

A Decision Problem of Hotels on Setting Online-Exclusive-Rooms in E-Commerce Age

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Abstract

Due to the rapid development of information technology, customers' consumption pattern in hotel industry has dramatically changed, and many people book rooms through some professional online platforms. Hotel managers need to redesign their sale channel structures by incorporating online platforms, in order to increase profit. This paper focuses on a revenue maximization problem for a hotel which cooperates with an online platform. Two cooperating forms are investigated and compared. Specifically, one intuitive form is named *complete sharing*, in which all the hotel rooms are shared with the online platform for booking. The other form is named *setting online-exclusive-rooms* (OERs), in which the hotel set a fixed quantity of rooms for exclusively booking from the online platform. By building a loss queueing model, this paper finds some properties of the optimal quantity of OERs in the latter form. Factors affecting the selection and how they affect the selection are also investigated.

Keywords

Revenue Management, Hotel Room Management, Online Platform, Optimization

1. Introduction

During the last two decades, information technology has been developing rapidly. More and more people buy products or book service through the internet [1]. A typical example is that many customers prefer to booking hotel rooms via online platforms (e.g. Expedia, CTRIP, etc.), rather than through the front desk of the hotel. Customers' consumption pattern has changed and is still changing hotel industry. According to CTRIP's survey in 2010, more than 40% of hotel customers book rooms through such online platform in China, and more than 1 million rooms are booked through CTRIP per year [2]. Professional hotel booking platforms nowadays have become an important channel for hotel room booking [3] [4].

With this emerging phenomenon, hotel managers have to redesign their sale channel structures by incorporating online platforms, in order to reach different market segments, expand market share, and increase profit. One intuitive cooperating form between a hotel and an online platform is that, the hotel completely shares all rooms with the online platform for room booking. The online platform gets a commission from the hotel once it successfully accomplishes a room booking. We name this cooperating form as “*complete sharing*”. This form seems to be reasonable and can maximize the utilization of the hotel rooms. However, this cooperating form may lead to a foreseeable result for the hotel: more and more customers will choose to book rooms via the online platform, and then the hotel should strongly rely on the online platform for getting customers. In the long term, the profit of the hotel cannot be maximized. Actually, the cooperation between CTRIP and many hotels is in another form. The hotel offers a fixed quantity of rooms to the online platform (*i.e.* CTRIP) which is less than the total quantity of rooms the hotel has. This fixed quantity of rooms can only be booked from the online platform, so that we call them *Online-Exclusive-Rooms* (OERs). No more room can be booked from the online platform once all OERs have been booked out. We name this cooperating form as “*setting OERs*”.

In this paper, we focus on a profit maximization decision problem for a hotel when cooperating with an online platform. Specifically, we intend to answer the question that should a hotel set OERs, and if yes, how many OERs should be set. The rest of this paper is structured as follows. In Section 2, we review the related literature and show the difference between our work and the extant research, where the contributions of this research are addressed. In Section 3, the model is formally introduced, and some properties of the optimal quantity of OERs are discussed. In Section 4, we formally discuss how parameters affect the optimal quantity of OERs and the selection of cooperating forms, by analytical study and numerical study. We conclude this paper and give some future research directions in Section 5. All the proofs of propositions are relegated to the Appendix.

2. Literature Review

Our work is relevant to the research on hotel yield management and the dual-channel supply chain management. However, there are some significant differences between this work and the extant literature. Next we will review the literature and the differences accordingly.

2.1. Hotel Yield Management

Much of the early literature in hotel yield management focuses on the overbooking problem, *i.e.* to set appropriate overbooking limits, e.g., see Rothstein [5], Liberman and Yechiali [6]. Later on, the literature on yield management focuses on maximizing the profit generated by a given capacity for service in the long run [7] [8]. In hotel industry, the capacity is the number of rooms the hotel has and the operational objective in hotel yield management is to sell the right rooms to the right customers at the right price. Bitran and Mondschein study optimal strategies for renting hotel rooms when there is a stochastic and dynamic arrival of customers from different market segments [9]. They formulate the problem as a stochastic and dynamic programming model and characterize the optimal policies as functions of the capacity and the time left until the end of the planning horizon. Bitran and Gilbert formulate a room allocation problem as a stochastic dynamic program and develop simple heuristic procedures for accepting reservations [10]. Badinelli also presents a dynamic programming formulation of the hotel yield management problem allowing for general demand patterns and a policy that is based on time and the number of vacancies [8]. Baker and Collier investigate a new Price Setting Method (PSM) to manage demand in the hotel yield management [11].

The main difference between the classical hotel yield management research and our work is that, the capacity (rooms) is usually regarded as a given resource constraint in classical hotel yield management research but a decision variable in our problem. Accompanied by the coming of E-commerce age, the problem of designing an optimal cooperating mechanism for hotel room booking service has emerged, but not investigated yet in the literature. In addition, an operational difficulty in hotel yield management is the customer's multiple night stays [9]. Generally, this problem is approximated by a series of independent single night reservations in extant hotel yield management literatures [10]. In our work, this difficulty does not exist, because the customers' stay time can be any number of days.

2.2. Dual-Channel Supply Chain Management

Most of the traditional dual-channel supply chain management research focuses on selling tangible product such

as computer, which can be stored for selling at different times. The rapid development of E-commerce has made it easier for tangible product manufacturers to move part of sales online. About ten years ago, about 42% of top-ranked suppliers in a variety of industries (e.g., IBM, Pioneer Electronics, Cisco System, Estee Lauder, and Nike) have begun to sell products directly to consumers over the Internet [12]. Manufacturers nowadays are increasingly adopting a dual-channel way to sell their products, *i.e.*, a traditional retail channel and an online channel.

Many problems regarding Dual-channel supply chain management have been extensively investigated in the literature, including the pricing and ordering decision problems in each channel, the coordination and competition problems between the dual channels [13]–[15]. Different from the extant research on traditional dual-channel supply chain management, this work considers the allocation problem of sellable “product” (the room booking service) and the corresponding profit maximizing problem.

In summary, up to now, little is known about how the hotel should use their limited room resource effectively to cooperate with the online platform. This is the main motivation for our work. The main contributions of our paper are three-folds. 1) We study the hotel room allocation problem considering the cooperation between hotel and the online platform. 2) We creatively formulate the loss-queueing model for the hotel service system which consists of a hotel and an online platform. 3) We investigate whether a hotel should set OERs and the property of the optimal quantity of OERs to maximize the hotel’s revenue, which provides a theoretical guide for the management of hotel rooms in E-commerce age.

3. Model Formulation and Analysis

3.1. Problem Description

Consider a hotel service system which is composed of a hotel and an online platform. The hotel provides room service for customers, while any customer can book the room service from either the front desk or the online platform.

Here we only describe the decision problem of the hotel in “*setting OERs*” case. Take CTRIP as an example. As an online platform, CTRIP provides room booking service to customers for many hotels. Each hotel sets a certain quantity of rooms for booking exclusively from CTRIP, and the remaining quantity of rooms for booking exclusively from the front desk. As addressed in the first section, the former type of rooms are named “Online-Exclusive-Rooms” or “OERs”, and the latter “hotel exclusive rooms” or “HERs”. Once CTRIP successfully accomplishes a booking task to one customer, the hotel pays a commission to CTRIP. If CTRIP did not “book out” all its OERs, it does not have any penalty. If CTRIP has “booked out” all its OERs, no extra customer can book a room from CTRIP. Also, if the hotel has “booked out” all its HERs, it cannot accept more customers from the front desk. From this way, it will be preferable for CTRIP to have more OERs, because more OERs bring more opportunities to make money. However, the hotel needs to think about an appropriate quantity of OERs. If the quantity of OERs is too large, the quantity of HERs will be less, which leads to potential loss of the hotel’s profit. If the quantity of OERs is too few, it will also lead to potential loss of customers and profit from online platform.

In this paper, we focus on the decision problem of a hotel which cooperates with an online platform. Specifically, we intend to answer the following two questions. First, within the above mentioned two cooperating forms (complete sharing and setting OERs), which one is better and when? In other words, when should a hotel set OERs? Second, in the latter form, how many OERs should the hotel set to maximize its profit? This is actually an optimization problem of the hotel. Since the second question is only related to the second cooperating form (setting OERs), we set the “complete sharing” case as benchmark, and mainly discuss the “setting OERs” case.

For simplicity and convenience of discussion, we make the following assumptions in this work.

- 1) All the hotel rooms are homogeneous, or of a single type. Actually, hotels usually have multiple types of rooms. For these hotels, different type of rooms can be analyzed separately.
- 2) Each customer only books one room at one time. This assumption is feasible, since it is the number of rooms booked that affects the revenue of the hotel, not the number of customers who make the booking.
- 3) There is no refund for the booking. Making this assumption is for simplicity.
- 4) The online platform does not set the lower threshold of OER quantity, since it will make money even if there is only one OER. In actual case, there may be a lower threshold set by the online platform. However, this constraint can be easily added to the hotel’s optimal OER quantity to give the final solution.

3.2. Model for “Setting OERs”

The hotel service system investigated in this paper can be modeled by a queuing service system with multiple servers. The rooms (including OERs and HERs) can be regarded as the servers, and the whole process that customers stay in the hotel rooms can be regarded as the serve process. In our problem, there are two customer flows, one arriving at the hotel’s front desk and the other arriving at the online platform. We assume that these two customer flows are independent, and each of them arrives according to a Poisson processes [5] [9] [10]. Assume that the time (days) a customer stay in the hotel (either in an OER or an HER) is an independent and identically distributed (i.i.d.) random variable obeying a general distribution, and independent of the arrival process [16] [17]. Here are the notations used in this paper.

- Q : the total quantity of rooms in the hotel;
- q : the total quantity of OERs;
- w_e : the price when a customer books a room from the online platform;
- w_h : the price when a customer books a room from the front desk;
- r : the commission the hotel pays to the online platform for successfully booking out a OER; we suppose that the commission is a proportion of the booking price from the online platform;
- θ : the proportion of the booking price as the commission ($0 < \theta < 1$); thus $r = \theta \cdot w_e$;
- λ_h : customers’ arriving rate at the front desk (obey a Poisson process); independent of the booking prices;
- λ_e : customers’ arriving rate at the online platform (obey a Poisson process); independent of the booking prices;
- u : service rate in the hotel (obey a general distribution); thus $1/\mu$ is the expected customer stay time (days) in the hotel.

Following the assumptions and notations above, there are $Q - q$ rooms (HERs) that can be booked from the front desk, and q rooms (OERs) from the online platform. We also note that new arriving customers will leave immediately when all the rooms (either OERs or HERs) are occupied (*i.e.*, all servers are busy). Therefore, the hotel front desk can be regarded as an $M/G/Q - q/Q - q$ queuing system [18], and the online platform can be regarded as an $M/G/q/q$ queuing system. These two systems are independent. We illustrate them in **Figure 1**.

To answer the two questions raised in Section 3.1, first we need to compute the revenue of the hotel under any quantity of OERs. According to the classical queueing theory [19], the revenue of the service system depends on the expected number of customers in the service system, which in turn depends on the probabilities of all rooms taken in our problem.

Let $P_h(q)$ and $P_e(q)$ be the probabilities of all rooms taken at front desk and at online platform, respectively, when the quantity of OERs is q . Actually, $P_h(q)$ and $P_e(q)$ also represent the customer loss probability in the $M/G/Q - q/Q - q$ and $M/G/q/q$ systems, respectively. Thus $P_h(q)$ and $P_e(q)$ can be easily computed as follows [20].

$$P_h(q) = \frac{\lambda_h^{Q-q}}{\mu^{Q-q} (Q-q)!} \cdot \frac{1}{\sum_{k=0}^{Q-q} \frac{\lambda_h^k}{\mu^k k!}}, \quad (1)$$

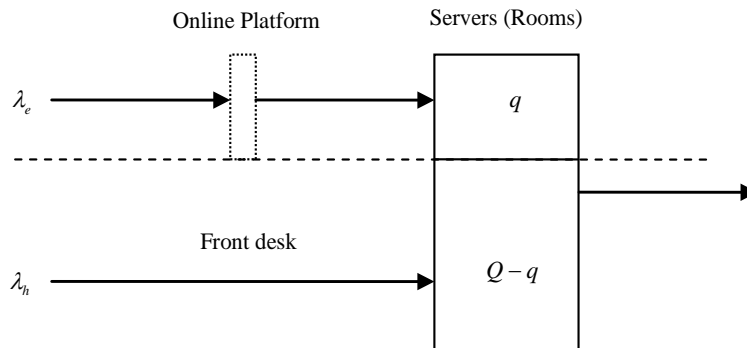


Figure 1. The front desk and online platform queuing service systems.

$$P_e(q) = \frac{\lambda_e^q}{\mu^q q!} \cdot \frac{1}{\sum_{k=0}^q \frac{\lambda_e^k}{\mu^k k!}}. \quad (2)$$

Let $L_h(q)$ and $L_e(q)$ be the expected number of customers in the $M/G/Q-q/Q-q$ and $M/G/q/q$ system, respectively, when the quantity of OERs is q . Then $L_h(q)$ and $L_e(q)$ can be calculated as follows [21].

$$L_h(q) = \frac{\lambda_h}{\mu} [1 - P_h(q)], \quad (3)$$

$$L_e(q) = \frac{\lambda_e}{\mu} [1 - P_e(q)]. \quad (4)$$

The revenue of the hotel comes from two parts: one from the $M/G/Q-q/Q-q$ system and the other from $M/G/q/q$ system (removing the commission paid to the online platform). The expected revenue of the hotel, $\pi_h(q)$, is as follows.

$$\pi_h(q) = w_h L_h + (1-\theta) w_e L_e = \frac{w_h \lambda_h}{\mu} [1 - P_h(q)] + \frac{(1-\theta) w_e \lambda_e}{\mu} [1 - P_e(q)]. \quad (5)$$

By substituting (1) and (2) into the Equation (5), we get

$$\pi_h(q) = \frac{w_h \lambda_h}{\mu} \left[1 - \frac{\lambda_h^{Q-q}}{\mu^{Q-q} (Q-q)!} \frac{1}{\sum_{k=0}^{Q-q} \frac{\lambda_h^k}{\mu^k k!}} \right] + \frac{(1-\theta) w_e \lambda_e}{\mu} \left[1 - \frac{\lambda_e^q}{\mu^q q!} \frac{1}{\sum_{k=0}^q \frac{\lambda_e^k}{\mu^k k!}} \right]. \quad (6)$$

Since the cost of providing room service can be regarded as a constant, it does not affect the optimal decision of the hotel. In other words, *revenue maximization* has the same meaning of *profit maximization*. In the latter part of this paper, we use revenue maximization as the operational objective of the hotel.

Before searching for the optimal decision, we need to understand the property of the expected revenue of the hotel. We proved that the expected revenue of the hotel has the following property, as shown in Proposition 1. All the proofs of propositions are arranged in the Appendix part.

Proposition 1. *The expected revenue of the hotel, $\pi_h(q)$, is a concave function of the quantity of OERs, q .*

Proposition 1 implies that, over any interval of the quantity of OERs (q), there is a maximal value of the expected revenue of the hotel (i.e. $\pi_h(q)$). The optimal quantity of OERs corresponding to the maximal value of $\pi_h(q)$ is what we are searching for.

3.3. The Optimal Quantity of OERs

Based on Proposition 1, we can formulate the following model to find the optimal quantity of OERs to maximize the hotel's revenue.

$$\max \pi_h(q) \quad s.t. \quad 0 \leq q \leq Q, \quad q \in \mathbf{Z} \quad (7)$$

Because q is discrete, it is not easy to directly solve the above model. Denote the solution of Model (7) (i.e. the optimal quantity of OERs) to be q^* . According to Proposition 1, $\pi_h(q)$ is a concave function of q , therefore we can use the marginal analysis method to find q^* [22]. In Model (7), q^* must satisfy the following conditions:

$$\begin{cases} \pi_h(q^*) \geq \pi_h(q^* - 1) \\ \pi_h(q^*) \geq \pi_h(q^* + 1), \end{cases} \quad (8)$$

where $\pi_h(q^*)$ is the expected revenue of the hotel when the quantity of OERs is q^* . Based on the conditions

in (8), we have Proposition 2 for determining q^* .

Proposition 2. *The optimal quantity of OERs (q^*) can be determined by the following conditions*

$$\frac{P_h(q^*) - P_h(q^* - 1)}{P_e(q^* - 1) - P_e(q^*)} \leq \frac{(1 - \theta)w_e\lambda_e}{w_h\lambda_h} \leq \frac{P_h(q^* + 1) - P_h(q^*)}{P_e(q^*) - P_e(q^* + 1)}. \quad (9)$$

The conditions in Proposition 2 are not intuitive, but the implications are reasonable and practical. Let $g(q) = \frac{P_h(q) - P_h(q-1)}{P_e(q-1) - P_e(q)}$, then $g(q+1) = \frac{P_h(q+1) - P_h(q)}{P_e(q) - P_e(q+1)}$. $g(q)$ means the ratio of the increase of customer loss probability at front desk to the decrease of customer loss probability at online platform, if the quantity of OERs is increased from $q-1$ to q . In (9), $\frac{(1-\theta)w_e\lambda_e}{w_h\lambda_h}$, which is equal to $\frac{(1-\theta)w_e\lambda_e/\mu}{w_h\lambda_h/\mu}$, means the ratio of

the expected revenue per day from the online platform to that from the front desk with no customer loss. Because $P_e(q)$ and $P_h(q)$ are decreasing and increasing in q , respectively [23], and $P_e(q)$ and $P_h(q)$ are convex functions of q (see the proof of Proposition 1), then $g(q)$ is increasing in q . The left inequality in (9), $g(q^*) \leq \frac{(1-\theta)w_e\lambda_e}{w_h\lambda_h}$, is equivalent to $[P_h(q^*) - P_h(q^* - 1)]w_h\lambda_h \leq [P_e(q^* - 1) - P_e(q^*)](1-\theta)w_e\lambda_e$. It implies

that the increase of expected revenue loss per day from the front desk is less than the decrease of expected revenue loss from the online platform, if the quantity of OERs is increased from $q^* - 1$ to q^* . The right inequality in (9), $\frac{(1-\theta)w_e\lambda_e}{w_h\lambda_h} \leq g(q^* + 1)$, is equivalent to $[P_h(q^* + 1) - P_h(q^*)]w_h\lambda_h \geq [P_e(q^*) - P_e(q^* + 1)](1-\theta)w_e\lambda_e$.

It implies that the increase of expected revenue loss per day from the front desk is larger than the decrease of expected revenue loss from the online platform, if the quantity of OERs is increased from q^* to $q^* + 1$. Since $g(q)$ is increasing in q , the two inequalities in (9) ensure that q^* is the optimal quantity of OERs which maximizes the expected revenue of the hotel.

Up till now we are able to answer the second question proposed in Section 3.1. However, to answer the first question, we need to investigate the “complete sharing” case, and do some analysis on the property of the optimal solution of Model (7).

3.4. Model for “Complete Sharing”

In the “complete sharing” case, the available room in the hotel will be provided to the customer whose booking request comes first, either from the front desk or from the online platform. To the hotel, these two customer flows can be integrated to one new customer flow. Based on the assumptions that these two customer flows are independent Poisson arrival process, with the arrival rate of λ_h and λ_e , respectively, the new customer arrival process is also a Poisson process and the arrival rate is $\Lambda (= \lambda_h + \lambda_e)$ [24]. Since it is assumed that the time customers stay in the hotel is an i.i.d. random variable and follows a general distribution, the front desk together with the online platform is an $M/G/Q/Q$ queue system.

Let \tilde{P} be the probability of all rooms taken in the $M/G/Q/Q$ system, then

$$\tilde{P} = \frac{(\lambda_h + \lambda_e)^Q}{\mu^Q Q!} \frac{1}{\sum_{k=0}^Q \frac{(\lambda_h + \lambda_e)^k}{\mu^k k!}}. \quad (10)$$

For the convenience of comparison, we let the booking prices and the commission proportion remain the same for both the “setting OERs” case and the “complete sharing” case. Let $\tilde{\pi}$ be the expected revenue of the hotel in the “complete sharing” case. $\tilde{\pi}$ can be computed as follows.

$$\tilde{\pi} = \frac{w_h\lambda_h + (1-\theta)w_e\lambda_e}{\mu} (1 - \tilde{P}) = \frac{w_h\lambda_h + (1-\theta)w_e\lambda_e}{\mu} \left(1 - \frac{(\lambda_h + \lambda_e)^Q}{\mu^Q Q!} \frac{1}{\sum_{k=0}^Q \frac{(\lambda_h + \lambda_e)^k}{\mu^k k!}} \right). \quad (11)$$

It is obvious from Equation (11) that $\tilde{\pi}$ has no relationship with q . That's why we set the “complete sharing” case as the benchmark.

4. Property Analysis

To find out when should a hotel set OERs and when a hotel should share all room resources with the online platform, we check the relative magnitude of the expected revenue of the hotel under “setting OERs” and “complete sharing” cases. Define a variable α to be the ratio of these two expected revenues, i.e.,

$$\alpha = \frac{\pi_h(q^*)}{\tilde{\pi}}. \quad (12)$$

when $\alpha > 1$, the hotel is more willing to set OERs. And when $\alpha < 1$, the hotel is more willing to share all room resources with the online platform.

Since $\pi_h(q^*)$ depends on q^* , α also depends on q^* . Thus in the remaining part of this section, we try to finish two tasks. One is to find out what variables affect the optimal quantity of OERs, q^* , and what the effects are. The other is to find out what variables affect the revenue ratio, α , and what the effects are.

Specifically, we check the effects of the following four factors on q^* and α : the booking prices (w_h and w_e), the proportion of the booking price as the commission (θ), the customer arriving rates (λ_h and λ_e), and the expected customer stay time in the hotel ($1/\mu$).

4.1. The Factor of Booking Prices (w_h and w_e)

For the effects of booking prices (w_h and w_e) on q^* , we have the following proposition.

Proposition 3: *The optimal quantity of OERs, decreases in w_h , and increases in w_e .*

Proposition 3 can be explained as follows. When the other parameters are given, if w_h is increased, the potential revenue from a room booked at the hotel's front desk is increased. Thus the hotel is more willing to limit rooms for booking at the online platform, i.e. setting less OERs. However, if w_e is increased, the revenue from a room booked at the online platform front desk is increased. Thus the hotel is willing to offer more OERs to the online platform.

Proposition 3 addresses how q^* will be changed when one booking price (w_h or w_e) is changed given the other fixed. However, when the two prices are changed simultaneously, it is difficult to find conclusions analytically. Thus we use numerical studies to analyze it. Let $\lambda_h = 30$, $\lambda_e = 20$, $Q = 100$, $\mu = 0.5$ and $\theta = 0.2$. The effects of w_h and w_e on q^* are illustrated in Figure 2.

Figure 2 shows that the conclusions in Proposition 3 are valid. Furthermore, it can be observed from Figure 2 that, if the ratio of w_h to w_e is increased, q^* tends to be lower. If the ratio of w_h to w_e is decreased, q^* tends to be higher. These two observations are reasonable, because higher ratio of w_h to w_e means less revenue from the online platform. Thus the hotel tends to decrease OERs.

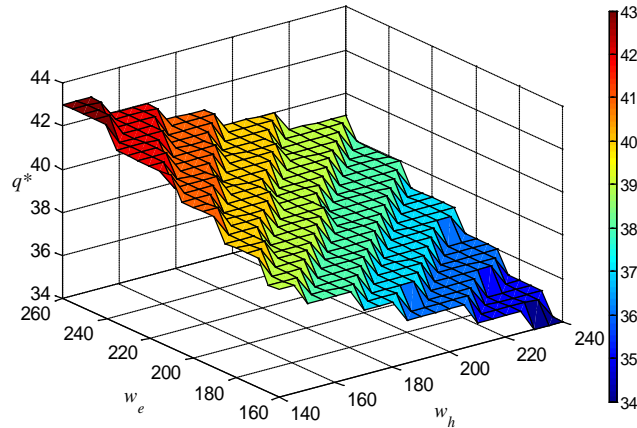


Figure 2. The effect of w_h and w_e on q^* .

It is rather difficult to analytically identify the effects of w_h and w_e on α , since $\pi_h(q^*)$ is difficult to be analytically solved. We use numerical studies to check the effects. Let $\lambda_h = 30$, $\lambda_e = 20$, $Q = 100$, $\mu = 0.5$ and $\theta = 0.2$. The effects of w_h and w_e on α are illustrated in Figure 3.

It is shown in Figure 3 that α increases in w_h , and decreases in w_e . Furthermore, α increases in the ratio of w_h to w_e . These observations are reasonable, because on one hand, as w_e increases, the hotel can get more revenue from booking out one room at the online platform. Thus the hotel is more willing to share more rooms with the online platform, and then the “complete sharing” case is welcome. As w_e decreases, the hotel is more willing to set OERs. It is important to note that the expected revenue of the hotel in “setting OERs” case can be higher than that in “complete sharing” case, when the ratio of w_h to w_e is rather large. This indicates that in particular case, “complete sharing” form may not be better than “setting OERs” form for the hotel.

4.2. The Factor of the Proportion of Booking Price as Commission (θ)

For the effect of the proportion of booking price as commission (θ) on q^* , we have the following proposition.

Proposition 4. *The optimal quantity of OERs, q^* , decreases in θ .*

The conclusion in Proposition 4 can be interpreted as follows. Larger θ means the hotel should give more commission to the online platform, which implies less revenue from booking out one room at the online platform. Thus the hotel tends to set less OERs when θ is increased.

To show the effect of θ on q^* numerically, we also do some computation. Let $\lambda_h = 30$, $\lambda_e = 20$, $w_h = 200$, $w_e = 180$, $Q = 100$, and $\mu = 0.5$. The effect of the proportion of the booking price as the commission, θ , on q^* is illustrated in Figure 4.

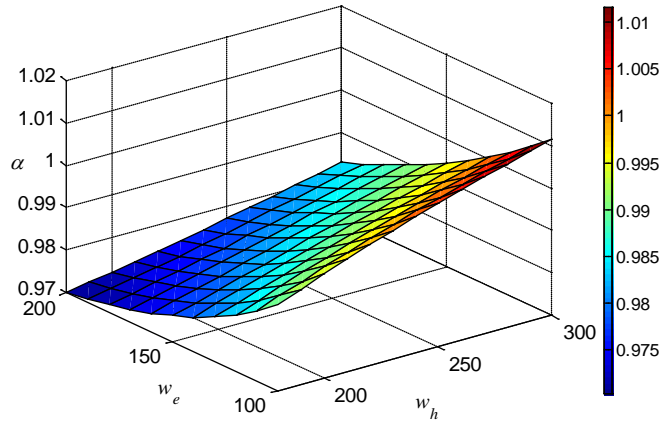


Figure 3. The effect of w_h and w_e on α .

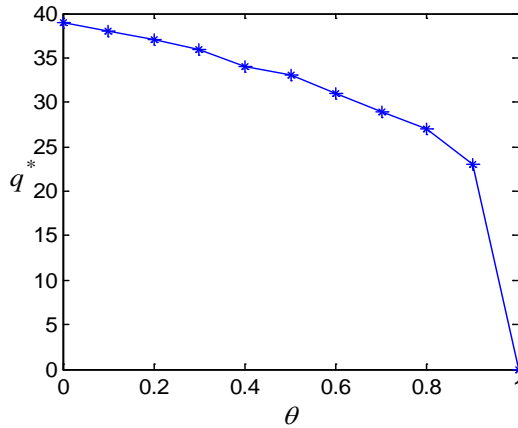


Figure 4. The effect of θ on q^* .

Figure 4 shows that q^* decreases in θ , which verifies the conclusion in Proposition 4. Let $\lambda_h = 30$, $\lambda_e = 20$, $w_h = 200$, $w_e = 180$, $Q = 100$, and $\mu = 0.5$. We check the effect of θ on α , as shown in **Figure 5**.

Figure 5 shows that α increases in θ , which means as θ increases, the hotel is less willing to share all room resources with the online platform but like to set OERs. This is reasonable because higher θ means less revenue from booking out one room at the online platform.

4.3. The Factor of Customer Arriving Rate (λ_h and λ_e)

It is difficult to analytically check the effect of λ_h and λ_e on q^* and α . We use numerical studies to check these effects. Let $w_h = 200$, $w_e = 180$, $Q = 100$, $\mu = 0.5$ and $\theta = 0.2$. The effects of λ_h and λ_e on q^* are illustrated in **Figure 6**.

Figure 6 shows that q^* decreases in λ_h and increases in λ_e . Furthermore, q^* decreases in the ratio of λ_h to λ_e . This is because larger λ_e means more customers arriving at online platform, then it will be better to share more rooms with the online platform. However, larger λ_h means more customers arriving at the front desk. Since the total rooms are fixed, it will be better to set smaller OERs.

For the effect of λ_h and λ_e on α , it is not easy to make a speculation. We also resort to numerical studies. Let $w_h = 200$, $w_e = 180$, $Q = 100$, $\mu = 0.5$ and $\theta = 0.2$. The effects of λ_h and λ_e on α are illustrated in **Figure 7**.

As shown in **Figure 7**, the effects of λ_h and λ_e on α are a little complicated. When both λ_h and λ_e are low, α is close to 1. It means the expected revenue of the hotel is nearly the same in both the “setting OERs” case and the “complete sharing” case, because there is little probability that all rooms are taken. When

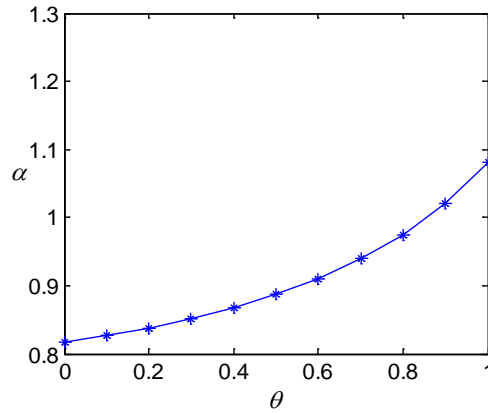


Figure 5. The effect of θ on α .

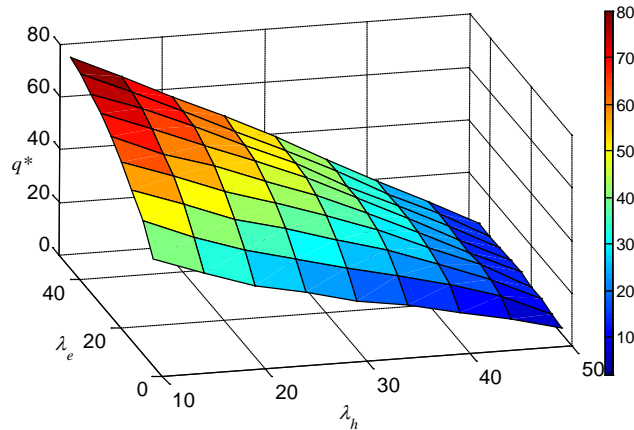


Figure 6. The effects of λ_h and λ_e on q^* .

one of λ_h and λ_e is high and the other is low, α is less than 1. This is because in the “complete sharing” case, all room resources can be fully utilized. But the resource utilization may not be so high in the “setting OERs” case. Thus the “complete sharing” case is better than the “setting OERs” case. If both λ_h and λ_e is high, α is higher than 1. This is because in this situation, the hotel can get highest revenue by setting a small quantity of OERs. However, if the hotel shares all room resources with online platform, many rooms may be occupied by online customer and the hotel will have potential revenue loss.

4.4. The Factor of Expected Customer Stay Time in the Hotel ($1/\mu$)

We resort to numerical studies to check the effects of $1/\mu$ on q^* and α . Let $\lambda_h = 40$, $\lambda_e = 30$, $w_h = 200$, $w_e = 180$, $Q = 100$ and $\theta = 0.2$. The effect of the expected customer stay time ($1/\mu$) on q^* is illustrated in Figure 8.

It can be observed from Figure 8 that q^* decreases in the expected customer stay time, $1/\mu$. This is because, as the expected customer stay time increases, the probability of all room taken in front desk is increasing. It is better to set more HERs and less OERs. When the expected customer stay time is rather long (3 or more days in this example), the optimal quantity of OERs is 0, since the probability of all room taken in front desk is close to 1, and there is no need to provide OERs to online platform. This is actually the third case, i.e. the “operating alone” case. In this case, the hotel operates alone, without cooperating with the online platform. This case is a special one of the “setting OERs” case.

The corresponding effect of $1/\mu$ on α is illustrated in Figure 9.

It can be observed from Figure 9 that α is increasing in the expected customer stay time, $1/\mu$. This can be explained similarly. As the expected customer stay time increases, the probability of all room taken in front desk

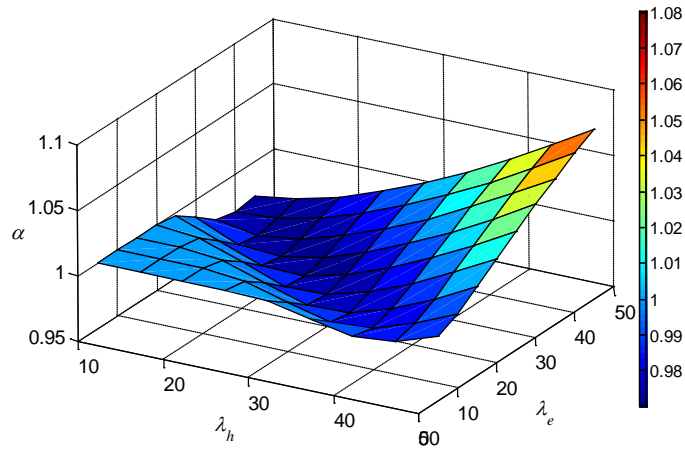


Figure 7. The effects of λ_h and λ_e on α .

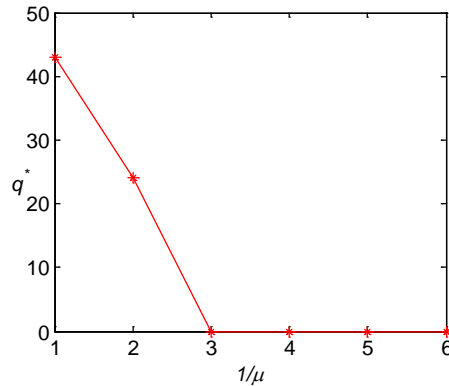


Figure 8. The effect of $1/\mu$ on q^* .

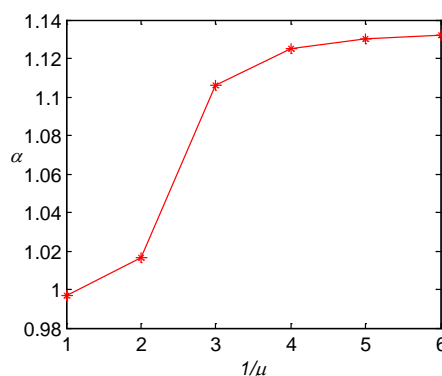


Figure 9. The effect of $1/\mu$ on α .

is increasing. It is better to share less room with the online platform, *i.e.* “complete sharing” is not welcome to the hotel.

5. Conclusions

In this paper, we have investigated a new type of hotel room management problem in the coming E-commerce age: deciding whether to set a fixed quantity of Online-Exclusive-Rooms (OERs) for booking at the online platform, and if yes, then how many OERs should be set? This “setting OER” form is compared with another intuitive cooperating form, complete sharing, *i.e.* the hotel shares all room resources with the online platform for booking. We formulate the hotel and the online platform as a loss queueing service system. Properties of the expected revenue of the hotel and the optimal quantity of OERs to maximize the hotel’s revenue are identified.

In the “setting OER” form, the expected revenue of the hotel is proved to be a concave function of the quantity of OERs, which insures that there is an optimal quantity of OERs which maximizes the expected revenue of the hotel. The method of determining the optimal quantity of OERs is presented. It is found by analytical and numerical studies that: 1) the optimal quantity of OERs decreases in the booking price at the front desk of the hotel, and increases in the booking price at the online platform; 2) the optimal quantity of OERs decreases in the proportion of booking price as commission; 3) the optimal quantity of OERs decreases in the customer arrival rate at the front desk, and increases in the customer arrival rate at the online platform; 4) the optimal quantity of OERs decreases in the expected customer stay time. These properties can provide suggestions on how to set and change the optimal quantity of OERs to maximize the expected revenue of the hotel.

It seems that the “complete sharing” form can utilize the capacity of the hotel more fully. The numerical studies in this paper verified this point. However, it is also shown by numerical studies that completely sharing rooms with online platform is not always a good idea for the hotel. It is better to set OERs when the online booking price is too low, or the commission part is too high, or both online and offline customer arrival rates are large, or the expected customer stay time is long.

There are several limitations in this paper. First, the booking price of the OERs is exogenously fixed but not a decision variable; we do not consider the pricing behavior of the online platform. Second, since this paper focuses on the profit optimization of the hotel, we do not consider the behaviors of the online platform, such as advertizing and ranking which can attract customers to book from the online platform. Third, we assume that no more customers can book rooms from the online platform once all OERs are booked out. Actually, some customers still can get rooms in the above case, but with a lower probability. These limitations are all interesting future research directions. Furthermore, if the hotel adopts the “complete sharing” form, how should the expected revenue of the hotel change in the long run? This should also be an interesting topic in the future.

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Appendix

Proof of Proposition 1

In an $M/G/c/c$ queueing system, the probability of new customer rejected because all servers are busy, is a convex function of the number of servers [25]. That means in our problem, $P_e(q)$ is a convex function of q .

Let $f(q) = \frac{\lambda_h^q}{\mu^q q!} \frac{1}{\sum_{k=0}^q \frac{\lambda_h^k}{\mu^k k!}}$, then $f(q)$ is a convex function of q [25]. According to the property of convex

function [26], $f(Q-q)$ is also a convex function of q . Thus $P_h(q)$, which is equal to $f(Q-q)$, is a convex function of q . Because the additive inverse of a convex function is a concave function [26], $-P_h(q)$ and $-P_e(q)$ are concave functions of q . In Equation (5), w_h , w_e , λ_h , λ_e , θ and μ are fixed positive constants. Thus as a linear combination of $-P_h(q)$ and $-P_e(q)$, $\pi_h(q)$ is a concave function of the quantity of OERs, q .

Proof of Proposition 2

From (5), we have $\pi_h(q) = \frac{w_h \lambda_h}{\mu} [1 - P_h(q)] + \frac{(1-\theta) w_e \lambda_e}{\mu} [1 - P_e(q)]$. Submitting this Equation into (8), we have

$$\begin{cases} \frac{w_h \lambda_h}{\mu} (1 - P_h(q^*)) + \frac{(1-\theta) w_e \lambda_e}{\mu} (1 - P_e(q^*)) \geq \frac{w_h \lambda_h}{\mu} (1 - P_h(q^* - 1)) + \frac{(1-\theta) w_e \lambda_e}{\mu} (1 - P_e(q^* - 1)) \\ \frac{w_h \lambda_h}{\mu} (1 - P_h(q^*)) + \frac{(1-\theta) w_e \lambda_e}{\mu} (1 - P_e(q^*)) \geq \frac{w_h \lambda_h}{\mu} (1 - P_h(q^* + 1)) + \frac{(1-\theta) w_e \lambda_e}{\mu} (1 - P_e(q^* + 1)) \end{cases}$$

Simplify the above Equation, and we get

$$\begin{cases} \frac{w_h \lambda_h}{\mu} P_h(q^*) + \frac{(1-\theta) w_e \lambda_e}{\mu} P_e(q^*) \leq \frac{w_h \lambda_h}{\mu} P_h(q^* - 1) + \frac{(1-\theta) w_e \lambda_e}{\mu} P_e(q^* - 1) \\ \frac{w_h \lambda_h}{\mu} P_h(q^*) + \frac{(1-\theta) w_e \lambda_e}{\mu} P_e(q^*) \leq \frac{w_h \lambda_h}{\mu} P_h(q^* + 1) + \frac{(1-\theta) w_e \lambda_e}{\mu} P_e(q^* + 1) \end{cases}$$

After further simplification, we have

$$\begin{cases} \frac{P_h(q^*) - P_h(q^* - 1)}{P_e(q^* - 1) - P_e(q^*)} \leq \frac{(1-\theta) w_e \lambda_e}{w_h \lambda_h} \\ \frac{P_h(q^* + 1) - P_h(q^*)}{P_e(q^*) - P_e(q^* + 1)} \geq \frac{(1-\theta) w_e \lambda_e}{w_h \lambda_h} \end{cases}$$

which implies that $\frac{P_h(q^*) - P_h(q^* - 1)}{P_e(q^* - 1) - P_e(q^*)} \leq \frac{(1-\theta) w_e \lambda_e}{w_h \lambda_h} \leq \frac{P_h(q^* + 1) - P_h(q^*)}{P_e(q^*) - P_e(q^* + 1)}$.

Proof of Proposition 3

Denote that $\Delta\pi(q) = \pi_h(q) - \pi_h(q-1)$, $\Delta P_h(q) = P_h(q) - P_h(q-1)$, and $\Delta P_e(q) = P_e(q) - P_e(q-1)$. Since $P_h(q)$ strictly increases in q and $P_e(q)$ strictly decreases in q [25], we have $\Delta P_h(q) > 0$ and $\Delta P_e(q) < 0$.

According to Equation (5), we have

$$\Delta\pi(q) = -\Delta P_h(q) \frac{w_h \lambda_h}{\mu} - \Delta P_e(q) \frac{(1-\theta) w_e \lambda_e}{\mu}. \quad (\text{A.1})$$

Consider two prices of an HER, w_{h1} and w_{h2} ($w_{h1} > w_{h2}$). Let q_1^* and q_2^* be the optimal quantity of OERs when the prices of an HER are w_{h1} and w_{h2} , respectively.

It is easy to verify that $\Delta\pi(q)|_{w_h=w_{h1}} - \Delta\pi(q)|_{w_h=w_{h2}} = \Delta P_h(q) \frac{\lambda_h(w_{h2} - w_{h1})}{\mu} < 0$, which implies

$$\Delta\pi(q_1^*)|_{w_h=w_{h1}} - \Delta\pi(q_1^*)|_{w_h=w_{h2}} < 0, \quad (\text{A.2})$$

and

$$\Delta\pi(q_2^*)|_{w_h=w_{h1}} - \Delta\pi(q_2^*)|_{w_h=w_{h2}} < 0. \quad (\text{A.3})$$

According to Proposition 1, $\pi_h(q)$ is a concave function of q , thus $\Delta\pi(q)$ decreases in q . Since $\pi_h(q)$ achieves maximum at q^* , then $q^* = \arg \min_{0 \leq q \leq Q} \{\Delta\pi(q) | \Delta\pi(q) > 0\}$. Because q_1^* and q_2^* are the optimal quantities of OERs when the prices of an HER are w_{h1} and w_{h2} , respectively, we have $\Delta\pi(q_1^*)|_{w_h=w_{h1}} > 0$. Furthermore, according to (A.2), $\Delta\pi(q_1^*)|_{w_h=w_{h2}} > 0$.

Since $q_2^* = \arg \min_{0 \leq q \leq Q} \{\Delta\pi(q) | \Delta\pi(q) > 0; w_h = w_{h2}\}$, we have $\Delta\pi(q_2^*)|_{w_h=w_{h2}} \leq \Delta\pi(q_1^*)|_{w_h=w_{h2}}$. Because $\Delta\pi(q)$ decreases in q , we have $q_2^* \geq q_1^*$, which implies q^* decreases in w_h .

Following the similar way, q^* increases in w_e . Thus we omit it.

Proof of Proposition 4

According to the proof of Proposition 3 (Equation A.1), we have

$$\Delta\pi(q) = -\Delta P_h(q) \frac{w_h \lambda_h}{\mu} - \Delta P_e(q) \frac{(1-\theta) w_e \lambda_e}{\mu}. \quad (\text{A.4})$$

Consider two proportions of the booking price as the commission, θ_1 and θ_2 ($\theta_1 > \theta_2$). Let q_1^* and q_2^* be the optimal quantity of OERs when the proportions are θ_1 and θ_2 , respectively.

Since $\Delta P_e(q) < 0$, It is easy to verify that $\Delta\pi(q)|_{\theta=\theta_1} - \Delta\pi(q)|_{\theta=\theta_2} = \Delta P_e(q) \frac{w_e \lambda_e (\theta_1 - \theta_2)}{\mu} < 0$, which implies

$$\Delta\pi(q_1^*)|_{\theta=\theta_1} - \Delta\pi(q_1^*)|_{\theta=\theta_2} < 0, \quad (\text{A.5})$$

Because $q^* = \arg \min_{0 \leq q \leq Q} \{\Delta\pi(q) | \Delta\pi(q) > 0\}$, we have $\Delta\pi(q_1^*)|_{\theta=\theta_1} > 0$. Furthermore, according to (A.5), $\Delta\pi(q_1^*)|_{\theta=\theta_2} > 0$.

Since $q_2^* = \arg \min_{0 \leq q \leq Q} \{\Delta\pi(q) | \Delta\pi(q) > 0; \theta = \theta_2\}$, we have $\Delta\pi(q_1^*)|_{\theta=\theta_2} \leq \Delta\pi(q_1^*)|_{\theta=\theta_2}$.

Because $\Delta\pi(q)$ decreases in q , we have $q_2^* \geq q_1^*$, which implies q^* decreases in θ .

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