

A Quantity Discount Pricing Model Based on the Standard Container under Asymmetric Information

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ABSTRACT

A supply chain system is studied, in which the information about the retailer's storage cost is usually asymmetric. This paper studies the inventory control in the system and presents a quantity discount pricing model for inventory coordination based on the standard container, a transport tool from the supplier to the retailer with a fixed size. First, it investigates the inventory models under full information. Before inventory coordination, the supplier and the retailer myopically choose their lot sizes, which is not the optimal decision for the whole system. Then, it presents an incentive scheme under asymmetric information from the point of view of the supplier. It also discusses the solution and the distribution of the incremental profits after the incentive scheme is adopted by both the supplier and the retailer. A constant in the model, which affects the distribution of the incremental profits, is optimized for the supplier by using numerical analysis. Finally, an example illustrates the application of the model. After inventory coordination, both the supplier and the retailer have a positive incremental profit.

Keywords: *inventory coordination, asymmetric information, quantity discounts, standard container*

1. Introduction

In order to improve managerial effectiveness, people have traditionally focused their efforts on making effective decisions within an organization. Since the 1980s, the supply chain management has become people's focus. A supply chain is a very complicated system; therefore, coordination is the key point of the supply chain management.

Thomas & Griffin classifies supply chain coordination into strategic coordination and operational coordination [1]. In terms of operational coordination, the quantity discount pricing model is a basic method of coordinating in the supply chain, both in study and in practice.

Traditionally, quantity discount pricing models have been studied from the point of view of the buyer, which focus on the problem of determining the economic order quantities for the buyer, given a quantity discount schedule set by the supplier [2]. Goyal, one of the first scholars studying the buyer-vendor inventory coordination proposes an integrated inventory model from both the point of view of the buyer and supplier [3]. Monahan proposes a quantity discount pricing model to increase the supplier profits [4]. Lee & Rosenblatt relax the im-

plicit assumption of a lot-for-lot policy adopted by the supplier in Monahan's model [2]. Weng studies the all-unit quantity discount and the incremental quantity discount, and shows that both these discounts are equivalent in achieving channel coordination [5].

Those traditional quantity discount models are studied under full information, and both the supplier and the buyer exactly know the cost structure of each other. However, in practice, such full information assumption is very hard to satisfy [6,7]. Corbett & Groote propose a quantity discount policy under asymmetric information and compare it with the model under full information [6]. Guo Min & Wang Hongwei present a quantity discount mechanism for the cooperation and coordination between the two sides. The mechanism can make the buyer share its information about storage holding cost with the supplier [7].

On the other hand, quantity discounts are often related to transport tools in practice. Usually, the transport tools are containers with standard sizes, and any lot size adjustment may force the buyer to carry less-than-truck loads, resulting in hidden freight costs that were not con-

sidered [8]. Pantumsinchai & Knowles propose algorithms for solving an SPP in which Q is made up of a number of containers with standard sizes. The news-vendor can choose any combination of container sizes and the larger the container, the smaller the unit cost [9]. Shin & Benton consider that the lot-size should be augmented by an integer multiple, and propose a quantity discount approach to coordination [10]. But these researches are all assumed that the supplier has full information.

In this paper, we study a supply chain system with a standard transport tool under asymmetric information. In the system, there are a single supplier and a single retailer, and the transport tools are standard containers with same size. The supplier places quantity discount scheme under asymmetric information about the retailer's storage cost, and the retailer chooses the lot-size according to the scheme.

The study is based on the following assumptions. First, the exogenous demand is constant and continuous. Second, the supplement lead time is determinate. Third, no shortages are allowed. The main symbols are listed as follows:

- D : the annual demand;
- P : the retailer's selling price;
- P_1 : the retailer's purchase price;
- P_2 : the supplier's purchase price;
- h_1 : the retailer's yearly unit holding cost;
- h_2 : the supplier's yearly unit holding cost;
- S_1 : the retailer's fixed ordering cost per order;
- S_2 : the supplier's fixed ordering cost per order;
- Q_S : the size of the standard container;
- Q : the retailer's lot-size;
- d : the unit discount under full information;
- d_A : the unit discount under asymmetric information;
- YN_1 : the retailer's yearly profit without discount;
- YN_2 : the supplier's yearly profit without discount;
- YND_1 : the retailer's yearly profit with discount;
- YND_2 : the supplier's yearly profit with discount.

2. The Inventory Model under Full Information

2.1 The Inventory Model with Independent Decision

Before coordination, both the supplier and the retailer

myopically choose the optimal lot-size according to their own cost structure. The retailer's yearly profit is

$$YN_1(Q) = D(P - P_1) - DS_1 / Q - h_1 Q / 2,$$

and the optimal lot-size is $Q = \sqrt{2DS_1 / h_1}$. Since the transport tools are standard containers with same size, the order should be transported with full container for economic purpose. If Q isn't an integer multiple of the standard size, i.e., $Q \neq k_1 Q_S$, the retailer will compare the two integer multiples of the standard size directly above and below of Q to determine the optimal lot-size $Q^* = k_1^* Q_S$ to maximize his yearly profit.

Lee & Rosenblatt [2] show that, when the retailer's lot-size is $Q^* = k_1^* Q_S$, the supplier's optimal lot-size should be $K_1 Q^* = K_1 k_1^* Q_S$ (K_1 is a positive integer), and the supplier's average storage is $(K_1 - 1)k_1^* Q_S / 2$. Then the supplier's yearly profit is

$$YN_2(K_1 Q^*) = D(P_1 - P_2) - DS_2 / K_1 k_1^* Q_S - h_2 (K_1 - 1) k_1^* Q_S / 2$$

Since the second derivative of $YN_2(K_1 Q^*)$ with respect to K_1 is

$$\frac{d^2 YN_2(K_1 Q^*)}{dK_1^2} = -\frac{2DS_2}{K_1^3 k_1^* Q_S} < 0$$

there is a maximum of $YN_2(K_1 Q^*)$. So the parameter K_1 could be derived by taking the first derivative of $YN_2(K_1 Q^*)$ with respect to K_1 and setting it to zero. K_1 can be derived as:

$$K_1 = \frac{1}{k_1^* Q_S} \sqrt{\frac{2DS_2}{h_2}} \tag{1}$$

If K_1 isn't an integer, it will compare the two integers directly above and below of K_1 to determine the factor K_1^* to maximize the supplier's yearly profit. If K_1 is an integer, the supplier's yearly profit will be

$$YN_2(K_1^* Q^*) = D(P_1 - P_2) - \sqrt{2DS_2 h_2} + k_1^* Q_S h_2 / 2.$$

Thus, as the retailer's lot-size increases, the supplier's yearly profit increases as well.

2.2 The Integrated Inventory Model

Based on the above discussion, we know that the supplier hopes that the retailer chooses the largest possible lot-size. On the other hand, it can be derived that $YN_1(Q)$ will be decreased as the retailer's lot-size Q increases (where $Q > Q^* = k_1^* Q_S$). For the whole system, the optimal lot-size can be derived as following.

When the retailer's lot-size is $k_2 Q_S$ and the supplier's optimal lot-size is $K_2 k_2 Q_S$, the system's yearly profit is:

$$YN(kQ_s, KkQ_s) = YN_1(k_2Q_s) + YN_2(K_2k_2Q_s) = D(P - P_2) - \frac{DS_1}{k_2Q_s} - \frac{h_1k_2Q_s}{2} - \frac{DS_2}{K_2k_2Q_s} - \frac{h_2(K_2 - 1)k_2Q_s}{2} \tag{2}$$

Accordingly, the parameter K_2 could be derived by taking the first derivative from (2) with respect to K_2 . So K_2 can be derived as:

$$K_2 = \frac{1}{k_2Q_s} \sqrt{\frac{2DS_2}{h_2}} \tag{3}$$

If K_2 is an integer, $K_2^* = K_2$; otherwise, $K_2^* = [K_2] - 1$ or $K_2^* = [K_2] + 1$. Here, $[K_2]$ is the integer part of K_2 .

The steps for solving above question are as follows. First, choosing a positive integer k_2 within the interval of $[1, D/Q_s]$. Second, adding the k_2 into (3), and getting K_2^* to maximize the system's yearly profit. Then, spreading all over k_2 and choosing the optimal lot-size ($k_2^*Q_s$, $K_2^*k_2^*Q_s$) to maximize the total profit.

It is obvious that $k_2^* > k_1^*$, and

$$\begin{aligned} \Delta YN &= YN(k_2^*Q_s, K_2^*k_2^*Q_s) - YN_1(k_1^*Q_s) \\ &\quad - YN_2(K_1^*k_1^*Q_s) \geq 0 \end{aligned}$$

Therefore there is a positive incremental profit in the system. But $k_1^*Q_s$ is the retailer's optimal lot-size for his yearly profit. When the retailer's lot-size increases from $k_1^*Q_s$ to $k_2^*Q_s$, its incremental profit will be:

$$\begin{aligned} \Delta YN_1 &= YN_1(k_2^*Q_s) - YN_1(k_1^*Q_s) \\ &= \frac{DS_1}{k_1^*Q_s} - \frac{DS_1}{k_2^*Q_s} + \frac{h_1k_1^*Q_s}{2} - \frac{h_1k_2^*Q_s}{2} < 0 \end{aligned} \tag{4}$$

Therefore, the inventory optimization with the integrated decision will decrease the retailer's benefit. In order to make the retailer accept the optimal lot-size, it is necessary to give the retailer a quantity discount to compensate it. With the integrated decision, the system's total profit is augmented by ΔYN . If the divisions of the incremental profit between the retailer and the supplier are α and $1 - \alpha$ (where $\alpha \in [0, 1]$), the unit quantity discount amount will be $d = (\alpha \Delta YN - \Delta YN_1) / D$.

3. The Inventory Model under Asymmetric Information

3.1 The Model

It is known from (4) that, the bigger the retailer's yearly unit holding cost h_1 , the bigger the compensation given to the retailer. Under asymmetric information, the retailer could hide h_1 in order to gain more benefit. Therefore, the supplier must devise a rational compensating mechanism to make the retailer give the true information.

As the information is asymmetric, the retailer knows

h_1 , but the supplier doesn't know it. In order to coordinate, the supplier gives a quantity discount scheme according to the forecast value \hat{h}_1 of h_1 . It gives the lot-size of discount point ($k_A^*(\hat{h}_1)Q_s$) and the unit discount ($d_A(\hat{h}_1)$). The retailer decides the lot-size according to the scheme. Since the optimal lot-size for the whole system is $k_2^*Q_s$, the supplier could devise the unit discount as following ($k_A^* = k_2^*(\hat{h}_1)$):

$$d_A = \begin{cases} d_A(\hat{h}_1) & \forall Q \geq k_2^*(\hat{h}_1)Q_s \\ 0 & \forall Q < k_2^*(\hat{h}_1)Q_s \end{cases} \tag{5}$$

Before coordination, the retailer's yearly profit is $YN_1(k_1Q_s) = D(P - P_1) - DS_1/k_1Q_s - h_1k_1Q_s/2$, and the supplier's yearly profit is

$YN_2(K_1k_1Q_s) = D(P_1 - P_2) - DS_2/K_1k_1Q_s - (K_1 - 1)k_1Q_s h_2/2$. After coordination, the retailer's yearly profit is

$$YN_{D1}(k_AQ_s) = D(P - P_1 + d_A) - DS_1/k_AQ_s - h_1k_AQ_s/2,$$

and the supplier's yearly profit is

$$\begin{aligned} YN_{D2}(K_Ak_AQ_s) &= D(P_1 - P_2 - d_A) - DS_2/K_Ak_AQ_s \\ &\quad - (K_A - 1)k_AQ_s h_2/2 \end{aligned}$$

To make the retailer give the true h_1 , it needs to let $YN_{D1}(k_AQ_s)$ be the maximum at $\hat{h}_1 = h_1$. Calculating the first derivative of $YN_{D1}(k_AQ_s)$ with respect to h_1 , and setting it to 0 at $\hat{h}_1 = h_1$, then

$$\frac{\partial d_A}{\partial h_1} = \frac{\partial}{\partial h_1} \left[\frac{S_1}{k_AQ_s} + \frac{k_AQ_s h_1}{2D} \right]$$

or

$$d_A = \frac{S_1}{k_AQ_s} + \frac{k_AQ_s h_1}{2D} + b \tag{6}$$

Here b is a constant. The supplier can choose a certain constant b for a quantity discount scheme. Different b 's result in different schemes. When the retailer accepts the quantity discount scheme, the constant b doesn't affect the retailer giving the true h_1 .

Therefore, from the point of view of the supplier, we can derive the inventory coordination model under asymmetric information as follows:

$$\max YN_{D2}(K_Ak_AQ_s) \tag{7}$$

$$\text{s.t.} \begin{cases} YN_{D1}(k_AQ_s) \geq YN_1(k_1^*Q_s) \end{cases} \tag{8}$$

$$\begin{cases} YN_{D2}(K_Ak_AQ_s) \geq YN_2(K_1^*k_1^*Q_s) \end{cases} \tag{9}$$

where (8) is the condition for the retailer to accept the scheme, and (9) is the condition for the supplier to accept it.

From (6), the quantity discount scheme can ensure the retailer giving the true information. Hereafter we assume that $\hat{h}_1 = h_1$.

3.2 The Solution

It can be derived from the function of $YN_{D1}(k_A Q_S)$ that, if $k_A > k_1^*$, the retailer's yearly profit will be decreased with the lot-size increasing. From Section 2, we know that $k_2^* > k_1^*$. Thus, if the retailer accepts the scheme as (5), $k_2^*(\hat{h}_1)Q_S$ must be his optimal lot-size. In the following discuss, we assume that $k_A^*(\hat{h}_1) = k_2^*(\hat{h}_1)$.

Adding (6) into (7), the supplier's yearly profit will be:

$$YN_{D2}(K_A k_A Q_S) = D(P_1 - P_2 - \frac{S_1}{k_A Q_S} - \frac{k_A Q_S h_1}{2D} - b) - \frac{DS_2}{K_A k_A Q_S} - \frac{(K_A - 1)k_A Q_S h_2}{2} \tag{10}$$

As discussed in Subsection 2.2 for obtaining the parameter K_2^* , we can derive K_A^* by taking the first derivative from (10) with respect to K_A , and setting it to zero. Then K_A^* can be derived as:

$$K_A = \frac{1}{k_A Q_S} \sqrt{\frac{2DS_2}{h_2}} \tag{11}$$

If K_A is an integer, $K_A^* = K_A$. If K_A isn't an integer, it will compare the two integers directly above and below of K_A to determine K_A^* to maximize the supplier's yearly profit. So

$$K_A^* = [K_A] - 1 \text{ or } K_A^* = [K_A] + 1.$$

Adding (6) into (8) and (9), we can derive the boundary of b as follows:

$$b \geq -\frac{S_1}{k_1^* Q_S} - \frac{h_1 k_1^* Q_S}{2D} \tag{12}$$

$$b \leq \frac{S_2}{K_1^* k_1^* Q_S} + \frac{(K_1^* - 1)k_1^* Q_S h_2}{2D} - \frac{S_1}{k_A Q_S} - \frac{k_A Q_S h_1}{2D} \tag{13}$$

$$\frac{S_2}{K_A k_A Q_S} - \frac{(K_A - 1)k_A Q_S h_2}{2D}$$

Let $\underline{b} = -\frac{S_1}{k_1^* Q_S} - \frac{h_1 k_1^* Q_S}{2D}$,

$$\bar{b} = \frac{S_2}{K_1^* k_1^* Q_S} + \frac{(K_1^* - 1)k_1^* Q_S h_2}{2D} - \frac{S_1}{k_A Q_S} - \frac{k_A Q_S h_1}{2D} - \frac{S_2}{K_A k_A Q_S} - \frac{(K_A - 1)k_A Q_S h_2}{2D}$$

then

$$b \in [\underline{b}, \bar{b}].$$

The solution procedure of the model is as follows:

First, calculate k_1^* , K_1^* and k_2^* as Section 2.

Second, choose the constant b by experience or by using numerical analysis as following discuss.

Third, calculate K_A according to (11), and add $[K_A] - 1$ or $[K_A] + 1$ to (6) for choosing K_A^* .

Then, calculate \underline{b} and \bar{b} according to (12) and (13), and validate the constant $b \in [\underline{b}, \bar{b}]$. Skip this step when choose b by using numerical analysis.

In the end, calculate d_A from (6), and get the quantity discount scheme.

3.3 The Division of Profit

The model we discussed above could solve the problem of inventory coordination under asymmetric information. But the constant b in the model has a direct effect on the discount amount and then on the division of incremental profit between the retailer and the supplier. In fact, the retailer and supplier are both cooperator and rival in the supply chain. Therefore the choice of constant b is the result of cooperation and competition between the two partners.

Here the constant $b \in [\underline{b}, \bar{b}]$. When $b = \underline{b}$, the discount amount compensates for the retailer's storage cost increment, but the supplier claims the entire incremental profit. On the other hand, when $b = \bar{b}$, the system's incremental profit is completely claimed by the retailer.

Now we derive the retailer's and the supplier's incremental profit without compensation when the retailer's lot-size is $k_2^* Q_S$ and the supplier's lot-size is $K_A^* k_2^* Q_S$.

Without compensation, the retailer's incremental profit is ΔYN_{D1} and the supplier's incremental profit is ΔYN_{D2} .

$$\Delta YN_{D1} = YN_{D1}(k_2^* Q_S) - YN_1(k_1^* Q_S) = DS_1/k_1^* Q_S - DS_1/k_2^* Q_S + h_1 k_1^* Q_S/2 - h_1 k_2^* Q_S/2 \tag{14}$$

$$\Delta YN_{D2} = YN_{D2}(K_A^* k_2^* Q_S) - YN_2(K_1^* k_1^* Q_S) = DS_2/K_1^* k_1^* Q_S - DS_2/K_A^* k_2^* Q_S + (K_1^* - 1)k_1^* Q_S h_2/2 - (K_A^* - 1)k_2^* Q_S h_2/2 \tag{15}$$

In (14), $YN_{D1}(k_2^* Q_S)$ is the retailer's profit when his lot-size is $k_2^* Q_S$, and $YN_1(k_1^* Q_S)$ is the retailer's profit at his optimal lot-size $k_1^* Q_S$. In (15), $YN_{D2}(K_A^* k_2^* Q_S)$ is the supplier's maximum profit when the retailer's lot-size is $k_2^* Q_S$, and $YN_2(K_1^* k_1^* Q_S)$ is the supplier's maximum profit when the retailer's lot-size is $k_1^* Q_S$.

As we know from above discussion, $\Delta YN_{D1} < 0$ and $\Delta YN_D = \Delta YN_{D1} + \Delta YN_{D2} > 0$.

Therefore the discount amount should satisfy $-\Delta YN_{D1} \leq Dd_A \leq \Delta YN_{D2}$. It is shown in the Figure 1.

When the retailer's yearly unit storage cost is h_1 , the biggest discount amount is d_A^H and the least discount amount is d_A^L .

When the supplier has given the discount amount d_A ,

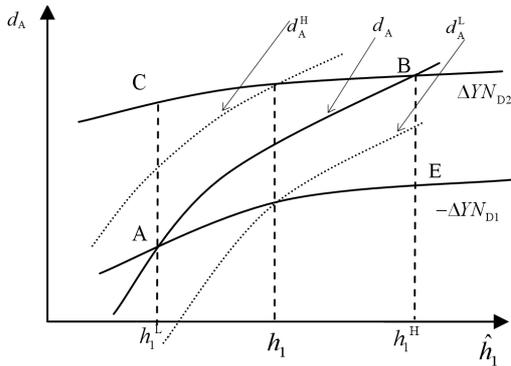


Figure 1. The discount amount changes with h_1

the feasible interval, in which the quantity discount scheme could be accepted by both the supplier and the retailer, is $[h_1^L, h_1^H]$.

When h_1 isn't in the interval, the two partners will go back to the state without coordination.

3.4 The Optimal Discount for the Supplier

In general, the supplier can decide the constant b to maximize his expected profit according to the forecast of h_1 . In order to obtain the optimal b for the expected profit, here we suppose that the supplier knows h_1 as normal distribution with the mean μ and the variance σ^2 (the solution process is same when it is another distribution), see Figure 2.

When $h_1 < h_1^L$ or $h_1 > h_1^H$, the quantity discount scheme can't be accepted by both the supplier and the retailer, and the supplier's incremental profit is zero. When h_1 is in the interval of $[h_1^L, h_1^H]$, the supplier has a positive incremental profit, which will be changed with h_1 . On the other hand, h_1^L and h_1^H will be changed with b , and the expected profit will also be changed with b . Therefore, the supplier can choose an optimal b to maximize his expected profit. In order to get the optimal b we could search it in the interval that will be met at the point C and E in Figure 1.

4. Application

Here we give an example of application as a supplier provides one kind of product to a retailer. The yearly demand of the product is 100000 pieces.

The supplier places an order from its upper supplier and order cost is 800 dollars per order. The supplier's unit purchase price is 8 dollars and his yearly unit holding cost is 6 dollars.

The retailer's order cost is 400 dollars per order. The retailer's unit purchase price is 10 dollars and his unit selling price is 12 dollars. The retailer's yearly unit holding cost is 8 dollars.

The transport tool from the supplier to the retailer is

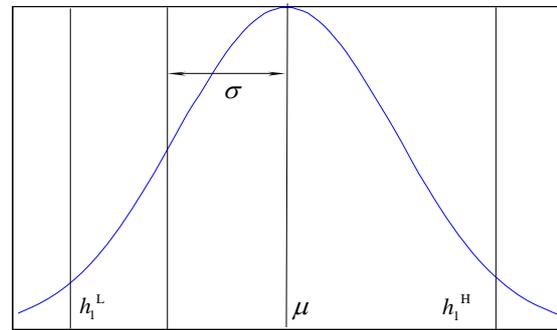


Figure 2. The distribution of h_1

the standard container with 500 pieces.

1) The quantity discount under full information

The retailer's lot-size is 3000 pieces ($k_1^* = 6$) without discount. The supplier's lot-size multiple factor $K_1^* = 2$. The retailer's yearly profit is 174667 dollars and the supplier's yearly profit is 177667 dollars.

In order to maximize the system's yearly profit, the retailer's lot-size should be 5500 pieces ($k_2^* = 11$), and the supplier's lot-size is also 5500 pieces. The system's yearly profit is 356182 dollars.

2) The quantity discount under asymmetric information

Under asymmetric information, the supplier knows h_1 as normal distribution with the mean 8 dollars and the standard deviation 0.5 dollars.

First, we calculate $k_1^* = 6$, $K_1^* = 2$, and $k_2^* = 11$,

Second, we use Mat lab programs to calculate the incremental profit of the supplier with a fixed b , and the relationship between b and the supplier's incremental profit is shown in Figure 3.

From Figure 3, we know that the optimal b for the supplier is -0.2437 .

Then, we can calculate that $k_A^* = 11$, $K_A^* = 110$.

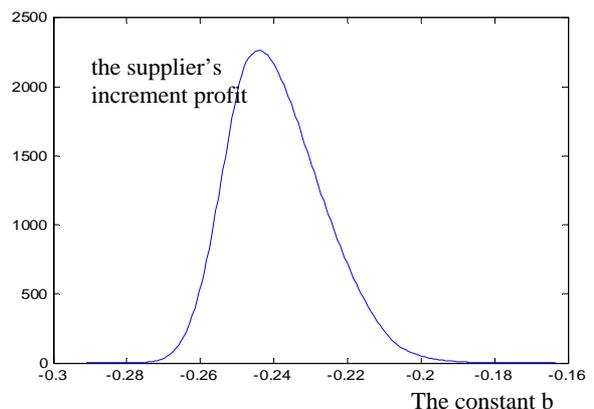


Figure 3. The changes of the supplier's increment profit

In the end, we calculate $d_A = 0.049$. So the supplier could devise the quantity discount scheme as:

$$d_A = \begin{cases} 0.049 & Q \geq 5500 \\ 0 & Q < 5500 \end{cases}.$$

The retailer's yearly profit is 176253 dollars and the supplier's is 179929 dollars after the quantity discount scheme is accepted. In other words, the retailer's total inventory cost decreases from 25333 dollars to 23747 dollars, and the supplier's total inventory cost decreases from 22333 dollars to 20071 dollars.

5. Conclusions

This paper has studied the inventory coordination of a supply chain system with a single supplier and a single retailer, in which the information about the retailer's storage holding cost is asymmetric. In tradition, the supplier and the retailer decide their lot sizes based on their cost structure respectively. But this is not optimal for the whole system. This paper proposes an inventory coordination model based on standard container under asymmetric information, and studies on the division of the incremental profit in the system from the point of view of the supplier. This model could be used widely for some reasons. First, although the supplier doesn't know the retailer's storage holding cost exactly, he can also devise the quantity discount scheme in order to maximize his profit. Second, the quantity discount based on the standard container is natural in practice for saving the freight costs. Third, the optimal discount for the supplier can be obtained by using the Mat lab programs, and it makes the supplier devise the quantity discount scheme easily. By using this model, both the supplier and the retailer have a positive incremental profit after inventory coordination.

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