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An Analytical Optimal Strategy of the Forest Asset Dynamic Management under Stochastic Timber Price and Growth: A Portfolio Approach

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ABSTRACT

Considering the valuation of forest stands based on revenue from wood sales, concession policy (such as carbon subsidies) and associated costs, the paper focuses on the stochastic control model to study the forest asset dynamic management. The key contribution is to find the optimal dynamic strategy about harvesting quantity in the continual and multiple periods in conditions of stochastic commodity price and timber growth by using portfolio approach. Finally, an analytical optimal strategy is obtained to analyze the quantification relations through which some important conclusions about the optimal forest management can be drawn.

Keywords: *Forest Management, Analytical, Stochastic Price and Growth, Portfolio, Carbon Subsidies*

1. Introduction

Forest ecosystem harbors a large potential for carbon sequestration and biomass production. When the public good benefits of carbon sequestration are considered, the cash flows from forest management include not only timber value but also carbon subsidies. It is necessary to study how the carbon credit payment influences the decisions of forest harvesting.

Graeme Guthrie and Dinesh Kumareswaran [1] considered the effect of carbon credit payment schemes on forest owners' harvest decisions by using a real options model. They studied two possible payment schemes: one where the government rents the carbon sink, in which case the carbon credit payment is proportional to the current carbon stock and another where the government buys the carbon sink, in which case the carbon credit payment will be proportional to the change in the carbon. They referred to rental scheme as the tree-based carbon credit payment scheme but did not give the detail of contrast.

According to this classification, we found an analytical optimal dynamic strategy about harvesting quantity in conditions of stochastic commodity price and timber growth under the buying scheme by using portfolio approach [2]. In this paper, we will focus on the rental scheme to found the strategy about harvesting quantity and compare with the tow results to draw some conclusions about the selection of carbon subsidies schemes.

The paper is organized as follows. A portfolio model, including carbon sequestration under stochastic wood prices and growth, is introduced in Section 2. In Section 3 we solve the model and obtain the analytical optimal strategy of the forest asset dynamic management by applying stochastic control method in portfolio field. Finally, Section 4 contains some conclusions.

2. The Stochastic Control Model

2.1. The Stochastic Prices

Under stochastic prices, if the harvest is delayed until the next period, the owner will face uncertainty over whether prices will be higher or lower than the current period. According to the geometric Brownian motion, suppose the price of timber, P (\$/m^3), characterized by the following stochastic differential equation:

$$dP = \alpha P dt + \sigma_1 P d\omega_1, \tag{1}$$

where both the expected percentage growth rate α (drift term) and the volatility coefficient σ_1 are exogenously given positive constants and ω_1 is Brownian motion, t is the current time [3-5].

2.2. The Stochastic Growth

Define I (m^3) as inventory of timber (or biomass volume), I_{t+1} and I_t as the stocking levels at age of $t + 1$ and t, $q(P,I,t)$ as the control variable (the quantity of cutting) of the time t which is depended on the market price P and stocking level I, $g(t,I_t - q(P,I,t))$ as the timber growth function with I and q. The relationship of all these variables follows:

$$I_{t+1} - I_t = g(t,I_t - q(P,I,t)) - q(P,I,t), \tag{2}$$

If we assume that growth is governed by the stochastic process, the timber volume I satisfies the stochastic differential equation:

$$\begin{aligned} dI &= \mu(I-q)dt + \sigma_2 d\omega_2 - qdt \\ &= [\mu I - (1+\mu)q]dt + \sigma_2 d\omega_2, \end{aligned} \tag{3}$$

The inventory growth rate, $[\mu I - (1+\mu)q]$, depends on the cutting rate policy $q(P, I, t)$ and can be either negative or positive. The parameter μ corresponds to the inventory growth rate as a percentage of the residual inventory. The coefficient σ_2 is the volatility parameter representing the uncertainty over the inventory growth rate and ω_2 is Brownian motion [6-8].

Under these assumptions, the forest owner's optimal problem is to determine the control strategies $q^*(P,I,t)$ at the different age t for the maximization of a so-called value function V during the period $t \in (t_0, T)$, where t_0 is the age of the mature forest that is permitted to be cut, and can be noted as *0* for simplification, and T is the final time when forest is harvested in one time or the expiration year of the forest lease (T_e), $T = \min\{T_e, t(q_t \ge I_t)\}$. We assume that management decisions can be implemented only at fixed time points $t_0, t_1, \ldots\ldots, t_n = T$, e.g., only on a yearly basis, and action payments are also received at these specified time points only.

2.3. The Value Function

Under the optimality portfolio principle, the value of forest is composed of log market value and standing forest value, and the value function V about time t can be decomposed into the sum of the immediate profit π and the expected discounted continuation value $E(V)$:

$$\begin{aligned} V(P,I,t) &= \max_{q^* \in [0,\min(I,\bar{q})]} e^{-\rho t}\{\int_0^T \pi(P,q,t)dt + \\ &E[V(P_{t+\Delta t}, I_{t+\Delta t}, t+\Delta t) | P_t, I_t]\}. \end{aligned} \tag{4}$$

where $\bar{q}$ is the maximum annual cutting permission of policy, ρ is the resource manager's risk-free discount rate, and the immediate profit π is composed of amenity value of standing forest (A) and log value (B) minus the operating costs of harvesting and management (C) (including the appropriate amount of carbon credits that must be purchased back once the harvest is performed).

Especially, if we consider the payments arising from carbon sequestration and suppose that carbon payments are received from any increment in standing volume, the CO_2 prices is a constant number P_C, and the parameter γ (a conversion factor) states how many tons of CO_2 are sequestered in 1 m^3 of wood, the value of standing forest will be as follows:

$$A(P_C, \Delta I) = \gamma P_C (I - q) \tag{5}$$

The log value is

$$B(\tau, P, q) = (1-\tau)Pq, \tag{6}$$

where τ corresponds to the tax rate of revenues.

The cost function is given by the quadratic equation

$$C(q) = a_0 + a_1 q + a_2 q^2, \tag{7}$$

where a_0 is the fixed cost, a_1 is the variable cost, and a_2 is the quadratic term reflecting increasing marginal cost. The quadratic functional form is not critical, and is chosen solely for its algebraic simplicity.

Then, the cash flow from forest [4,8,9] is

$$\begin{aligned} \pi(P,q,t) &= A(P_C, \Delta I) + B(\tau,P,q) - C(q) \\ &= \gamma P_C(I-q) + (1-\tau)Pq - (a_0 + a_1 q + a_2 q^2), \end{aligned} \tag{8}$$

So, the value function under the above assumptions follows the equation:

$$\begin{aligned} &V(P,I,t) = \\ &\max_{q^* \in [0,\min(I,\bar{q})]} e^{-\rho t}\{\int_0^T [\gamma P_C(I-q) + (1-\tau)Pq - \\ &(a_0 + a_1 q + a_2 q^2)]dt + E[V(P_{t+\Delta t}, I_{t+\Delta t}, t+\Delta t) | P_t, I_t]\}. \end{aligned} \tag{9}$$

3. An Analytical Solution

According to the above model, the classical tools of stochastic optimal control and maximum principle [10,11] lead to the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \rho V &= V_t + [\mu I - (1+\mu)q]V_I + \alpha P V_P + \\ &\frac{1}{2}\sigma_2^2 P^2 V_{PP} + \frac{1}{2}\sigma_1^2 V_{II} + \sigma_1\sigma_2 P V_{PI} + \\ &e^{-\rho t}\{(\gamma P_C I - a_0) - [\gamma P_C - (1-\tau)P + a_1]q - a_2 q^2\}, \end{aligned} \tag{10}$$

We denote it simply as:

$$\begin{aligned} \rho V &= V_t + [\mu I - (1+\mu)q]V_I + \alpha P V_P + \\ &\frac{1}{2}\sigma_2^2 P^2 V_{PP} + \frac{1}{2}\sigma_1^2 V_{II} + \sigma_1\sigma_2 P V_{PI} + \\ &e^{-\rho t}(\lambda_0 - \lambda_1 q - \lambda_2 q^2), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \lambda_0 &= \gamma P_C I - a_0, \\ \lambda_1 &= \gamma P_C - (1-\tau)P + a_1, \\ \lambda_2 &= a_2, \end{aligned} \tag{12}$$

From (11), the optimal policy q^* satisfies

$$q^* = -\frac{1}{2\lambda_2}[(1+\mu)V_I e^{\rho t} + \lambda_1], \quad (13)$$

Putting (13) in (11), we obtain another partial differential equation:

$$\rho V = V_t + \mu I V_I + \alpha P V_P + \frac{1}{2}\sigma_2^2 P^2 V_{PP} + \frac{1}{2}\sigma_1^2 V_{II} + \sigma_1\sigma_2 P V_{PI} + \lambda_0 e^{-\rho t} + \frac{(1+\mu)^2}{4\lambda_2} e^{\rho t} V_I{}^2 + \frac{\lambda_1(1+\mu)}{2\lambda_2} V_I + \frac{\lambda_1{}^2}{4\lambda_2} e^{-\rho t}. \quad (14)$$

We try to find a solution to (14) in the following way:

$$V = e^{-\rho t} f(P, I), \quad (15)$$

Introducing this in (14), we obtain:

$$2\rho f = \mu I f_I + \alpha P f_P + \frac{1}{2}\sigma_2^2 P^2 f_{PP} + \frac{1}{2}\sigma_1^2 f_{II} + \sigma_1\sigma_2 P f_{PI} + \frac{(1+\mu)^2}{4\lambda_2} f_I{}^2 + \frac{\lambda_1(1+\mu)}{2\lambda_2} f_I + \frac{\lambda_1{}^2}{4\lambda_2} + \lambda_0, \quad (16)$$

Namely, as that:

$$2\rho f = \mu I f_I + \frac{1}{2}\sigma_1^2 f_{II} + \gamma P_C I + \frac{(1+\mu)^2}{4a_2} f_I{}^2 + \frac{(\gamma P_C + a_1)(1+\mu)}{2a_2} f_I + \sigma_1\sigma_2 P f_{PI} - \frac{(1+\mu)(1-\tau)}{2a_2} P f_I + \alpha P f_P + \frac{1}{2}\sigma_2^2 P^2 f_{PP} + \frac{(1-\tau)^2}{4a_2} P^2 - \frac{(\gamma P_C + a_1)(1-\tau)}{2a_2} P + \frac{2\gamma P_C a_1 + \gamma^2 P_C{}^2 + a_1{}^2 - 4a_0 a_2}{4a_2} \quad (17)$$

Decompose $f(P, I)$ as:

$$f = g(I) + h(P), \quad (18)$$

Introducing this in (17), we obtain:

$$\mu I g_I + \frac{1}{2}\sigma_1^2 g_{II} + \frac{(1+\mu)^2}{4a_2} g_I{}^2 + \frac{(\gamma P_C + a_1)(1+\mu)}{2a_2} g_I + \gamma P_C I - \frac{(1+\mu)(1-\tau)}{2a_2} P g_I - 2\rho h + \alpha P h_P + \frac{1}{2}\sigma_2^2 P^2 h_{PP} + \frac{(1-\tau)^2}{4a_2} P^2 - \frac{(\gamma P_C + a_1)(1-\tau)}{2a_2} P + \frac{2\gamma P_C a_1 + \gamma^2 P_C{}^2 + a_1{}^2 - 4a_0 a_2}{4a_2} = 2\rho g. \quad (19)$$

Assume that

$$g(I) = k_1 I + l_1, . h(P) = k_2 P^2 + l_2 P + m_2, \quad (20)$$

Introducing this in (18), we obtain:

$$(-2\rho k_1 + \mu k_1 + \gamma P_C) I + [\frac{(1+\mu)^2}{4a_2} k_1{}^2 + \frac{[\gamma P_C + a_1](1+\mu)}{2a_2} k_1 - 2\rho l_1] + [\frac{(1-\tau)^2}{4a_2} + \sigma_2^2 k_2 + 2\alpha k_2 - 2\rho k_2] P^2 + [\alpha l_2 - \frac{(1+\mu)(1-\tau)}{2a_2} k_1 - 2\rho l_2 - \frac{[\gamma P_C + a_1](1-\tau)}{2a_2}] P - 2\rho m_2 + \frac{2\gamma P_C a_1 + \gamma^2 P_C{}^2 + a_1{}^2 - 4a_0 a_2}{4a_2} = 0, \quad (21)$$

It is not difficult to decide $k_1, l_1, .k_2, l_2, m_2$ by the method of undeterminated coefficients, such as

$$k_1 = \frac{\gamma P_C}{2\rho - \mu}, (2\rho - \mu \neq 0) \quad (22)$$

We are not usually interested in the value function, but rather in the optimal strategy. From (12), (13), (15), (18) and (22), it is given by:

$$q^* = \frac{1}{2a_2}[(1-\tau)P - \frac{2\rho + 1}{2\rho - \mu}\gamma P_C - a_1] \quad (23)$$

4. Conclusions

It is not difficult to draw some conclusions about the quantity of harvesting from (22):

1) The quantity is decided by the log market price (*P*), the concession value or the carbon price (P_C), the timber growth average speed (μ), the discount rate (ρ), the tax rate (τ), the carbon transform coefficient (γ), and the costs coefficients (a_1 and a_2), but it has nothing with the gross of present forest (*I*), the fixed cost (a_0), the price volatility (σ_1), and the timber growth volatility (σ_2).

2) It is obvious that the more expensive timber price is and the lower tax rate is, the more trees will be cut down.

3) Normally, two times of the discount rate is larger than the timber growth rate ($2\rho > \mu$), so the more concession value there is, the less trees will be cut down. It means that the forest concession or the carbon credit payment schemes can discourage deforestation. Otherwise, if the timber growth average speed is faster than two times of the discount rate, the owner will harvest more forest to pursuit more wood sales value, with the fast-growing forestry as an example. In another word, only policies such as subsidies from government are not able to satisfy the forest owners' profits. So it is important to improve the management and technology to increase the growth rate of forest.

4) Obviously, the more cost of harvesting, the less

trees will be cut down. But we find that the fixed cost does not influence it. The cost is also can be regarded as the penalty for the destruction.

5) As said above, the harvesting strategy has nothing with the gross of present forest, so the control of the portfolio is suitable for any scope of forest regardless of the amount.

In a word, we believe many more interesting conclusions will be brought under the different assumptions by using the portfolio approach, and welcome more researchers to take part in the field.

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