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Dynamic Option Pricing Model Based on the Realized-GARCH-NIG Approach

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Abstract

In this paper, we take the advantage of high frequency data to develop option pricing model and select the Realized GARCH model to describe the volatility of assets, use NIG distribution to describe the distribution of underlying assets, and also build the Realized-GARCH-NIG model to price the option. Finally, we obtain the dynamic option pricing model based on the Realized-GARCH-NIG approach. To verify the effect of the dynamic option pricing model based on the Realized-GARCH-NIG approach, this paper provides the empirical analysis between the dynamic option pricing model based on the Realized-GARCH-NIG approach and the B-S option pricing model. The results show that the option value obtained from the dynamic option pricing model based on the Realized-GARCH-NIG approach is more accurate and effective than the B-S option pricing model.

Keywords

Option Pricing Model, Realized GARCH Model, High Frequency Data, NIG Distribution

1. Introduction

Option is a derivative financial tool, which now has become an important risk management tool for investors. The estimation and forecast of asset volatility have a great influence on option value, which have become the important direction of current option pricing research. Black and Scholes [1] derived the B-S option pricing model with risk neutral conditions, which lay a solid foundation for modern option pricing theory. Merton and Robert [2] considered the jump process in the market price volatility, and proposed the option pricing model based on market price jump-diffusion process. Hull, John, and Alan White [3] built random price volatility model by modeling the volatility with random process method, and proposed the option pricing model with random volatility diffusion. Duan [4] built an option pricing model by estimating the time-varying characteristics of asset volatility with GARCH model. Melick, William and Thomas [5] considered the mixed distribution assumption in the price dynamics process, presented the option pricing model that assets return follows the mixed normal distribution, and conducted empirical analysis to obtain better results. Duan [6] considered the characteristics of fat tail, sharp peak and skewness distribution, and built option pricing model based on GARCH-GED. But GED distribution selected by Duan is still symmetric, which can't describe the asymmetric distribution characteristics of assets. Chorro [7] selected more general generalized hyperbolic distribution to de-

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scribe the distribution characteristics of assets, described the dynamic process of asset price with GARCH process, built option pricing model based on GARCH-GH, and improved the accuracy of option pricing model by conducting empirical analysis on CAC40 and SP500 index options.

Efficient estimation on asset volatility and distribution characteristics is an important factor affecting the option pricing model. As high frequency data is widely used in recent years, many econometricians start to take the advantages of high frequency data into account, and directly model the volatility of asset with high frequency data. The realized volatility model proposed by Andersen [8] is representative, and the realized volatility model with high frequency data adopted by them has included plenty of intraday trading information. Hansen [9] presented Realized GARCH model to estimate the volatility, and obtained better results than other models in volatility estimation. Since all previous option pricing researches are based on low frequency data research results, high frequency data hasn't been utilized for option pricing research yet.

Different from previous option pricing research, we take the advantages of high frequency data into account, and use high frequency data to make research of option pricing model to describe the volatility of assets with Realized GARCH model integrating high frequency data. To capture the skewness, sharp peak and fat tail distribution characteristics, we use the normal inverse gaussian (NIG) distribution to describe the distribution of assets rather than the normal distribution and build Realized-GARCH-NIG model to price the option. Final, we obtain the dynamic option pricing model based on the Realized-GARCH-NIG approach. In this paper we use the NASDAQ 100 Index Option data as research example, give an empirical analysis between the dynamic option pricing model based on the Realized-GARCH-NIG approach and the B-S option pricing model.

The structures of this article are shown in the order as follows: Section 2 is Research design, Section 3 is Empirical Methods, Section 4 is results and discussion, and Section 5 is conclusion.

2. Research Design

2.1. The Option Pricing Model

The B-S option pricing model proposed by Black and Scholes [1] is the beginning of option pricing research, and in the meanwhile, scholars represented by John [10] have developed risk neutral (martingale) option pricing theory, and derive option pricing model with risk neutral assumption. Option price model can be obtained by constructing equivalent martingale measure, and risk neutral (martingale) pricing method is now the mainstream option pricing method. So risk neutral (martingale) pricing method is utilized to research option pricing model in this paper. Let $S(t)$ as the price of asset, and $g(S,t)$ as the price of derivative product. Let $B(t)$ as the price of risk-free asset of a unit. If risk neutral measure Q exists, then $g(S,t)/B(t)$ is the martingale**,** this paper can obtain

$$\frac{g(S,t)}{B(t)} = E^Q\left(\frac{g(S,T)}{B(T)}\right) \tag{1}$$

As the B-S option model,

$$C_t = S_t N(d_1) - K\exp(-r_f T) N(d_2) \tag{2}$$

This paper can obtain

$$g(S,t) = B(t) E^Q\left(\frac{g(S,T)}{B(T)}\right) = \frac{B(t)}{B(T)} E^Q\left(g(S,T)\right) \tag{3}$$

If the return of risk-free asset r_f is assumed to be fixed, the price $B(t)$ of risk-free asset of a unit at t is

$$B(t) = \exp(r_f t) \tag{4}$$

So, this paper can obtain

$$g(S,t) = \exp\left(-r_f(T-t)\right) E^Q\left(g(S,T)\right) \tag{5}$$

The function shows that the current price of derivative product is the discounted value of the price under risk neutral measure Q at T. According to equivalent martingale measure principle, the option value can be regarded as the function of the underlying asset, so the option that can be obtained under the underlying asset condition is as follows:

$$Call_t = e^{-r_f(T-t)} E^Q\left(\max\left\{g\left(S_T\right)-K,0\right\}\right) \tag{6}$$

where, r_f is risk-free interest rate, E^Q is the expectation under risk neutral measure, T is the expiration date, $g(\cdot)$ is the function of underlying asset, S_T is the price of the underlying asset i at the expiration date , and K is the exercise price of option. $g(\cdot)$ is the function of underlying asset.

2.2. The Dynamic Option Pricing Model Based on the Realized-GARCH-NIG Approach

Duan [4] utilizes GARCH process to describe the volatility of assets, but the data selected by such GARCH model is low frequency data. As high frequency data is widely used in recent years, many econometricians start to directly model the volatility with high frequency data. Andersen [8] set up a realized volatility model with the high frequency data. Hansen [9] integrated the advantages of high frequency data and low frequency data in volatility estimation, and presented the Realized GARCH model. Such Realized GARCH model is as follows:

$$\begin{aligned} r_t &= \mu_0 + \sqrt{h_t} z_t,\ \ z_t \sim N(0,1) \\ h_t &= \omega + \beta h_{t-1} + \gamma x_{t-1} \\ x_t &= \xi + \mu h_t + \tau(z_t) + \sigma_\varepsilon \varepsilon_t \\ \tau(z_t) &= \tau_1 z_t + \tau_2\left(z_t^2 - 1\right),\ \ \varepsilon_t \sim N(0,1) \end{aligned} \tag{7}$$

where μ_0 represents a mean equation, and residual error z_t of the return follows standard normal distribution, and x_t can be any one of realized volatility measure, and errors caused by market intraday microstructure noise and non-trading time (closing) are adjusted by coefficients ξ and μ. $\tau(z_t)$ is the lever function which is used to represent the asymmetric influence of return on volatility, and parameters τ_1 and τ_2 are used to describe different influences of positive and negative returns to the volatility. But in Hansen's research, they adopt normal distribution assumption, but actual financial data is abnormal distribution. Eberlein [11] first applied the generalized hyperbolic distribution into financial research. Generalized hyperbolic distribution is now widely used in financial research field. Probability density function of generalized hyperbolic distribution is defined as follows:

$$f_{GH}(x,\lambda,\alpha,\beta,\delta,\mu) = h(\lambda,\alpha,\beta,\delta,\mu)\left(\delta^2 + (x-\mu)^2\right)^{\lambda/2-0.25} K_{\lambda-0.5}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right) e^{\beta(x-\mu)} \tag{8}$$

where, K_λ is the adjusted Bessel function of the third kind, and,

$$h(\lambda,\alpha,\beta,\delta,\mu) = \frac{\left(\alpha^2-\beta^2\right)^{\lambda/2}}{\sqrt{2\pi}\alpha^{\lambda-0.5}\delta^\lambda K_\lambda\left(\delta\sqrt{\alpha^2-\beta^2}\right)} \tag{9}$$

when $\lambda > 0$, $\delta \ge 0$, $|\beta| < \alpha$; when $\lambda = 0$, $\delta > 0$, $|\beta| < \alpha$; when $\lambda < 0$, $\delta > 0$, $|\beta| \le \alpha$. when $\lambda = -0.5$, GH distribution is called the normal inverse gaussian (NIG) distribution, and the density function of NIG distribution is as follows:

$$f_{NIG}(x,\alpha,\beta,\delta,\mu) = \frac{\alpha\delta}{\pi}\exp\left(\delta\sqrt{\alpha^2-\beta^2}\right)\frac{K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\sqrt{\delta^2+(x-\mu)^2}}\exp\left(\beta(x-\mu)\right),\ x\in R \tag{10}$$

If X follows *NIG* distribution, it is simplified as:

$$X \sim NIG(\alpha,\beta,\delta,\mu) \tag{11}$$

where in, β is deviation parameter, α and δ are scale parameters, and μ is location parameter. In view of above analysis reasons, this paper describes the volatility of assets with Realized-GARCH-NIG model. Hence, this complete Realized-GARCH-NIG equation is as follows:

$$\begin{aligned} r_t &= \mu_0 + \sqrt{h_t} z_t, \ z_t \sim NIG \\ h_t &= \omega + \beta h_{t-1} + \gamma x_{t-1} \\ x_t &= \xi + \mu h_t + \tau(z_t) + \sigma_\varepsilon \varepsilon_t \\ \tau(z_t) &= \tau_1 z_t + \tau_2 (z_t^2 - 1), \ \varepsilon_t \sim N(0,1) \end{aligned} \tag{12}$$

This paper utilizes Realized-GARCH-NIG model to describe the volatility process and distribution of assets. Duan provided a method of converting asset distribution into risk neutral distribution in his research, and this paper utilizes Realized-GARCH-NIG model to price the option by reference to Duan's method. According to the risk neutral pricing principle, option price can be expressed as:

$$Call_t = \exp(-r_f (T-t)) E^Q \left(\max\left\{ g\left(S_0 \cdot \exp\left(\sum_{t=1}^{T} r_t \right) \right) - K, 0 \right\} \right) \tag{13}$$

3. Empirical Methods

3.1. Data Sources

This paper selects the data from the database of the Chicago Board Options Exchange, and selects the NASDAQ 100 Index Option data as the research samples from June 01, 2012 to May 30, 2013. To verify the effectiveness of option pricing model proposed in this paper, this paper selects the B-S option pricing model to compare with the dynamic option pricing model based on the Realized-GARCH-NIG approach. Because of the complicated model that we used in this paper, we use the Monte-Carlo method (simulation times: 10000) to analyse the option price.

3.2. The Estimation Results

Table 1 shows the Realized-GARCH-NIG model parameter results, and **Table 2** shows parameter results of the NIG distribution.

It can be seen from **Table 1** that the estimated values Log, AIC and BIC can reflect the effectiveness of parameter results to a certain extent, and it can be seen from **Table 2** that NIG distribution parameters are significant, and the estimated values Log, AIC and BIC can reflect the effectiveness of parameter results to a certain extent.

3.3. Comparison of Empirical Results

To quantitatively compare the pricing effect of model, this paper compares the model with three different error measurement methods. These three errors are root mean square error (*RMSE*), average absolute error (*AAE*) and average relative error (*APRE*) respectively. Calculation functions are as follows:

Table 1. Realized GARCH model parameters of NASDAQ 100 index option.

	ω	β	γ	ξ	μ	τ_1	τ_2	σ_ε	Log	AIC	BIC
Mean	0.0461	0.0170	0.2603	0.0594	0.0261	0.0611	0.0575	0.0154	375.72	480.44	334.53
Median	0.0407	0.0136	0.2210	0.0456	0.0187	0.0589	0.0587	0.0111	345.61	410.23	324.32
Std	0.0870	0.0299	0.0631	0.0059	0.0558	0.0049	0.0571	0.0382	24.76	74.43	84.93
Minimum	0.0356	0.0010	0.0470	0.0285	0.0232	0.0507	0.0461	0.0097	305.33	368.66	263.74
Maximum	0.0691	0.0296	0.7317	0.0933	0.0771	0.2019	0.1578	0.2677	510.94	652.44	552.98

Table 2. NIG distribution parameter of NASDAQ 100 index options.

	υ_i	ζ_i	δ_i	μ_i	Log	AIC	BIC
Mean	0.1527	0.5507	0.0645	0.0489	334.4	279.1	356.1
Median	0.1838	0.5541	0.0512	0.0474	364.7	205.5	285.9
Std	0.0645	0.0456	0.0285	0.4547	51.3	48.5	54.5
Minimum	0.0655	0.0943	0.0176	0.0182	258.7	208.1	227.6
Maximum	0.2558	0.9012	0.1052	0.0970	585.6	355.6	471.6

Table 3. Pricing errors of different models.

	RMSE	AAE	ARPE
the B-S option pricing model	53.07	62.19	0.94
the dynamic option pricing model based on the Realized-GARCH-NIG approach	40.62	42.85	0.29

$$RMSE = \sqrt{\sum_{n=1}^{N} \frac{\left(Call_{N1IO_t}^{\zeta} - Call_{N1IO_t}\right)^2}{N}} \tag{14}$$

$$AAE = \sum_{n=1}^{N} \frac{\left(Call_{N1IO_t}^{\zeta} - Call_{N1IO_t}\right)^2}{N} \tag{15}$$

$$ARPE = \sum_{n=1}^{N} \frac{\left(Call_{N1IO_t}^{\zeta} - Call_{N1IO_t}\right)^2}{Call_{N1IO_t} \cdot N} \tag{16}$$

where, $Call_{N1IO_t}$ is the market price of NASDAQ 100 Index Option, and $Call_{N1IO_t}^{\zeta}$ is the estimation of option value.

Table 3 shows the error comparison results. From three indexes, it can be seen that estimation error of dynamic option pricing model based on the Realized-GARCH-NIG approach is less than the B-S option pricing mode.

Above empirical results indicate that the B-S option pricing model can't accurately describe the volatility and the characteristics of fat tail, sharp peak and skewness distribution. These defects result the deviation of option price. In this paper, the Realized GARCH model integrating high frequency data selected reflects unique advantages and the empirical results indicate that the Realized-GARCH-NIG model built in this paper is more accord with actual market situation and it can effectively describe the volatility of assets. Compared with the B-S option pricing mode, the dynamic option pricing model based on the Realized-GARCH-NIG approach is slightly deviated from actual option price and more accord with actual market situation, and can improve the accuracy and effectiveness of option pricing model.

4. Conclusion

In previous option pricing researches, low frequency data and normal distribution are often used to estimate the dynamic process of assets, those inaccurate estimation methods have a larger influence on option value estimation. Based on the advantages of high frequency data, NIG distribution, this paper utilizes the Realized-GARCH-NIG model to describe the volatility and the characteristics of fat tail, sharp peak and skewness distribution, and gets the dynamic option pricing model based on the Realized-GARCH-NIG approach. Choice the NASDAQ 100 Index Option data as research example, give an empirical analysis between the dynamic option pricing model based on the Realized-GARCH-NIG approach and the B-S option pricing model. Empirical results indicate that the dynamic option pricing model based on the Realized-GARCH-NIG approach is more accord with actual market situation, and can effectively improve the accuracy of option pricing model. With the development of financial markets, the demand of option products will certainly continue to emerge, and the option pricing model proposed in this paper provides a good reference in option pricing research. This paper will further research the application of high frequency data in option pricing model, and research high frequency nonlinear option pricing model and realization method in a dynamic market process.

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