Journal of Signal and Information Processing, 2013, 4, 14-18
doi:10.4236/jsip.2013.43B003 Published Online August 2013 (http://www.scirp.org/journal/jsip)

SAR Imaging of Moving Target Based on Quadratic Phase Function

Chao Wang, Shaobin Li

School of Electronics information Engineering Harbin Institute of Technology Harbin, China.
Email: wangchao517@hit.edu.cn, lishaobin411@163.com

Received March, 2013.

ABSTRACT

In this paper, a novel signal processing technique has been developed to refocus moving targets image from their smeared responses in the Synthetic Aperture Radar (SAR) image according to the characteristics of the received signals for moving targets. Quadratic Phase Function is introduced to the parameters estimation for moving target echo and SAR imaging. Our method is available even under a low SNR environment and acquiring an exact SAR image of moving targets. The simulated results demonstrated the validity of the algorithm proposed.

Keywords: Synthetic Aperture Radar; Moving Targets; Quadratic Phase Function

1. Introduction

Airborne synthetic aperture radar (SAR) detection and imaging of moving targets on the ground has been a subject of long-standing interest in both civil and military applications such as battlefield reconnaissance, fire assessment, traffic management and monitoring ocean currents. If there are moving targets in the observed scene, SAR cannot simultaneously produce clear images of both stationary and moving targets because they have different Doppler frequency respectively, the target motion induced phase errors, which interact with the matched filter processing or cross-range compression, cause these images to be mislocated in the cross-range dimension and smeared in both the cross-range and the range domains, we can only focus on either the stationary target or the moving target. How to detect moving targets in the background of stationary objects called clutter is premise for SAR imaging of moving targets, this problem is usually solved by applying the displaced phase center antenna (DPCA) [1] or its modern extension, the space-time adaptive processing (STAP) [2]. Radar returns from terrain and stationary objects can be suppressed after using DPCA or STAP, only the returns from moving targets are used to reconstruct radar images.

Moving targets echo signal is generally characterized by the linear frequency modulated (LFM) signal, before obtain the focused images of moving targets, the parameters of LMF should be estimated firstly, and then construct correct match filter of range and azimuth directions for imaging. In which, initial frequency and chirp rate as the basic parameters of LMF, their estimation problem has been an important research content of the signal processing. In previous investigations, several parameter estimation methods based on Maximum Likelihood (ML) [3] estimation cannot be implemented in engineering because of a huge amount of computation. Some signal processing methods such as Radon-Wigner Transform (RWT) [4], Radon-Ambiguity Transform (RAT) [5], Fractional Fourier Transform (FrFT) [6] have been usually used in estimation of LMF parameters. It is worth mentioning that WVD combine with Radon Transform as a commonly used method of detection and estimation for LMF can achieve an idea result. However, there is cross-term interference associated with the WVD. When the signal contains more than one component, the WVD will generate cross-term interference between components that occurs at spurious locations of the time-frequency plane.

In this paper the Quadratic Phase Function (QPF) [7] [8] is introduced to the SAR moving target imaging, and a new SAR imaging algorithm based on the Quadratic Phase Function is presented. This method can achieve an idea result for moving targets imaging. The simulated results demonstrate the validity of this algorithm.

2. Radar Returns of Moving Targets

In this section, we setablish the signal model for the received signal in SAR imaging of moving targets, here, take the ground coordinate system for the reference coordinate system and without considering the rotation of the earth. The scene of a side-looking SAR and a moving target is illustrated in **Figure 1**.

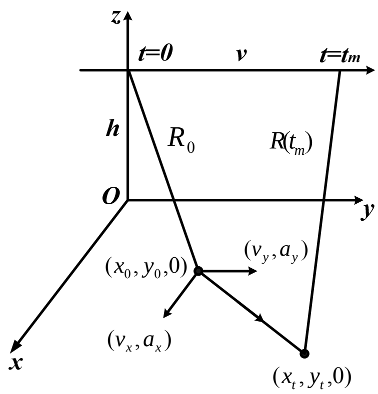

Figure 1. Geometry of an airborne SAR and a moving target

In the case of a moving point target, we assume that at $t=0$ a point target is located at $(x_0, y_0, 0)$, and the radar platform is located at $(0,0,h)$. Then at time *t* the target moves to $(x_0 + v_x t + a_x t^2/2, y_0 + v_y t + a_y t^2/2, 0)$ and the platform moves to $(vt, 0, h)$. Where v_x and a_x represent the azimuth velocity and acceleration of the point in azimuth direction respectively, v_y and a_y represent the ground range velocity and acceleration of the point in ground range direction respectively. The range from the radar to the point target at time t can be written as

$$R(t) = [(vt - v_x t - \frac{a_x t^2}{2} - x_0)^2 + (y_0 + v_y t + \frac{a_y t^2}{2})^2 + h^2]^{\frac{1}{2}} \tag{1}$$

which can be seen by expanding and approximating [9]:

$$R(t) \cong R_0 + \frac{x_0 v_x + y_0 v_y - x_0 v}{R_0} t + \frac{v^2 + v_x^2 + v_y^2 + x_0 a_x + y_0 a_y - 2 v v_x}{2R_0} t^2 \tag{2}$$

where the range component due to the radar motion is:

$$R_{radar}(t) = [(vt - x_0)^2 + y_0^2 + h^2]^{\frac{1}{2}} \tag{3}$$

and the range component due to the target motion is:

$$R_{target}(t) = [(v_x t + \frac{a_x t^2}{2})^2 + (v_y t + \frac{a_y t^2}{2})^2 - 2(vt - x_0)]^{\frac{1}{2}} \times (v_x t + \frac{a_x t^2}{2}) + 2 y_0 (v_y t + \frac{a_y t^2}{2})]^{\frac{1}{2}} \tag{4}$$

where R_0 is the initial range at $t=0$, because x_0 is much smaller than R_0 and y_0 is approximately equal to R_0, then the raw data can be described by:

$$s_0(t) = C_0 \exp\{-j4\pi \frac{R(t)}{\lambda}\} \cong C \exp\{-j2\pi \frac{2v_y}{\lambda} t\} \times \exp\{-j\frac{4\pi}{\lambda}[(v - v_x)^2 + v_y^2 + R_0 a_y]\frac{t^2}{2R_0}\} \tag{5}$$

In which

$$\Phi_{shift}(t) = 4\pi \frac{v_y}{\lambda} t \tag{6}$$

Equation (6) is a linear phase function due to the target velocity in the radial direction, and

$$\Phi_{defocus}(t) = \frac{4\pi}{\lambda}\left[(v - v_x)^2 + v_y^2 + R_0 a_y\right]\frac{t^2}{2R_0} \tag{7}$$

is a quadratic phase function determined by the relative velocity between the radar and the moving target in the x-direction---the radial velocity, and acceleration of the target.

From the above phase functions we can see that by making use of a matched filter designed to match the baseband returns from a stationary target, the linear phase change due to the target's radial velocity v_y in (6) causes the image of the moving target to be shifted in the cross-range direction, and the quadratic phase variation in (7) causes the image of the moving target to be defocused. So the present efforts in moving targets imaging require that estimations be made of targets motion before any imaging procedure can be performed.

3. Quadratic Phase Function

As to the LFM signal $s(t) = b\exp(a_1 t + a_2 t^2)$, where b represents amplitude, a_1 and a_2 represents initial frequency and chirp rate respectively. The Quadratic Phase Function Transform has been explained in detail by the reference [7, 8]:

$$QP(t,u) = \int_0^{+\infty} s(t+\tau)s(t-\tau)e^{-ju\tau^2} d\tau \tag{8}$$

Substitute $s(t)$ into equation (8), we can obtain:

$$QP(t,u) = b^2 e^{2j(a_1 t + a_2 t^2)} \int_0^{+\infty} e^{j(2a_2 - u)\tau^2} d\tau \tag{9}$$

The parameter a_2 can be found through peak search and the other parameters can be estimated by dechirp technology.

From equation (8), it is demonstrated that the quadratic phase function is bilinear transformation, therefore when the signal contains more than one component, the QPF will generate cross-term interference between components that occurs at spurious locations of the time-frequency plane. To reduce the cross-term interference, multiple type and integral type of QPF had been defined in the reference [10] and [11].

In which multi type of QPF is defined as:

$$MQP(u) = \prod_{l=1}^{L} QP(t_l, u) \quad (10)$$

And integral type of QPF is defined as:

$$IQP(u) = \int_t QP(t,u)dt \quad (11)$$

where t_l is of L time sample points. From the equations (10) and (11), we can see that multi type or integral type of QPF can reduce cross-term interference effectively and then suitable for the analysis of LMF signal.

4. Application of Quadratic Phase Function for SAR Imaging

In the section3, we have presented the Quadratic Phase Function algorithm to estimate the parameters of the LFM signal. In this section, we will discuss the SAR imaging techniques for moving targets based on Quadratic Phase Function algorithm, and the new approach for SAR imaging of moving targets can be illustrated explicitly as follows:

Step 1. Range compression. Definition of range compression reference function is described by:

$$sref(t) = \exp(j\pi K_r t^2) \quad (12)$$

where $K_r = 2v^2/\lambda r$ represents chirp rate, while t represents time value in range direction. In the case of point moving targets the acquired raw data after range compression can be written as in

$$s_1(t) = IFFT\{FFT[s(t)] \times FFT[sref(t)]\} \quad (13)$$

where $FFT[\cdot]$ and $IFFT[\cdot]$ represent Fourier Transform operator and Inverse Fourier Transform operator respectively.

Step 2. Azimuth compression function can be defined as:

$$sref_1(t_m) = \exp(j\pi K_r t_m^2) \quad (14)$$

where t_m represents the time value in azimuth direction. As to stationary targets, focused images could be achieved after equation (13) multiplying with equation (14), however the azimuth velocity of the moving targets will cause smearing effect in azimuth direction when processed by conventional image formation of stationary points. At this moment, we multiply every range cell with equation (14), the result can be written as in

$$s_2(t_m) = s_1(t_m) \times sref_1^*(t_m) \quad (15)$$

In order to counteract these smearing effects and get focused image, the parameters of $s_2(t_m)$ should be taken into account during processing and for this purpose, the parameters should be estimated firstly, this is achieved by the following method named Quadratic Phase Function in step 3.

Step 3. Estimate the chirp rate $\hat{\gamma}$ of $s_2(t_m)$, which can be written as

$$\hat{\gamma} = \arg\max_{(\omega,\gamma)}\{F(\omega,\gamma)\} \quad (16)$$

where $F(\omega,\gamma)$ is the Quadratic Phase Function of $s_2(t_m)$.

Step 4. According to the parameters which have already been estimated in step3, amended azimuth compression function is contrasted as in

$$sref_2(t_m) = \exp[j\pi(K_r + \hat{\gamma})t_m^2] \quad (17)$$

Step 5. Using equation (17) to reconstruct azimuth compression, we can obtain the focused SAR image of the moving targets finally. The detailed method is as follows:

$$S(t,t_m) = IFFT\{FFT[s_2(t_m)] \times FFT[sref_2(t_m)]\} \quad (18)$$

Based on the procedure above, we can obtain the focused image of moving targets with the Quadratic Phase Function technique. The flow chart of this algorithm is illustrated in **Figure 2**.

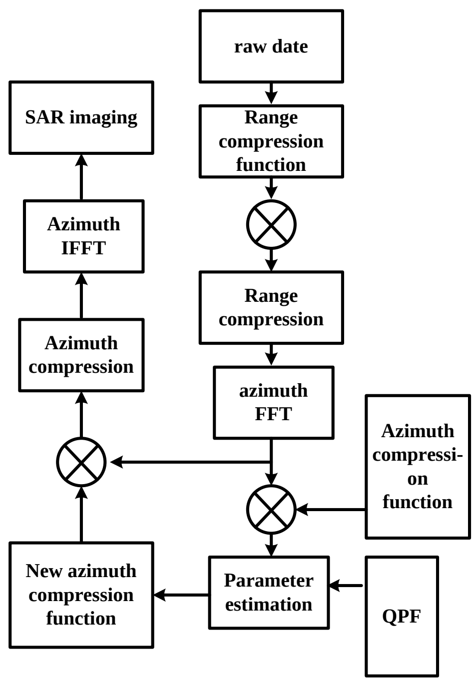

Figure 2. The flow chart of SAR imaging based on Quadratic Phase Function.

5. Computer Simulations

To demonstrate the effectiveness of the algorithm developed in the preceding section, we applied it to simulate data samples. We assume that there are five point targets in the same imaging plane, the radar operates at the frequency f_0=10 GHz, the bandwidth of LFM signals is B =

60 MHz, the pulse repetition frequency is PRF = 500, the pulse width is τ_w = 10 μs, the sampling frequency is Fs=70 MHz. The aircraft with the radar is moving along the x-axis with velocity v = 120 m/s, while the radar's ground distance from the origin of the coordinate system connected to the targets is 8 Km at t = 0. The motion parameters of the targets are given in **Table 1**.

Table 1 motion parameters of the targets used in experiment.

No.	1	2	3	4	5
x0(m)	0	30	5	-5	0
y0(m)	0	0	30	15	-30
Vx(m/s)	3	2.2	3.1	0	3
Vy(m/s)	0.7	1	0	0	1
ax(m/s^2)	0	0	0	0	0
ay(m/s^2)	0	0	0	0	0

Figure 3 is the SAR image of moving targets based on Range-Doppler algorithm without parameter estimation, we can see that the image is blurred in azimuth direction.

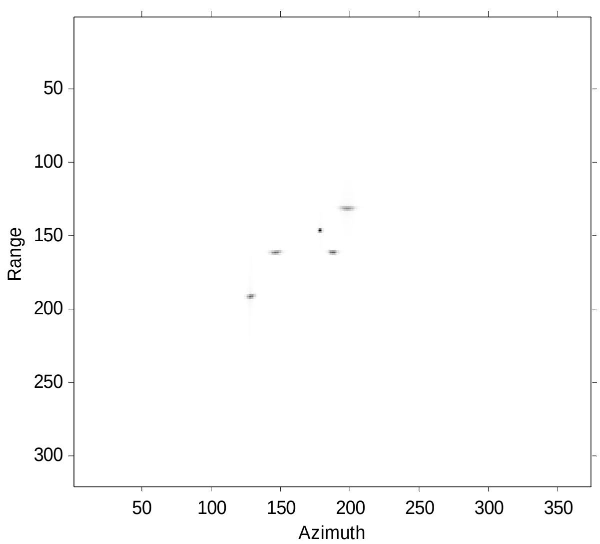

Figure 3. SAR imaging without parameters estimation.

Figure 4 is the result of SAR imaging after parameter estimation, compared with **Figure 3**, we can see that the blurred of azimuth direction has been removed and the moving targets has been focused.

Figure 5 and **Figure 6** are the result of SAR imaging when the SNR is -10 dB and -15 dB respectively, we can see that this method still effective when noise exists.

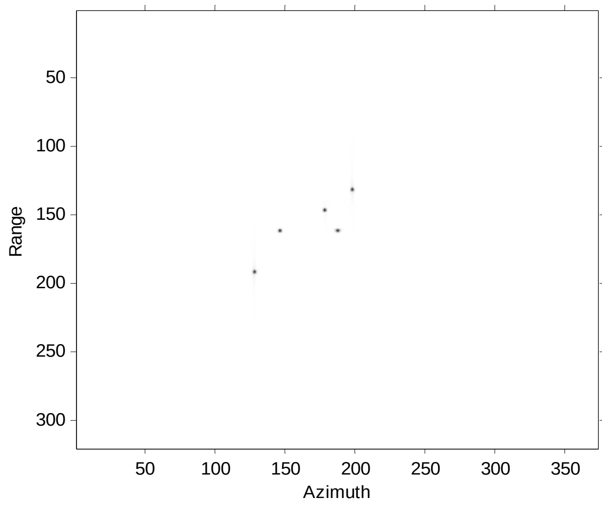

Figure 4. SAR imaging based on Quadratic Phase Function.

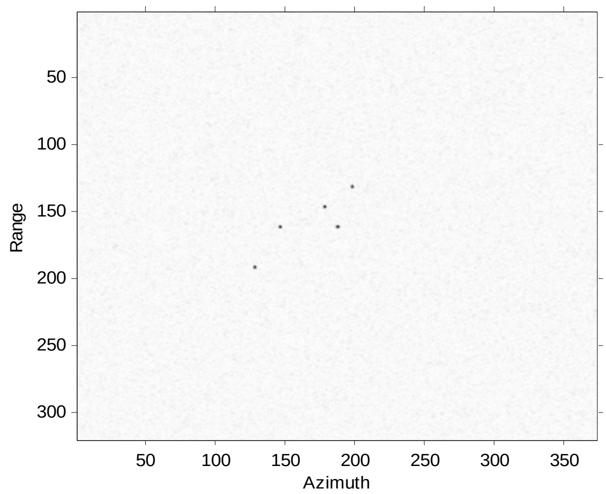

Figure 5. SAR imaging based on Quadratic Phase Function with the SNR is -10 dB.

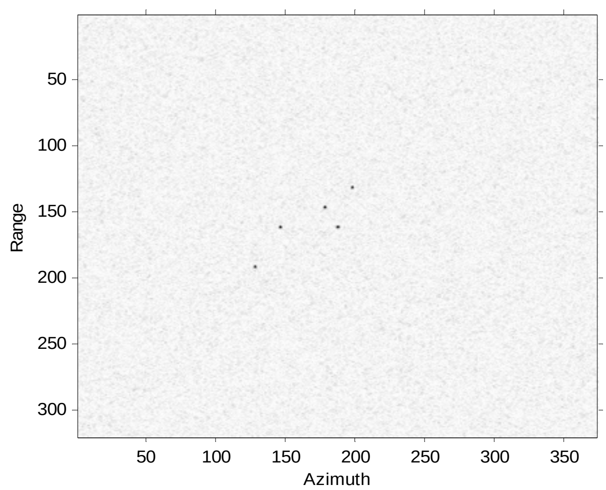

Figure 6. SAR imaging based on Quadratic Phase Function with the SNR is -15 dB.

6. Conclusions

In this paper, we introduce Quadratic Phase Function algorithm to estimate the parameters of the LFM signal and propose a new algorithm for moving targets imaging, which can acquire an exact SAR image even under a SNR environment. The effectiveness and practicability of this imaging approach are demonstrated by means of numerical experiments using simulated data.

REFERENCES

[1] H. S. Wang and C. Mainlobe, "Clutter Cancellation by DPCA for Space Based Radars," *IEEE Aero Space Applications Conference,* Crested Butte, CO, USA: IEEE, 1991. pp. 1- 128.

[2] R. Klemm, "Application of Space-time Adaptive Processing," The Institution of Electrical Engineers, London, 2004.

[3] T. J. Abatzoglou, "Fast Maximum Likelihood Joint Estimation of Frequency and Frequency Rate," *IEEE Transactions on Aerospace and Electronic System*, Vol. 22, No. 6, 1986, pp. 708-715. doi:10.1109/TAES.1986.310805

[4] J. C. Wood and D. T. Barry, "Linear Signal Synthesis Using the Radon-Wigner Transforms," *IEEE Transactions On Signal Processing*, Vol. 42, No. 8, 1994, pp. 2105-2111. doi:10.1109/78.301845

[5] M. S. Wang, A. K. Chan and C. K. Chui, "Linear Frequency-Modulated Signal Detection Using Radon-ambiguity Transform," Vol. 46, No. 3, 1998, pp. 571-586.

[6] R. Tao, B. Deng and Y. Wang, "Research Progress of the Fractional Fourier Transform in Signal Processing," *Science in China* (*Ser. F, Information Science*), Vol. 49, No. 1, 2006, pp. 1-25.

[7] P. O'Shea, "A New Technique for Estimating Instantaneous Frequency Rate," *IEEE Signal Processing Letters,* Vol. 9, No. 8, 2002, pp. 251-252. doi:10.1109/LSP.2002.803003

[8] P. O'Shea, "A Fast Algorithm for Estimating the Parameters of a Quadratic FM Signal," *IEEE Transactions On Signal Processing*, Vol. 52, No. 2, 2004, pp. 385-393. doi:10.1109/TSP.2003.821097

[9] V. C. Chen "Time-Frequency Transforms for Radar Imaging and Signal Analysis," Artech House. 2002.

[10] L. Hong, "Analysis of Multi-component LFM Signals by the Integrated Quadratic Phase Function," *Journal of Electronics & Information Technology*, Vol. 31, No. 6, pp. 2009, pp. 1363-1366.

[11] P. Wang and J. Y. Yang, "Multicomponent Chirp Signals Analysis Using Product Cubic Phase Function," *Digital Signal Processing*, Vol. 16, No. 6, 2006, pp. 654-669. doi:10.1016/j.dsp.2006.09.002

