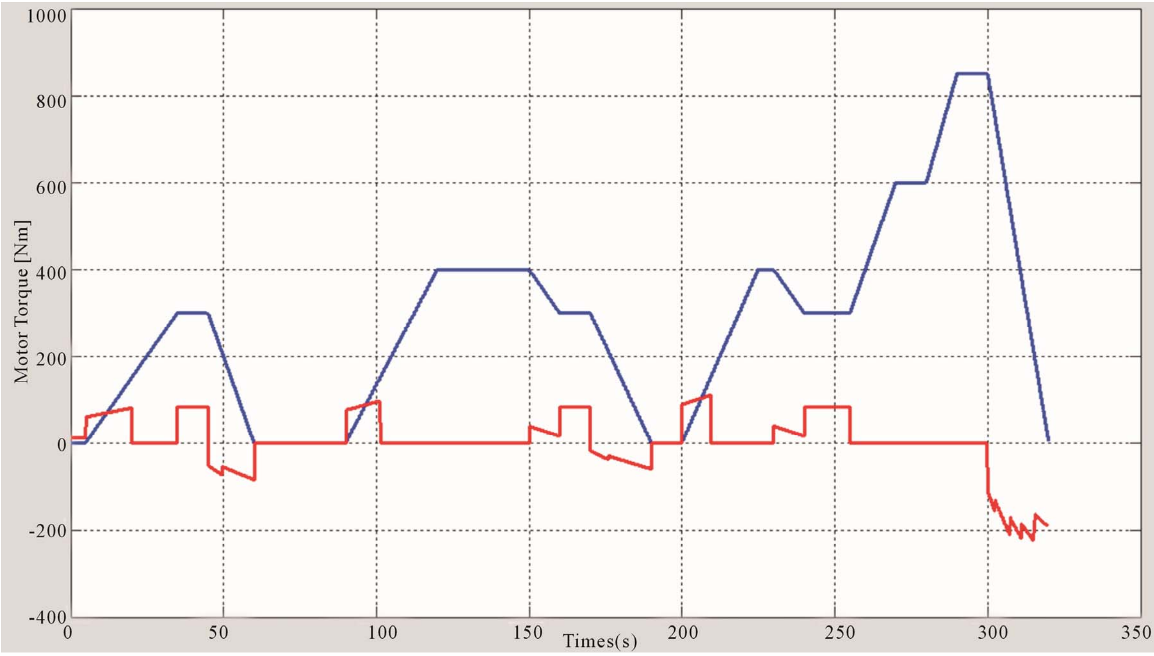

Figure 4. Motor torque distribution over the speed reference.

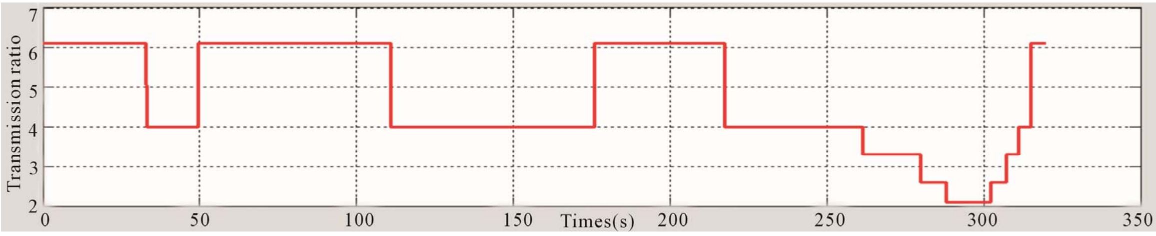

Figure 5. Transmission ratio distribution over the reference speed.

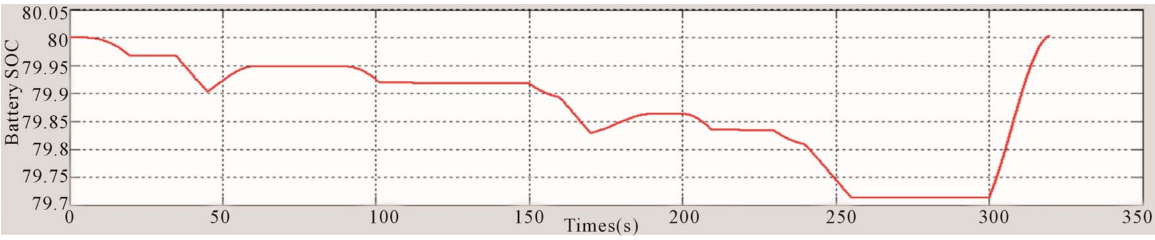

Figure 6. Battery SOC distribution over the reference speed.

found using the engine Map; thus by time integration we can find the fuel consumption. In other hand since we have battery SOCfinal equal to SOCinitial (3), this means that no fuel equivalent has to be transformed to electric energy. After integration we found a fuel consumption of (25.2 L/100 Km) which is an optimal value (benchmark) for this bus that cannot be reached by a real-time control strategy.

5. Conclusion

In this paper we used the dynamic programming method to solve the problem of optimal power management in a hybrid city bus; first we calculated the torque demand at each sample time all over the speed reference; then we specified a zone of acceptable solution (if a solution is not inside this zone that means at least one of the constraints discussed before is not satisfied) and finally we used a sweep method (dynamic programming) to sweep all the possible torque split and choose the solution that gives minimum fuel consumption (optimal solution).

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Nomenclature

Te: Diesel Engine Torque
Tm: PMSM Motor Torque
Td: Torque demand
we: Engine Speed (equal to motor speed)
wt: Speed after Transmission
w: Wheel Speed
V: Vehicle speed
M: Vehicle mass (11000 kg)
*J*1: Sum of inertia moving at the same speed as the engine
*J*2: Sum of inertia moving at the same speed as the transmission
*J*3: Sum of inertia moving at the same speed as the wheel
if, *it*: final drive and transmission ratios
Ts: sample time
*Cr*1, *Cr*2; *Cd*: rolling and aerodynamic coefficien
SOC: state of charge of the battery of (34 Ah, 42 V)
Pe: electric power
C(*Te*, *we*): engine fuel consumption at torque Te and speed we
X: energy stocked into the battery

