

PAPR Distribution Analysis at the Output of Nonlinear PAPR Reducers

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ABSTRACT

Nonlinear PAPR reducers, such as clipping and companding techniques, are some simple methods used to reduce the Peak-to-Average Power Ratio (PAPR). In this paper, assuming that the baseband OFDM signal is characterized as a band-limited complex Gaussian process, we investigate the PAPR distribution of an OFDM signal when it is passed through a nonlinear PAPR reducer. The obtained PAPR distribution depends on the nonlinear function which characterizes the PAPR reducer. Later in this paper, we apply the obtained PAPR distribution in the clipping case. The comparisons made between the proposed distribution and that obtained thanks to computer simulations show good agreement.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM); Peak-To-Average Power Ratio (PAPR); Distribution; Nonlinear PAPR Reducers

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is an attractive modulation technique for the next generation of high bit rate wireless transmission due to its high robustness to multipath fading and its great simplification of channel equalization [1]. However, one of the main problems of the OFDM modulation technique is the large peak-to-average power ratio (PAPR) of the transmitting signals. This high PAPR causes in-band and out-band interferences when the OFDM signals are passed through a high power amplifier (HPA) which does not have enough linear range. Several PAPR reduction techniques have been proposed [2] to reduce the PAPR of OFDM signals. To well understand this PAPR problem and to predict possible gain thanks to reduction techniques, many papers were interested in the PAPR distribution analysis. The pioneer work was the work of R. van Nee and A. de Wild in [3]. But this expression was obtained at the Nyquist frequency and therefore did not represent a realistic value of the continuous signal PAPR distribution. Then, using a probabilistic approach, Ochiai et Imai proposed a more realistic expression in [4]. Later, Zhou et Caffery in [5] proposed an upper bound of the Complementary Cumulative Distribution Function (CCDF) of the PAPR. Louet and Hussain in [6] proposed a new expression for continuous baseband OFDM signals. This latter expression was very close to the continuous signal PAPR simulation.

But, only few papers are dealing with PAPR distribu-

tion analysis at the output of PAPR reducers. Some researchers proposed a PAPR distribution analysis when there is unequal power allocation between carriers [7]. In [8], YOO *et al.* studied the PAPR distribution at the output of probabilistic PAPR reducer. In this paper we are interested in the PAPR distribution analysis at the output of PAPR reducers. We will, first of all, focus on the class of PAPR reduction techniques known as nonlinear PAPR reducers, *i.e.* the schemes for PAPR reduction that use spectrum distortion or spectral regrowth. This class of nonlinear PAPR reducers includes mainly clipping techniques [9] and companding techniques [10]. We derive a general expression of the CCDF at the output of non-linear reducers. Then, we apply this expression to the Soft Envelope Clipping (SEC) reducer.

The remainder of this paper is organized as follows: Section 2 briefly introduces nonlinear PAPR reducers. In Section 3, the PAPR distribution is analyzed through its Complementary Cumulative Distribution Function (CCDF). Then in Section 4 results of previous analysis is applied to Soft Envelope Clipping reducer. In this section, we provided some results which show good agreement between simulation and theoretical expressions. Finally in Section 5, a conclusion is drawn.

2. Characterization of Nonlinear PAPR Reducers

Let $x(t)$, be the baseband equivalent time-domain OFDM signal. $x(t)$ can be written as

$$x(t) = r(t)e^{j\phi(t)}, \quad (1)$$

$$0 \leq t \leq T_s,$$

where $r(t) = |x(t)|$ is the OFDM magnitude, $\phi(t)$ is the OFDM phase and T_s is the OFDM symbol period.

The PAPR of $x(t)$ may be defined as

$$\text{PAPR}_{[x]} = \frac{\max_{t \in [0, T_s]} |x(t)|^2}{P_x}, \quad (2)$$

where P_x is the signal $x(t)$ average power.

In nonlinear PAPR reducers (clipping, companding techniques), the data signal PAPR is reduced by a nonlinear function as shown in **Figure 1**.

Now, let us suppose $f[\cdot]$ the nonlinear function that characterizes the nonlinear PAPR reducer shown in **Figure 1**, the PAPR reduced signal $y(t)$ at the output of PAPR reduction scheme is expressed as

$$y(t) = f[r(t)]e^{j\phi(t)}, \quad (3)$$

$$0 \leq t \leq T_s,$$

where $f[\cdot]$ is nonlinear positive function also called function for PAPR reduction.

3. PAPR Distribution Analysis

In the literature, it is customary to use the Complementary Cumulative Distribution Function (CCDF) of the PAPR as a performance criterion. Let us consider x_n and y_n the discrete-time signals at the Nyquist rate of the OFDM signal $x(t)$ and its PAPR reduced version $y(t)$ respectively. For a large number of subcarriers, the OFDM envelope converges to a Rayleigh envelope distribution. Therefore, the probability density function (PDF) $p(r)$ of the OFDM envelope can be expressed as

$$p(r) = \frac{2r}{P_x} e^{-\frac{r^2}{P_x}}, \quad (4)$$

$$r \geq 0,$$

where P_x is the mean power OFDM signal.

Using (4), it was shown in [4] that, the OFDM PAPR CCDF could be approximated by the following expression:

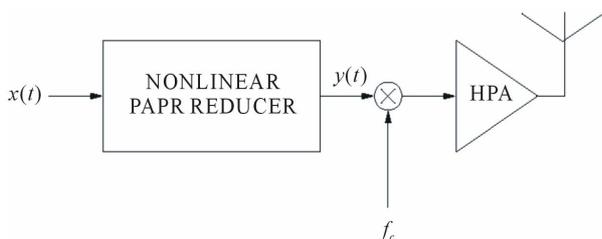


Figure 1. Nonlinear PAPR reducer scheme.

$$\text{CCDF}_{[x]}(\psi) = \Pr \left\{ \frac{\max_{0 \leq n < N} |x_n|^2}{P_x} > \psi \right\}$$

$$\approx 1 - \left(\Pr \left\{ |x_n| \leq \sqrt{\psi P_x} \right\} \right)^N \quad (5)$$

$$\approx 1 - \left(1 - e^{-\psi} \right)^N,$$

where, N is the number of samples per OFDM symbol period. This PAPR CCDF expression has been proved for the first time by R. van Nee and A. de Wild in [3].

In the same way as (5), we show that the PAPR distribution of the output signal y could be approximated by Equation (6):

$$\text{CCDF}_{[y]}(\tilde{\psi}) = \Pr \left\{ \frac{\max_{0 \leq n < N} |y_n|^2}{P_y} > \tilde{\psi} \right\} \quad (6)$$

$$\approx 1 - \left(\Pr \left\{ |y_n| \leq \sqrt{\tilde{\psi} P_y} \right\} \right)^N.$$

Using (2), it can be shown that, $|y_n| = f[|x_n|]$ and Equation (6) becomes

$$\text{CCDF}_{[y]}(\tilde{\psi})$$

$$\approx 1 - \left(\Pr \left\{ f[|x_n|] \leq \sqrt{\tilde{\psi} P_y} \right\} \right)^N \quad (7)$$

Equation (7) shows that, the expression of $\text{CCDF}_{[y]}(\tilde{\psi})$ depends on the function $f[\cdot]$ for PAPR reduction. In the following section of this paper, an exact expression of $\text{CCDF}_{[y]}(\tilde{\psi})$ is given in the soft envelop clipping's case [9].

4. PAPR Distribution in the Soft Envelope Clipping Technique's Case

In this section, in order to illustrate the theoretical results obtained for nonlinear PAPR reducers, we consider one nonlinear PAPR reducer which is commonly studied in the literature: The Soft Envelop Clipping (SEC) [9].

The nonlinear function $f(r)$ of SEC is expressed as

$$f(r) = \begin{cases} r & r \leq A \\ A & r > A \end{cases}, \quad (8)$$

where A is the magnitude threshold and commonly known as clipping threshold.

It is shown in [11] that the PDF $v(r)$ of the clipped signal envelope can be written as

$$v(r) = p(r) \cdot 1_{r \leq A} + \Pr \{ r > A \} \cdot \delta(r - A), \quad (9)$$

where $\delta(r)$ is the Dirac impulse. From (9), we show that,

$$\begin{aligned}
 \Pr\{f[|x_n|] \leq \sqrt{\tilde{\psi}P_y}\} &= \int_0^{\sqrt{\tilde{\psi}P_y}} \nu(r) dr \\
 &= \Pr\{|x_n| \leq \sqrt{\tilde{\psi}P_y}\} \cdot 1_{\tilde{\psi} \leq \frac{A^2}{P_y}} + 1_{\tilde{\psi} > \frac{A^2}{P_y}} \\
 &= \left(\int_0^{\sqrt{\tilde{\psi}P_y}} p(r) dr \right) \cdot 1_{\tilde{\psi} \leq \frac{A^2}{P_y}} + 1_{\tilde{\psi} > \frac{A^2}{P_y}} \\
 &= (1 - e^{-\tilde{\psi}\gamma}) \cdot 1_{\tilde{\psi} \leq \frac{\rho^2}{\gamma}} + 1_{\tilde{\psi} > \frac{\rho^2}{\gamma}},
 \end{aligned} \tag{10}$$

where $\rho = A/\sqrt{P_x}$ is the clipping ratio (CR) and γ is the output-to-input average power ratio defined as

$$\begin{aligned}
 \gamma &= \frac{P_y}{P_x} = \frac{1}{P_x} \int_0^\infty [f(r)]^2 p(r) dr \\
 &= 1 - e^{-\rho^2},
 \end{aligned} \tag{11}$$

where $f(r)$ is the function for PAPR reduction defined in (8) and $p(r)$ is the PDF of the OFDM signal expressed in (4).

Substituting (10) into (7), we show that, the expression of $CCDF_{[y]}(\tilde{\psi})$ for SEC is expressed as

$$CCDF_{[y]}(\tilde{\psi}) \approx 1 - \left[(1 - e^{-\tilde{\psi}\gamma}) \cdot 1_{\tilde{\psi} \leq \frac{\rho^2}{\gamma}} + 1_{\tilde{\psi} > \frac{\rho^2}{\gamma}} \right] \tag{12}$$

when ρ becomes great and tends to infinity, then γ tends to 1 and expression 12 is equal to classical expression 5 of the CCDF at the input of the clipping.

Figure 2 compares the theoretical CCDF of the signal's PAPR at the output of the PAPR reduction scheme expressed by (7), and this for simulation results obtained with three different values of ρ . The OFDM signal comprises $N = 64$ subcarriers and is simulated with an oversampling factor of $L = 4$. It should be noted that, the theoretical $CCDF_{[y]}(\tilde{\psi})$ shows good agreement with the simulation results. Nevertheless, where $\rho = 3$ dB and 5 dB, the theoretical $CCDF_{[y]}(\tilde{\psi})$ is less accurate with the simulation results where $\rho = 7$ dB. The reason for this is that, for low values of ρ , the theoretical $CCDF_{[y]}(\tilde{\psi})$ tends to be a Dirac and becomes very sensitive to approximation errors.

5. Conclusions

In this paper, assuming that the baseband OFDM signal is characterized as a band-limited complex Gaussian process, we have investigated the PAPR distribution of an OFDM signal at the output of a nonlinear PAPR reducer. The obtained PAPR distribution has been applied in the clipping case, which is a well-known example of nonlinear PAPR reducer used for OFDM PAPR reduction.

The comparisons made between the proposed PAPR

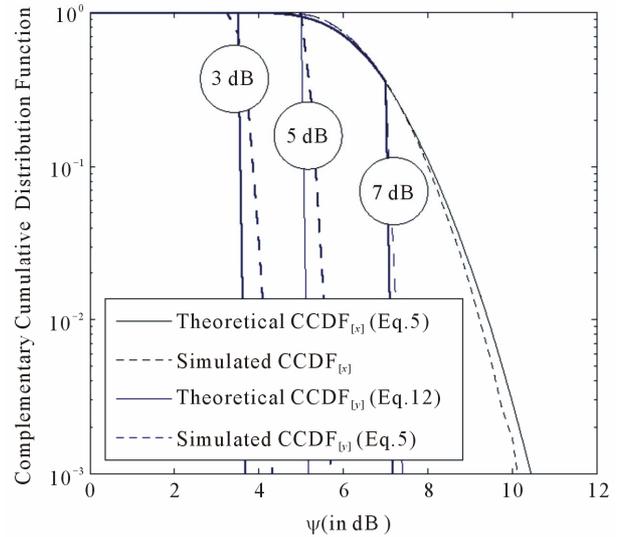


Figure 2. Comparison of the proposed distributions for the PAPR related to the clipping technique (12) with simulation results for $\rho = 3$ dB, 5 dB and 7 dB.

distribution at the output of clipping and with that obtained thanks to computer simulations show good agreement.

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