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The Equation of Real Option Value under Trinomial Tree Model

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Abstract

Trinomial tree model is excelled than binomial tree model in precision and calculation from an example. Triple Tree pricing error is much smaller, the result is simpler. By careful analysis, we conclude the real option value under trinomial tree model.

Keywords

Trinomial Tree Model, Binomial Tree Model, Equation of Real Option Value

1. Introduction

The numerical computation of real option value is very important in the evaluating of venture investment. We develop a trinomial tree pricing model of the real option, and prove that the equation of real option value under trinomial tree model is approximate to Black-Scholes equation. It is obvious that trinomial model is excelled than binomial tree model in precision and calculation from an example. What is more, the option price is determined in the Triple Tree Model, which is better to match with market realities.

At the beginning, the article introduces the binomial tree model. Although the binomial tree model is widely used in real option value, there are some problems it can't handle. Then some people come up with a new method-the trinomial tree pricing model of the real option.

The research status of this method is mentioned. B. Kamiad increased the possibility of each issue status in order to improve the accuracy of the calculation; Boyle came up with the trigeminal tree model, in which model the rise and fall of the two states is symmetrical. Tian obtained the corresponding risk neutral probability but its expression is the complex exponential function of time. Xie Chi used trinomial tree method to solve CEV procedure in Option pricing analysis. In a word, the trinomial tree model is wildly used. The following part will prove that the equation of real option value under trinomial tree model is approximate to Black-Scholes equation.

Crépey [3] Calibrated the local volatility in a trinomial tree by using Tikhonov regularization. Ding and Zeng [1] researched trinomial tree model of real option value. Jiang [4] get the convergence of the Binomial Tree Method for American Options in a Jump-Diffusion Model. Ma and Zhu [5] obtained the convergence rates of trinomial tree methods for option pricing under regime-switching models. Wu and Wang [2] arrived at Europin option pricing under trinomial tree model.

In this paper, we conclude the real option value under trinomial tree model. There are four Basic assumptions: the first one is that the expected benefits of investment projects $V(t)$ follows the geometric Brownian motion; the second one is that the markets are perfect, there is no risk-free arbitrage; the third one is that there is no need to pay transaction fees and taxes when the investment account transactions; the last one is that two value of investment projects expected return have nothing to do with the ordered sequence of movement.

The main part is to introduce the procedure in the paper. We need discretize continuous variables $V(t)$, we have

$$E(V) = V \circ e^{r\Delta t} \tag{1}$$

$$E(V^2) = V^2 \circ e^{(2r+\sigma^2)\Delta t} \tag{2}$$

$$E(V^3) = V^3 \circ e^{(3r+3\sigma^2)\Delta t} \tag{3}$$

$$u \circ d = 1 \tag{4}$$

$$E(V) = p_u \circ (u \circ V) + p_m \circ V + p_d \circ (d \circ V) \tag{5}$$

$$E(V^2) = p_u \circ (u \circ V)^2 + p_m \circ V^2 + p_d \circ (d \circ V)^2 \tag{6}$$

$$E(V^3) = p_u \circ (u \circ V)^3 + p_m \circ V^3 + p_d \circ (d \circ V)^3 \tag{7}$$

Then we have

$$\begin{cases} p_u + p_m + p_d = 1 \\ p_u \circ u + p_m + p_d \circ d = e^{r\Delta t} \\ p_u \circ u^2 + p_m + p_d \circ d^2 = e^{(2r+\sigma^2)\Delta t} \\ p_u \circ u^3 + p_m + p_d \circ d^3 = e^{(3r+\sigma^3)\Delta t} \\ u \circ d = 1 \end{cases}$$

we get the solution as follows:

$$u = M + \sqrt{M^2 - 1}$$

$$d = M - \sqrt{M^2 - 1}$$

$$p_u = \frac{e^{r\Delta t}(1+d) - e^{(2r+\sigma^2)\Delta t} - d}{(d-u)(u-1)}$$

$$p_m = \frac{e^{r\Delta t}(u+d) - e^{(2r+\sigma^2)\Delta t} - 1}{(1-d)(u-1)}$$

$$p_d = \frac{e^{r\Delta t}(1+u) - e^{(2r+\sigma^2)\Delta t} - u}{(1-d)(d-u)}$$

Next, we use the same method as binomial tree model to price real option. One has

$$F(V, i\Delta t) = e^{-r\Delta t}\left[p_u F(u^\circ V, (i+1)\Delta t) + p_m F(V, (i+1)\Delta t) + P_d F(d^\circ V), (i+1)\Delta t\right]$$

So, when $V = V_0$, we obtain

$$F(V_0, 0) = e^{-rn\Delta t}\sum \frac{n!}{i!\,j!(n-i-j)!} p^i{}_u \circ p_m^j \circ p_d^{n-i-j} \circ F\left(u^i \circ d^{n-i-j} \circ V_0, n^\circ \Delta t\right)$$

Finally, there is a brief introduction about the relationship between binomial tree model and trinomial tree model. As we all know, Black-Scholes equation is the limit of the binomial tree model. Trinomial tree model has the same result. We prove that the equation of real option value under trinomial tree model is first order approximate to Black-Scholes equation.

It is proved that Trinomial model is excelled than binomial tree model in precision and calculation from an example. Triple Tree pricing error is much smaller, the result is simpler.

What is more, real option value under trinomial tree model is more suitable for real market. Take an example: We assume that volatility and interest rate are constant, in real market, volatility and interest rate is a random variable. In order to better meet market pricing option and real life, it is necessary to put forward trinomial tree model. Wu Bin introduces the risk neutral measure in his article, he said that the interest rate volatility and the premise of random with trigeminal tree model of option pricing are studied by assuming risk--neutral measure and he used martingale measure to price the stochastic volatility and random interest rate of European option.

Wu Bin supposed that H represents the stock price rises, M represents the stock prices steady, T represents the stock price down; u, m, d are on behalf of the proportion of rise, smooth, decline. If initial wealth is X_0, we buy Δ_0 share of stock, the stock price is S_0, we have the derivative securities prices at 1 time period:

$$V_1(\omega_1)\begin{cases} V_1(H), \omega_1 = H \\ V_1(M), \omega_1 = M \\ V_1(T), \omega_1 = T \end{cases}$$

Then we get

$$X_0 + \Delta_0\left(\frac{1}{1+r_0} S_1(H) - S_0\right) = \frac{1}{1+r_0} V_1(H)$$

$$X_0 + \Delta_0\left(\frac{1}{1+r_0} S_1(M) - S_0\right) = \frac{1}{1+r_0} V_1(M)$$

$$X_0+\Delta_0\left(\frac{1}{1+r_0}S_1(T)-S_0\right)=\frac{1}{1+r_0}V_1(T)$$

We make $p_0+\tilde{m}_0+\tilde{q}_0=1$, then we get

$$X_0+\Delta_0(\frac{1}{1+r_0}(p_0S_1(H)+\tilde{m}_0S_1(M)+\tilde{q}_0S_1(T))-S_0)$$
$$=\frac{1}{1+r_0}(p_0V_1(H)+\tilde{m}_0V_1(M)+\tilde{q}_0V_1(T))$$

Risk neutral measure reads as

$$p_0=\frac{1+r_0-d_0}{u_0-m_0}-\lambda_0\frac{(u_0-1-r_0)(m_0-d_0)}{(u_0-m_0)(u_0-d_0)}$$
$$\tilde{m}_0=\lambda_0\frac{(u_0-1-r_0)}{(u_0-m_0)}$$
$$\tilde{q}_0=(1-\lambda_0)\frac{(u_0-1-r_0)}{(u_0-d_0)}.$$

So we obtain the price of zero moment derivatives

$$V_0=\frac{1}{1+r_0}[p_0V_1(H)+\tilde{m}_0V_1(M)+\tilde{q}_0V_1(T)]$$

If we consider trigeminal tree model of option price in n time period, the risk neutral measure reads as

$$p_n(\omega_1\omega_2...\omega_n)=\frac{1+r_n(\omega_1\omega_2...\omega_n)-d_n(\omega_1\omega_2...\omega_n)}{u_n(\omega_1\omega_2...\omega_n)-d_n(\omega_1\omega_2...\omega_n)}-\lambda_n\frac{(u_n-1-r_n)(m_n-d_n)}{(u_n-m_n)(u_n-d_n)}$$
$$\tilde{m}_n(\omega_1\omega_2...\omega_n)=\lambda_n\frac{u_n(\omega_1\omega_2...\omega_n)-1-r_n(\omega_1\omega_2...\omega_n)}{u_n(\omega_1\omega_2...\omega_n)-m_n(\omega_1\omega_2...\omega_n)}$$
$$\tilde{q}_0(\omega_1\omega_2...\omega_n)=(1-\lambda_n)\frac{u_n(\omega_1\omega_2...\omega_n)-1-r_n(\omega_1\omega_2...\omega_n)}{u_n(\omega_1\omega_2...\omega_n)-d_n(\omega_1\omega_2...\omega_n)}.$$

So, we can concluded that real option value under trinomial tree model as follows

$$V_n=\frac{1}{1+r_n}[p_nV_{n+1}+\tilde{m}V_{n+1}+\tilde{q}_nV_{n+1}].$$

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