Open Journal of Social Sciences, 2015, 3, 97-102
Published Online September 2015 in SciRes. http://www.scirp.org/journal/jss
http://dx.doi.org/10.4236/jss.2015.39015

A Game-Theoretic Model for Bystanders' Behaviour in Classes with Bullying

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Received 29 August 2015; accepted 15 September 2015; published 18 September 2015

Abstract

In this paper, the behaviour of bystanders in a classroom in which bullying is occurring is analyzed using Game theory. We focus on bystander's behaviour and formulate a threshold model. Our analysis shows that as class sizes become smaller, the probability of bullying being stopped increases.

Keywords

Bullying, Free Rider, Non-Corporative Game

1. Introduction

According to a survey on problematic behaviours by the Ministry of Education, Culture, Sports, Science and Technology (MEXT) in Japan, the number of recognized incidents of bullying at all grade levels nationwide in 2012 was 198,108, 2.82 times than that of the previous year. This represents the largest number of recorded incidents since the survey began [1]. Additionally, there were 196 student suicides in 2012, of which 3.1% were understood to have resulted from bullying; this made the issue of bullying a serious social problem that must be solved.

Morita [2] emphasised the importance of bystanders' behaviour. Shibata *et al*. [3] conducted an economic analysis on the behaviour of bystanders. Glass and Smith [4] established that a reduced class size can be expected to produce academic achievement. Additionally, Smith and Glass [5] have shown that small class size is effective in improving student attitudes and behaviour.

This paper comprises several sections in which different aspects of bullying analyses are discussed. Section 2 explains our model in detail. Section 3 analyses the Nash equilibrium within the model. Section 4 discusses numerical experiments with changes to class size and the impact of these changes on behaviour. Section 5 summarizes our results.

2. A Game-Theoretic Model for Bullying

There are three kinds of people in this situation: the bully, the bullied child and bystanders. In this paper we only focus on bystanders' behaviour in a class where there is bullying. Suppose that there are n bystanders in the

How to cite this paper: Isada, Y., Igaki, N. and Shibata, A. (2015) A Game-Theoretic Model for Bystanders' Behaviour in Classes with Bullying. *Open Journal of Social Sciences*, **3**, 97-102. http://dx.doi.org/10.4236/jss.2015.39015

class each bystander can take behaviour R, where a student reports bullying to a teacher, or behaviour S, where a student does not report the bullying. Bullying is resolved when more than t students report the bullying. All players are initially granted a utility level w. When bullying occurs, players incur a negative externality (dis-utility) b. Cost e is constantly incurred for student who selects behaviour R, regardless of whether bullying is stopped or not.

Then, a non-cooperative n-person game model [6] [7] is formulated, shown in **Table 1**. Each value shows the player's gain in each case, where X denotes the number of reporters other than himself of herself.

3. Nash Equilibrium in the Bullying Model

Each bystander play this game according to **Table 1**. Suppose each bystander has the same probability of reporting, q. When players other than oneself select behaviour R with a probability q, the probability of case 1, 2 and 3 are $p_1(q), p_2(q), p_3(q)$ respectively as shown below:

$$p_1(q) = p_1(q,n,t) = \sum_{i=0}^{t-2} {}_{n-1}C_i q^i (1-q)^{n-1-i} \tag{1}$$

$$p_2(q) = p_2(q,n,t) = {}_{n-1}C_{t-1} q^{t-1} (1-q)^{n-t} \tag{2}$$

$$p_3(q) = p_3(q,n,t) = \sum_{i=t}^{n-1} {}_{n-1}C_i q^i (1-q)^{n-1-i} \tag{3}$$

$$p_1(q) + p_2(q) + p_3(q) = 1. \tag{4}$$

The Expected utility of $E_R(q)$ when a player selecting behaviour R, and the Expected utility of $E_S(q)$ when a player selecting behaviour S are expressed with the following equations.

$$\begin{aligned} E_R(q) &= p_1(w-b-e) + p_2(w-e) + p_3(w-e) \\ &= w - e - b \cdot p_1(q) \end{aligned} \tag{5}$$

$$\begin{aligned} E_S(q) &= p_1(w-b) + p_2(w-b) + p_3 w \\ &= w - b \cdot (p_1(q) + p_2(q)). \end{aligned} \tag{6}$$

$q = 1$ is the state when all players select behaviour R and $q = 0$ is the state when all players select behaviour S. When $E_R(q) = E_S(q)$, the result is as $e/b = p_2(q)$.

From Equation (2), we have $p_2(0) = 0$, $p_2(1) = 0$ and

$$\begin{aligned} p_2'(q) &= {}_{n-1}C_{t-1} \left\{ (t-1) q^{t-2} (1-q)^{n-t} + q^{t-1} (n-t)(1-q)^{n-t} (-1) \right\} \\ &= {}_{n-1}C_{t-1} \left\{ q^{t-2} (1-q)^{n-t-1} \left((1-n) q + t - 1 \right) \right\}, \end{aligned} \tag{7}$$

It follows that

$$p_2'(q) \begin{cases} > 0 & \text{for } 0 < q < q_0 \\ = 0 & \text{for } q = 0, q_0, 1 \\ < 0 & \text{for } q_0 < q < 1 \end{cases} \tag{8}$$

where $q_0 = \dfrac{t-1}{n-1}$.

Figure 1 shows that there are two values of q which hold $E_s(q) = E_R(q)$ when $p_2(q_0) > e/b$. Let them denote $q_1 = q_1(n,t)$, $q_2 = q_2(n,t) (0 < q_1 < q_2 < 1)$.

Figure 2 shows the relationship between Expected utilities $E_R(q)$ and $E_S(q)$ and for player behaviours R and S where $p_2(q_0) > e/b$.

As we saw in **Figure 2**, $E_R(q)$ and $E_S(q)$ have two intersections for the range $q_2 < q$, $E_s(q) > E_R(q)$ occurs. This indicates that a free rider phenomenon occurs where many other players report bullying, but the player in question decides it is better not to report. Based on the above, we can make the following proposition.

Table 1. Changes to player gain by the number of reporters when selecting either behaviour R or S.

	Case 1 (X ≤ t−2)	Case 1 (X = t−1)	Case 1 (X ≥ t)
Behaviour R	w-b-e	w-e	w-e
Behaviour S	w-b	w-b	w

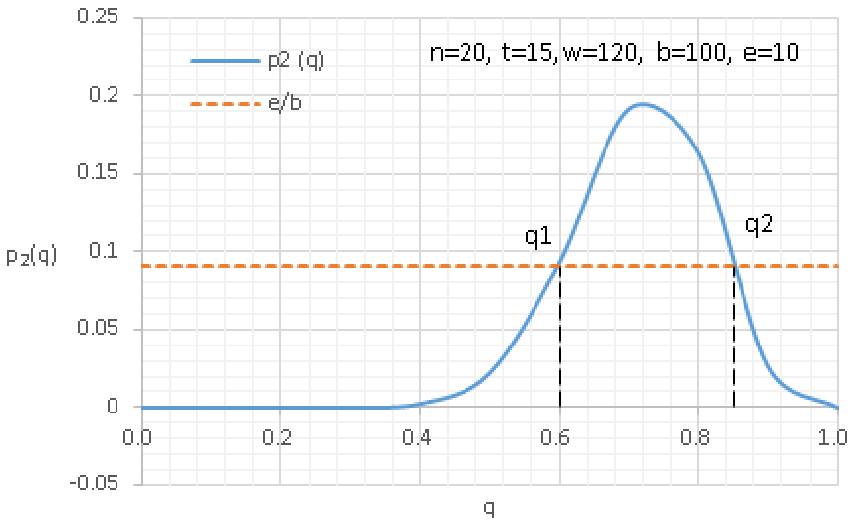

Figure 1. Two intersections of $\mathbf{y} = p_2(q)$ and $\mathbf{y} = e/b$.

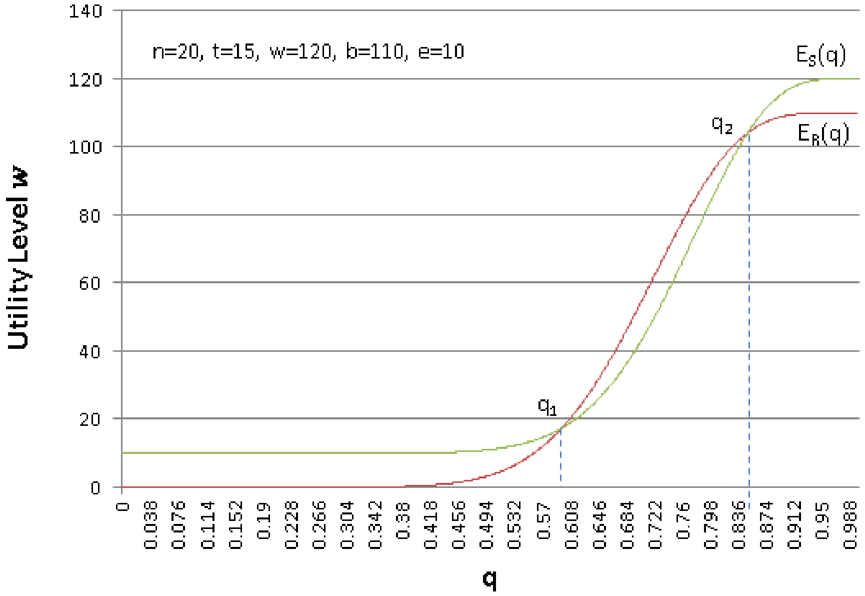

Figure 2. Expected utility when there are two intersections of $\mathbf{y} = p_2(q)$ and $\mathbf{y} = e/b$.

Proposition 1

1) A pure strategy Nash equilibrium always exists in which no player reports bullying. Only when $p_2(q_0) > e/b$ is true, two mixed strategies Nash equilibrium exists.
2) When $p_2(q_0) > e/b$, segment q_1, q_2 exists for q where $E_s(q) < E_R(q)$. Conversely, when $p_2(q_0) \le e/b, E_S(q) \ge E_R(q)$ it is always true.

When examining **Figure 1**, we see that we can expand the range $[q_1, q_2]$ of q by reducing e/b, since q_1 is decreased and q_2 is increased. Based on the above, we can make the following propositions.

Proposition 2

An increase in b or a decrease in e/b due to a decrease in e causes a decrease in q_1 and an increase in q_2.

<Proof of Proposition 2>

By differentiating both sides of $e/b = p_2(q)$ results in $dq/qb = -e/b^2 p_2'(q)$. Because q_1 satisfies $p_2'(q_1) > 0$ and q_2 satisfies $p_2'(q_2) > 0$, $dq_1/qb < 0$, $dq_2/qb > 0$, e and e/b are solved in similar fashion. <q.e.d.>

4. Behavior Resulting from Changing the Number of Bystanders in the Class

Let us examine changes in $y = p_2(q)$ and $y = e/b$ that occur at the two intersections with q_1 and q_2 when changing only n, the number of bystanders in the class, while the ratio of threshold to the number of bystanders is kept constant at t/n. **Figure 3** is a graph of $p_2(q)$ where the number of n is changing. As the value of n becomes smaller, q_1 becomes smaller and q_2 becomes larger. Thus, the range $[q_1, q_2]$ of q expands.

Figure 4 is a graph that shows the value of q_1 and q_2 for four cases, (*n*, *t*) = (20, 10), (40, 20)…, where the ratio of t/n keeps a constant $1/2$. As n becomes smaller, q_1 becomes smaller and q_2 becomes larger. Again, the range $[q_1, q_2]$ of q expands. On the other hand, as class size n becomes larger, q_1 becomes larger and q_2 becomes smaller. The values of the upper limit q_2 decrease, and the lower limit q_1 increase, and the range $[q_1, q_2]$ of q shrinks.

Proposition 3.

1) For $i = 1, 2, \cdots$, the following relations hold.

$$p_2(q, in, it) > p_2(q, (i+1)n, (i+1)t). \tag{9}$$

2) For *i* such as $p_2((t-1)/(n-1), (i+1)n, (i+1)t) < e/b,$ we have the following in equations.

$$q_1(in, it) < q_1((i+1)n, (i+1)t),$$
$$q_2(in, it) > q_2((i+1)n, (i+1)t). \tag{10}$$

<Proof of Proposition 3>

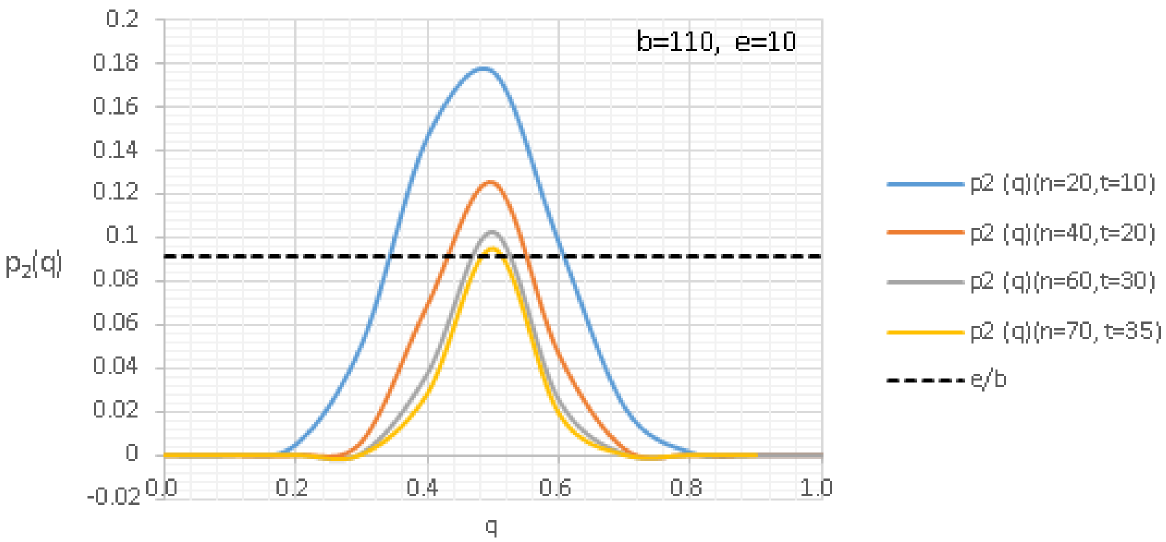

Figure 3. A graph of $p_2(q)$ when increasing *n* and *t* while maintaining t/n.

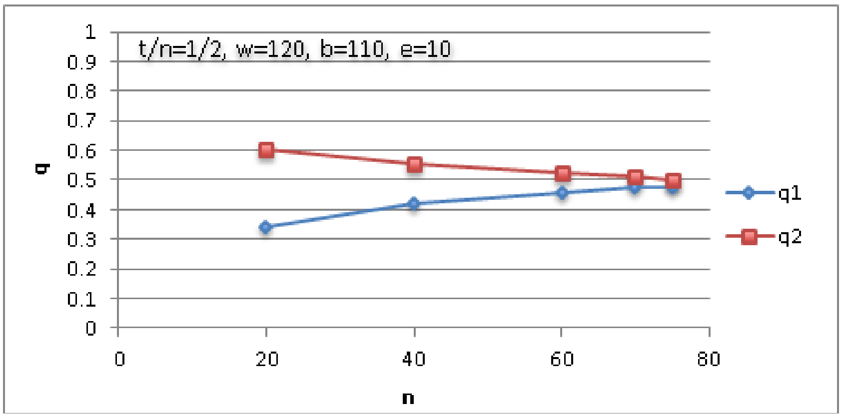

Figure 4. A graph of q_1, q_1 when increasing n and t while maintaining t/n.

Since

$$p_2\left(q,in,it\right)-p_2\left(q,\left(i+n\right)n,\left(i+1\right)t\right) = \frac{\left(in-1\right)!}{\left(it-1\right)!\left\{i\left(n-t\right)\right\}!}q^{it-1}\left(1-q\right)^{i(n-t)}\left\{1-f\left(n,t,q\right)\right\} \tag{11}$$

where $$f\left(n,t,q\right)=\frac{\prod_{j=1}^{n}\left\{\left(i+1\right)n-j\right\}}{\prod_{j=1}^{t}\left\{\left(i+1\right)t-j\right\}\prod_{j=0}^{n-t-1}\left\{\left(i+1\right)\left(n-t\right)-j\right\}}q^t\left(1-q\right)^{n-t}, \tag{12}$$

using the relation $$q^t\left(1-q\right)^{n-t}\le\left(\frac{t}{n}\right)^t\left(1-\left(\frac{t}{n}\right)\right)^{n-t}, \tag{13}$$

it is left for us to prove $$f\left(n,t,\frac{t}{n}\right)<1. \tag{14}$$

$f\left(n,t,\frac{t}{n}\right)$ can be written as

$$f\left(n,t,\frac{t}{n}\right)=\frac{\prod_{j=1}^{n}\left\{i+1-\frac{j}{n}\right\}}{\prod_{j=1}^{t}\left\{i+1-\frac{j}{t}\right\}\prod_{j=0}^{n-t-1}\left\{i+1-\frac{j}{n-t}\right\}}. \tag{15}$$

From relations

$$i+1-\frac{j}{n}<\begin{cases} i+1-\frac{j}{t}, & for\ j=1,2,\cdots,t \\ i+1-\frac{j-t-1}{n-t}, & for\ j=t+1,t+2,\cdots,n \end{cases} \tag{16}$$

we obtain $f\left(n,t,\frac{t}{n}\right)<1$, then the proof is completed. <q.e.d.>

Proposition 3 (2) shows that adopting smaller class sizes is effective for reducing bullying. It gives the effectiveness of small-group education.

5. Conclusion

In this paper we modelled the behaviour of bystanders of students in a non-cooperative n-player game. We showed by making t/n, the ratio of a threshold number of reporters to the number of bystanders, constant and decreasing n becomes possible to decrease the lower limit q_1 and to increase the upper limit q_2 of the probability of reporting bullying. If class sizes are smaller, the number of bystanders should be fewer. This shows the possibility of eliminating bullying by using the smaller number of bystanders. Note that the reason we insist that smaller classes are better, not because it is easier for a teacher to manage smaller classes. More intuitively, Proposition 3 shows that bystanders can report bullying more easily if they are in a smaller class.

Furthermore, to expand the range $[q_1, q_2]$ of q, it is useful to raise the disutility b associated with continued bullying and to reduce the cost e of reporting on bullies.

Finally, this study has demonstrated the existence of the "free riding" phenomenon: if the majority of other people report, it is advantageous for any given person not to do so.

Acknowledgements

This work was supported by JSPS KAKENHI Grant Numbers 25350468.

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