Journal of Signal and Information Processing, 2011, 2, 178-183
doi:10.4236/jsip.2011.23024 Published Online August 2011 (http://www.SciRP.org/journal/jsip)

Cosine Modulated Non-Uniform Filter Banks

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Received April 13th, 2011; revised May 19th, 2011; accepted May 29th, 2011.

ABSTRACT

Traditional designs for non-uniform filter bank (NUFB) are usually complex; involve complicated nonlinear optimization with a large number of parameters and lack of linear phase (LP) property. In this paper, we describe a simple design method for multirate near perfect reconstruction (NPR) cosine modulated filter banks with non-uniform frequency spacing and linear phase property that involves optimization of only single parameter. It is derived from the uniform cosine modulated filter bank (CMFB) by merging some relevant band pass filters. The design procedure and the structure of the uniform CMFB are mostly preserved in the non-uniform implementation. Compared to other design methods our method provides very good design and converges very rapidly but the method is applicable, only if the upper band edge frequency of each non-uniform filter is an integral multiple of the bandwidth of the corresponding band. The design examples are presented to show the superiority of the proposed method over existing one.

Keywords: *Cosine Modulation, Merging, Non-Uniform Filter Bank, Near Perfect Reconstruction*

1. Introduction

Multirate filter bank find wide applications in many areas of digital signal processing such as sub-band coding, transmultiplexer, image, video and audio compression, adaptive signal processing [1-3]. On the basis of time-frequency resolution, filter bank can be classified in two categories, *i.e.*, uniform and non-uniform filter bank. Uniform filter bank provides fixed and uniform time frequency decomposition [1]. However in some applications like audio analysis and coding, broadband array signal processing non-uniform and variable time-frequency resolution may lead to better performance and reduced arithmetic complexity, which is provided by non-uniform filter bank [NUFB] [4-6]. Therefore efficient structure and design procedures for NUFB are highly desirable. Over the years, a number of design methods have been proposed by different authors [6-10]. Among these, only few of them possess linear phase (LP) property. The tree structure method [1] is an easy way to design LP-NUFB *via* cascading uniform filter bank. However, the limitation of decimation factors and the long system delay are two major drawbacks of using this method. Most of the available approaches [6-10] for NUFBs, use standard constrained or unconstrained optimization techniques to obtain the design, which tend to be computationally expensive, when high order filters are used. In wideband audio signal analysis and coding, filter banks with high stop band attenuation greater than 100 dB is required. Moreover, it is difficult to design NUFBs with high stop band attenuation and LP property. In [11], a simple design method for NUFBs was proposed. It is based on the design of a uniform cosine modulated filter bank and is applicable only to non-uniform integer- decimated filter banks. Moreover, it still involves complicated nonlinear optimization with large number of parameters. Recently, Zing *et al*. [12] proposed interpolated FIR prototype filter to design the NUFB.

In this work, a simple design approach for linear phase NUFB is presented. The approach is based on uniform CMFBs] as shown in **Figure 1**. The constituent NUFB as shown in **Figure 2** is obtained by merging some relevant uniform filters in the associated uniform CMFB. The design procedure is therefore reduced to the design of the prototype filter in the associated uniform CMFB. With this approach NUFBs with high stop band attenuation up to 110 dB can be easily designed. A single variable optimization is used to obtain minimum value of amplitude ($E_{\max}$) and aliasing (E_a) distortions [1].

2. Cosine Modulated Filter Bank

2.1. Uniform

Cosine modulation is a cost effective technique for *M*-

-band filter bank [1]. In this approach all the filters of analysis and synthesis section are obtained by cosine modulation of single linear phase prototype low pass filter which normally has linear phase and a finite length impulse response as shown in **Figure 1**. Let $H(z)$ be the transfer function of the prototype filter. It is given as:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \quad (1)$$

The impulse responses of filters of analysis and synthesis sections are obtained from the closed form expressions as given by [1]:

$$h_k(n) = 2h(n)\cos\left[(2k+1)\frac{\pi}{2M}\left(n-\frac{N}{2}\right)+(-1)^k\frac{\pi}{4}\right]$$

$$f_k(n) = 2h(n)\cos\left[(2k+1)\frac{\pi}{2M}\left(n-\frac{N}{2}\right)-(-1)^k\frac{\pi}{4}\right] \quad (2)$$

for $0 \le k \le M-1,\ 0 \le n \le N-1$

The required prototype filter is designed by window technique using Kaiser Window function.

2.2. Non-uniform

In the case of non-uniform NPR filter banks, the concept of cosine modulating low pass filters is applied [11,13]. After designing the required uniform CMFB, the corresponding NUFB is obtained by merging the relevant band pass filters of analysis and synthesis section of the uniform filter bank as described below [13].

We define $\bar{H}_i(z)$, $i = 0,1,2,\cdots,\bar{M}-1$, to be the filters obtained by merging the $l_i(\ge 1)$ adjacent analysis filters, *i.e.*, $H_k(z)$'s, for $i = n_i$ through ($n_i + l_i - 1$) in a uniform M-channel CMFB. More specifically,

$$\bar{H}_i(z) = \sum_{k=n_i}^{n_i+l_i-1} H_k(z) \quad 0 \le i \le \bar{M}-1 \quad (3)$$

where $n_{i+1} = n_i + l_i$ .We define $\bar{F}_i(z)$, $i = 0,1,2,\cdots,\bar{M}-1$ in a similar manner for the synthesis filters $F_k(z)$'s of the M-channel CMFB. That is,

$$\bar{F}_i(z) = \frac{1}{l_i}\sum_{k=n_i}^{n_i+l_i-1} F_k(z) \quad 0 \le i \le \bar{M}-1 \quad (4)$$

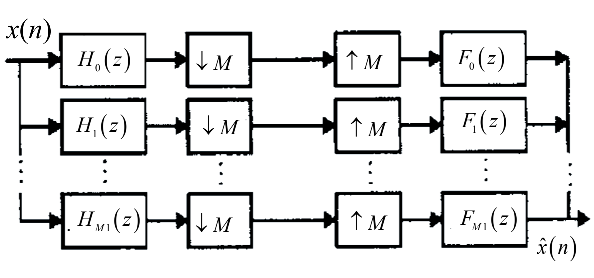

Figure 1. *M*-channel uniform filter bank.

Then $\bar{H}_i(z)$ and $\bar{F}_i(z)$, $i = 0,1,2,\cdots,\bar{M}-1$, form a new set of analysis and synthesis filters in the $\bar{M}$-channel non-uniform CMFB. Note that $n_0 = 0 < n_1 < n_2 < \cdots < n_{\bar{M}} = M$, and $l_0 < l_1 < l_2 < \cdots < l_{\bar{M}-1} = M$. **Figure 2** shows the resulting overall structure of the $\bar{M}$-channel non-uniform CMFB where $M_i(M/l_i)$, $i = 0,1,2,\cdots,\bar{M}-1$, amounts to the decimation ratio for the *i*th channel.

3. Optimization Technique

In NPR, perfect reconstruction condition is relaxed by allowing small amount of distortion. Three types of distortions occur at the reconstructed output, *i.e*, amplitude ($E_{\max}$), phase and aliasing (E_a) [1]. The aliasing and phase distortion can be eliminated by careful design of the linear phase FIR filter. However, amplitude distortion can not be eliminated completely but can be minimized by applying optimization technique [14]. Initially; Johnston [15] developed a nonlinear optimization technique. Later on many prominent authors such as Creusere *et al.* [16], Lin *et al.* [17], Jain *et al.* [18], have simplified it using linear optimization technique with objective function as given below:

$$\phi_a = \max\left\| H\left(e^{j\omega}\right)\right|^2 + \left|H\left(e^{j(\omega-\pi/M)}\right)\right|^2 - 1\right| \quad (5)$$

for $0 \le \omega \le \pi/M$

In this work same objective function in modified form is used for the design of non-uniform filter bank, as given below vaidynathan [1]:

$$\phi = \max\left|\sum_{i=0}^{\bar{M}-1}\left|\bar{H}_i\left(e^{j\omega}\right)\right|^2 - 1\right| \quad (6)$$

for $\bar{M} - 1 \le \omega \le \pi/\bar{M}$

where, $\bar{M}$ is number of channels in non-uniform filter bank and $\bar{H}_i\left(e^{j\omega}\right)$ is the frequency responses of the filters of the non-uniform section. Initially, input parameters, *i.e.*, sampling rate, number of band, pass band and stop band frequencies, pass band ripple and stop band attenuation of prototype filter are specified. Cutoff

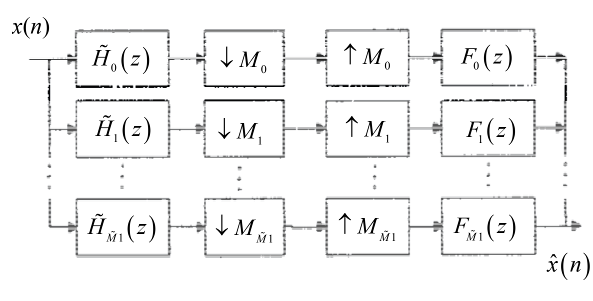

Figure 2. $\tilde{M}$-channel nonuniform filter bank.

frequency, transition band and filter length is than determined. Initialize, different optimization pointers like step size, search direction, flag and initial (perror) as well as expected minimum possible values (terror) of the objective function. Inside the optimization loop, design the prototype low pass filter and determine the band pass filters for analysis and synthesis sections using cosine modulation. Obtain the desired NUFB using merging of relevant band pas filters. In optimization routine cutoff frequency of the prototype filter is gradually changed as per the search direction and calculates the corresponding value of the objective function. Algorithm halts when it attains the minimum value of the objective function. The flowchart of optimization **Figure 3** is given below and

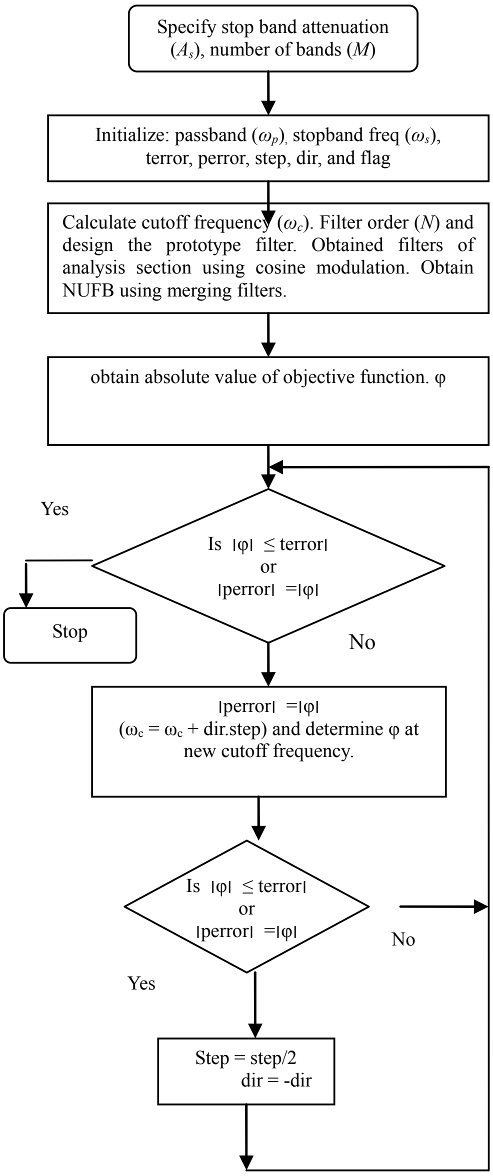

Figure 3. Flow chart of optimization algorithm.

simulated on MATLAB 7.0.

4. Design Examples

In this section 3-channel and 5-channel NUFBs are designed and the performance of proposed technique is compared with the earlier reported work [5,11,12,19].

In this example a 3-channel NUFB with decimation factor (4, 4, 2) has been designed using same specifications as given in Xie *et al.* [5] and Li *et al.* [11]. The design specifications of the filter are: stop band attenuation $A_s = 100$ dB, $N = 63$, $l_0 = 1$, $l_1 = 1$, $l_2 = 2$ and $n_0 = 0$, $n_1 = 1$, $n_2 = 2$. The band edge frequencies are $\omega_1 = \pi/4$, $\omega_2 = \pi/2$. The magnitude responses of prototype filter, NUFB, optimized value of amplitude distortion is shown in **Figures 4-6**. The obtained value of maximum amplitude distortion is $E_{\max} = 2.99 \times 10^{-3}$.

This example is quoted to compare the performance with recent work of Zing *et al.* [12]. In the work of [12], 5-channel NUFB with integer decimation factors (4, 4, 8, 8, 4) is designed using IFIR based prototype filter. The reported length of model and interpolator filters are N_m =

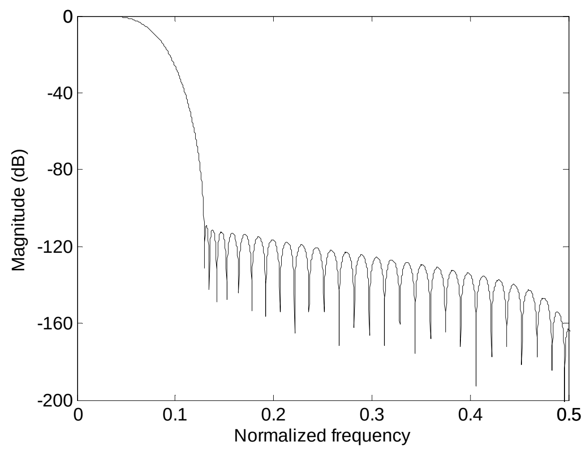

Figure 4. Magnitude response of optimized prototype filter.

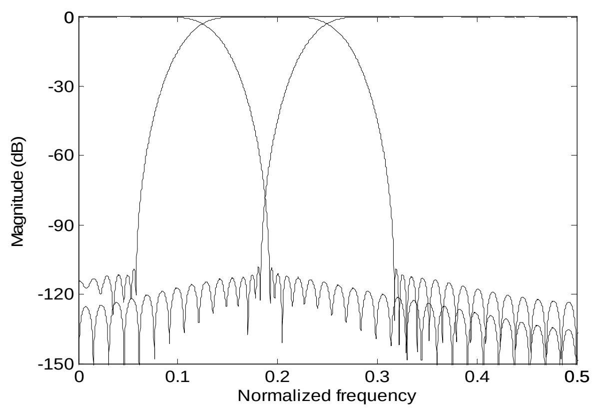

Figure 5. Magnitude response of three- channel filter bank.

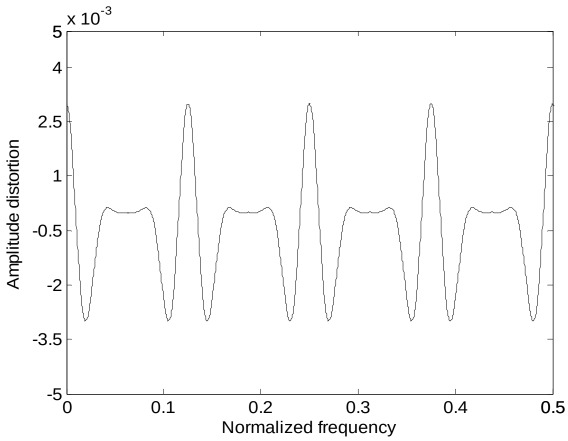

Figure 6. Amplitude distortion plot.

31 and N_i = 39, respectively. Therefore, the obtained overall filter length of IFIR prototype filter becomes $N = (L.N_m + N_i) = 163$ [14]. Here, L is the stretch factor. In this example, 5-band NUFB is designed with the following specifications as in [12]: The band edge frequencies are $\omega_1 = \pi/4$, $\omega_2 = \pi/2$, $\omega_3 = 5\pi/8$, $\omega_4 = 3\pi/4$. In this case, $n_0 = 0$. $n_1 = 2$, $n_2 = 4$, $n_3 = 5$, $n_4 = 6$, $n_5 = 8$; and $l_0 = 2$, $l_1 = 2$, $l_2 = 1$, $l_3 = 1$, $l_4 = 2$. The obtained prototype filter has the length 163, the stop band attenuation A_s = 110 dB**.** The magnitude responses of prototype filter, filter bank and distortion parameters are shown in **Figures 7-9**. The obtained resulting distortion parameter is maximum amplitude distortion $E_{\max}$ = 0.0065 dB.

5. Discussion

Two design examples for the NUFB are presented to demonstrate effectiveness of the design. A three and five channel symmetric non-uniform filter banks were designed and the amplitude characteristics of analysis filters are shown in **Figure 5** and **Figure 8**. The positive frequency range is clearly divided into three and five non-uniform bands. These filter banks have integer decimation factors. The filter lengths of analysis FIR filters are 63 and 85. For both the designs the Kaiser windowed LPF were used as initial filters for minimization of the performance function. And, as a tool for optimization, the linear iterative algorithm was utilized. Comparisons with Tree-Structure NUFBs show that the Tree-Structure can be either PR or NPR depending on the FBs used in the design. On the other hand proposed method can only design NPR FBs. The advantage of our method is that it can be used to design a feasible or non feasible partition NUFB with good performance. Since it is derived from uniform CMFBs by cosine modulating a prototype filter, its implementation also consists of one prototype filter and a discrete cosine transform (DCT). Since the number of parameter is reduced, the speed of convergence is faster, and filter bank with high attenuation can be designed. It is clear from **Table 1** and **Table 2** that for same decimation factors the proposed work provided better results for peak amplitude distortions ($E_{\max}$).

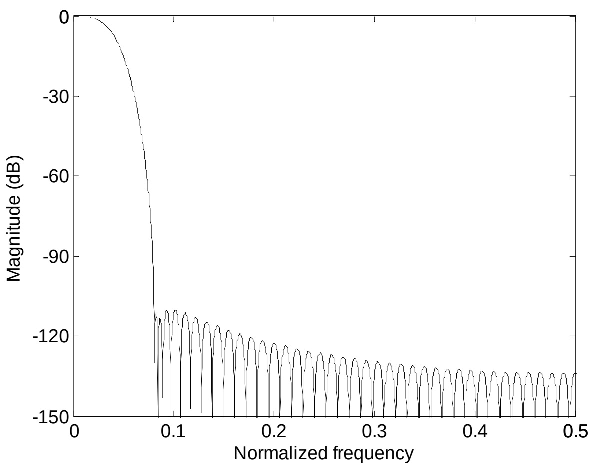

Figure 7. Magnitude response of optimized prototype filter.

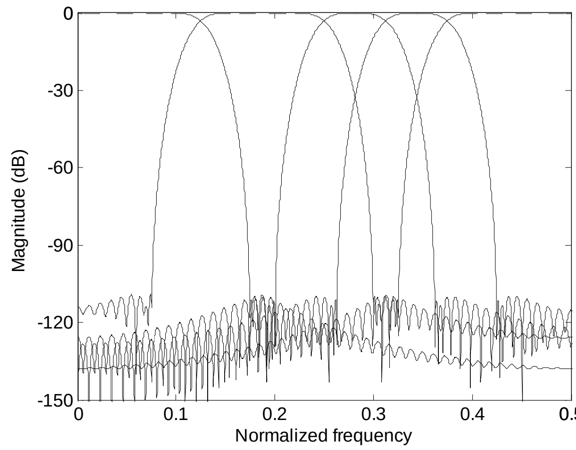

Figure 8. Magnitude response of five-channel filter bank.

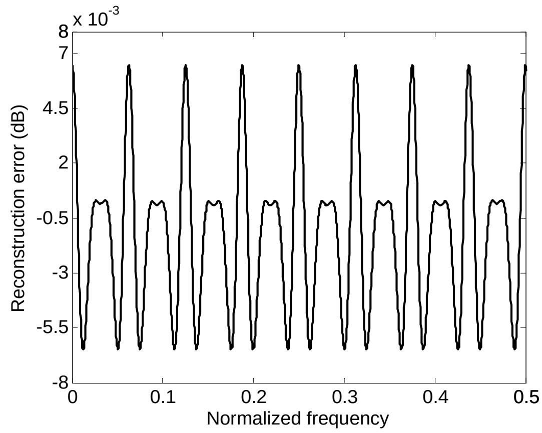

Figure 9. Amplitude distortion plot.

Table 1. Performance comparision with earlier reported works for three-channel NUFB.

Work	Channels /Decimation Factors	Technique used	A_s	Filter length	Amplitude distortion
Li *et al.* [11] (1997)	Three channels (4,4,2)	Cosine modulation	60 dB	64	7.803×10^{-3}
Xie *et al.* [5] (2006)	Three channels (4,4,2)	Recombination	110 dB	63	7.803×10^{-3}
Soni *et al.* [19] (2010)	Three channels (4,4,2)	Tree structure	110 dB	63	3.85×10^{-3}
Proposed	Three channels (4,4,2)	Cosine modulation	110 dB	63	2.99×10^{-3}

Table 2. Performance comparision with earlier reported works for five-channel NUFB.

Work	Channels Decimation Factors	Technique used	A_s	Filter order	Amplitude distortion
Lee *et al.* [13] (1995)	Five channels (4,4,8,8,4)	Cosine modulation (FIR)	46.3 dB	40	0.027 dB
Zijing *et al.* [12] (2007)	Five channels (4,4,8,8,4)	Cosine modulation (IFIR)	110 dB	$N_m = 31$, $N_i = 39$ $\text{N} = L \cdot N_m + N_i = 163$	0.0068 dB
Proposed	Five channels (4,4,8,8,4)	Cosine modulation (FIR)	110 dB	163	0.0065 dB

6. Conclusions

A simple and computationally efficient design of NUFB is presented. In traditional design approaches, it is difficult to design the NUFB at high stop band attenuation above 100 dB. The proposed work eliminated this constraint by exploiting the design process of cosine modulation and obtained NUFB with a feasible partition property. The performance comparison of proposed with previously reported work shows that the resulting overall distortion and aliasing errors are smaller than the previous reported work. In addition, this method has lower system delay compared with the LP NPR NUFBs by the indirect method. This method is suitable particularly for large number of channels where high order filters with unequal pass bands have to be designed with small distortion and aliasing. Such filter banks are needed in a wide variety of applications like speech coding and speech enhancement.

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