

# Estimation of the Yield Stress of Stainless Steel from the Vickers Hardness Taking Account of the Residual Stress

Osamu Takakuwa, Yusuke Kawaragi, Hitoshi Soyama

Department of Nanomechanics, Graduate School of Engineering, Tohoku University, Sendai, Japan.  
Email: o\_takakuwa@mm.mech.tohoku.ac.jp

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## ABSTRACT

In this paper, a method that uses the Vickers hardness to estimate the yield stress of a metallic material with taking account of residual stress is proposed. Although the yield stress of bulk metal can be evaluated by a tensile test, it cannot be applied to local yield stress varied by surface modification methods, such as the peening technique which introduces high compressive residual stress at the surface. Therefore, to evaluate the local yield stress employing a relatively easy way, the Vickers hardness test was conducted in this paper. Since the Vickers hardness depends on both the residual stress and the yield stress, the relationship between the residual stress and the Vickers hardness was experimentally examined. It was concluded that the yield stress of the surface treated by several peening techniques can be estimated from the Vickers hardness once this has been corrected for residual stress.

**Keywords:** Peening; Yield Stress; Hardness; Residual Stress

## 1. Introduction

Peening is one of the most effective surface modification techniques for introducing compressive residual stress and work hardening metallic materials [1]. The mechanical properties improved using this technique help to increase the fatigue strength of mechanical components and the resistance of structural materials to stress corrosion cracking. It is important to evaluate material properties such as the yield stress and the residual stress in the surface after peening.

Although the residual stress in a layer modified by peening can be easily measured by X-ray diffraction methods, evaluating the yield stress of the modified layer is much more difficult. The average yield stress of the material as a whole can be evaluated by a general tensile test, *i.e.*, from the stress-strain curve. However, the local yield stress altered by peening cannot be evaluated by this test since this is predominantly affected by the base material and less so by the modified layer.

Recently, indentation tests to evaluate the mechanical properties in local areas of a material, have come to the fore and several studies have been conducted on this topic, and some approximate equations for evaluating the yield stress from the load-displacement curve obtained by the indentation test have been proposed [2-5]. Dao

*et al.* proposed a method to estimate the yield stress and work hardening constant by an indentation test using a conical indenter [2], which is based on the concept of representative strain introduced by Tabor [6]. The representative strain represents a strain field under the indenter during the indentation test. This concept made it possible to evaluate the local mechanical properties. If the conical indenter is used, two conical indenters with different half apex angle at least would be needed. It is because that two different representative strains need to be obtained so as to determine the two unknown mechanical properties, *e.g.*, the yield stress and work hardening constant. Also, Yan *et al.* proposed a method using a plural indenter based on a reverse analysis to evaluate these parameters on the residual stress field [7]. To make it easier, a methodology employing a spherical indenter has been proposed by Nishikawa *et al.* By the methodology, the yield stress at the surface where residual stress exists can be evaluated by a combination of an indentation test and inverse analysis with response surface methodology using finite element analysis (FEA) [8]. However, this method is somewhat complex since much updating of the response surface needs to be done for an accurate estimation. For more practical use, a rapid and quantitative method needs to be developed. This study focuses on the rela

tion between the yield stress, residual stress and the Vickers hardness so as to rapidly and simply evaluate the local yield stress of high compressive residual stress field including treated materials by peening.

A simple method based on the Vickers hardness can be used to estimate the yield stress. The relationship between the Vickers hardness and the yield stress has been investigated [9,10]. Tabor [9] showed that the Vickers hardness of several metals is proportional to the yield stress, with a proportionality constant of approximately 3. In the case of austenitic stainless steel, Busby *et al.* [10] reported that the yield stress and Vickers hardness followed a linear relationship of the form Equation (1):

$$\Delta\sigma_y = K \Delta H_V \quad (1)$$

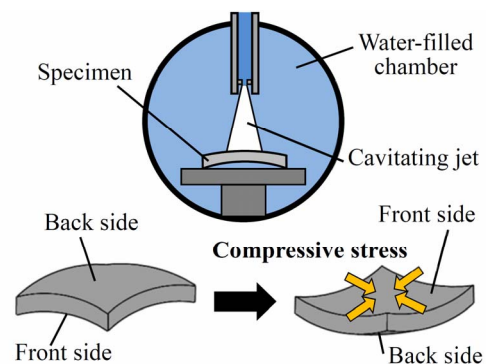
where,  $\Delta\sigma_y$  and  $\Delta H_V$ , are the changes in the yield stress  $\sigma_y$  and the Vickers hardness  $H_V$ , respectively.  $K$  is a constant determined experimentally, and has a value of  $309 \pm 18 \text{ MPa GPa}^{-1}$ . The yield stress can be determined from the measured value of the Vickers hardness using this linear relationship. However, the Vickers hardness is not only affected by material properties such as the yield stress and work hardening exponent but also the residual stress [11,12]. The residual stress is easily induced by heat treatment and/or mechanical processing. Therefore, in practical applications, the residual stress has to be taken into account when evaluating the yield stress. In addition, the relationship between yield stress and Vickers hardness is difficult to apply to materials which have a residual stress distribution induced by surface modification such as peening.

In this study, a simple and straight-forward method is proposed, using the Vickers hardness test to estimate the yield stress of JIS SUS316L stainless steel, in which compressive residual stress is extant. First, in order to establish the pure relationship between the equibiaxial compressive stress and the Vickers hardness, the Vickers hardness tests were performed on specimens having several value of the equibiaxial compressive stress. It was measured by X-ray diffraction method employing  $\sin^2\psi$  method. Second, in order to make sure that the regression coefficient obtained from the relationship between the compressive stress and the hardness was independent of the yield stress value, and simple finite element analysis was done. Finally, the Vickers hardness tests and the indentation tests employing the combination of inverse analysis with response surface methodology [8] were performed on specimens treated by several peening techniques. Then the relationship between the Vickers hardness and the yield stress irrespective of the compressive residual stress was determined. Therefore, the proposed method can be used to determine the yield stress from the Vickers hardness.

## 2. Experimental Apparatus and Procedures

### 2.1. Evaluation of Vickers Hardness Varied by Compressive Residual Stress

The material under test was austenitic stainless steel (Japanese Industrial Standards JIS SUS316L). The geometry of each specimen was square, 35 mm on each side and 4 mm thick. In order to introduce and control the equibiaxial compressive stress on the fronts of the specimens, a cavitating jet was applied to the backs as shown in **Figure 1** [13]. When the back side is exposed to the cavitating jet, plastic strain occurs due to impacts produced by cavitation bubble collapsing. Then an equibiaxial elastic reactive stress is generated against plastic deformation at the back side. The reactive stress corresponds to equibiaxial compressive stress induced by peening technique. So, at the back side, the compressive residual stress is introduced with increase in the yield stress due to work hardening caused by the jet. If the thickness of the specimen is certainly thin, curvature would be generated because of lower resistance to the deformation due to the plastic strain caused at the back side. Therefore, an equibiaxial compressive stress on the front side is introduced by stretching the specimen without any work hardening. In this case, neither plastic deformation nor a change in the crystalline structure occurs on the front side of specimens, since the cavitating jet is applied only to the back side. Therefore, using this method, different compressive stresses can be induced on the front sides of the specimens, while the other material properties, such as Young's modulus and the yield stress, are the same in each case. The equibiaxial elastic-compressive stress induced by this method depends on the equibiaxial curvature produced by the jet. The equibiaxial curvature can be controlled by the exposure time, *i.e.*, amount of the treatment, and the thickness of the specimen, *i.e.*, the resistance to the bending. By varying the exposure time to the cavitating jet, equibiaxial elastic-compressive stress on the front side can be controlled. A cavitating jet is generated when high-speed

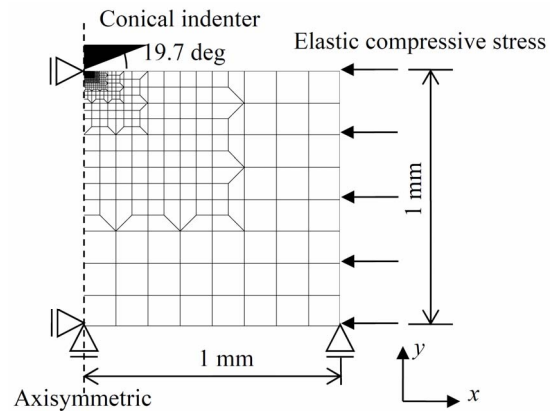


**Figure 1. Schematic illustration of cavitating jet apparatus to introduce equibiaxial elastic compressive stress.**

water is injected into water. To use a strong cavitating jet, the pressure at which the high-speed water jet was injected, *i.e.*, the upstream pressure, and the pressure of the water filled in tank, *i.e.*, the downstream pressure, were set to 30 MPa and 0.42 MPa, respectively [14]. The nozzle diameter for the high-speed water jet, *d*, was 2 mm, and the standoff distance between the nozzle throat and the specimen, *s*, was 85 mm. Exposure times of 10, 20, 40, 80, 160, and 320 sec. were used. The residual stress was measured in orthogonal directions at the centers on the front sides of the specimens. It was confirmed that the compressive residual stress introduced in the specimens was almost equibiaxial [13].

Both the equibiaxial elastic stress,  $\sigma_{ee}$ , and residual stress induced by peening,  $\sigma_r$ , were evaluated by the  $\sin^2\psi$  method using an X-ray diffraction system MSF-3M (Rigaku Corporation). The X-ray tube was a Cr tube operated at 30 kV and 8 mA. X-rays from the  $K\beta$  peak were used. The angle of the soller slit was 1 degree and the width was 4 mm. The diffractive plane was the (3 1 1) plane of  $\gamma$ -Fe. The reference diffractive angle  $2\theta_0$  was 148.52 deg and the diffractive angle  $2\theta$  ranged from 143 deg to 153 deg in increments of 0.2 deg with 8 s intervals. The stress factor for the X-ray diffraction measurements was  $-368.93$  MPa/deg. Vickers hardness tests were conducted five times at the center of each specimen using a micro hardness tester HMV-1 (Shimadzu Corporation). The maximum load was set to  $P_{max} = 1.961$  N.

The yield stress of SUS316L used in the above experiments was 300 MPa with 0.2% proof stress. In order to confirm that the regression coefficient of hardness as a function of residual stress does not depend on the variation in yield stress, it is necessary to investigate the relationship between the residual stress and the Vickers hardness at various yield stresses. So, finite element analysis FEA was done to just demonstrate that the relationship between hardness and residual stress does not depend on the value of the yield stress. The hardness at four values of yield stress, varied by rolling rates of 0 (not-treated), 10 (Condition 1), 30 (Condition 2) and 50% (Condition 3), was obtained from FEA using the stress-strain curve. The stress-strain curves of those specimens were measured by a tensile test using a precision universal tester AG-I 50 kN (Shimadzu Corporation). The tensile rate was 1 mm/min. Thus, the relationship at various yield stresses can be obtained by varying the residual stress. A finite element model for the indentation test was constructed to obtain the relationship between the residual stress and the hardness of a specimen with a yield stress other than 300 MPa. Based on this model, elastic-plastic incremental analyses using the commercial finite element code MSC. Marc was conducted. **Figure 2** illustrates the axisymmetric finite element model used in the analyses. The indenter was modeled as a rigid cone with a half-included tip



**Figure 2. Finite element model for indentation test with a conical indenter.**

angle,  $\alpha$ , of 70.3 deg, so as to conform the ratio of the contact area to depth with a Vickers indenter. The specimen was modeled as a homogeneous isotropic elastic-plastic material. The size of the specimen for the model was 1 mm radius  $\times$  1 mm depth, which was sufficiently large to exclude boundary effects. The indentation load was applied through analysis of the contact between the indenter and the specimen. Fixed boundary conditions were applied to the bottom of the specimen, as shown in **Figure 2**. Elastic-plastic analysis using an updated Lagrange configuration was conducted based on  $J_2$  flow theory with isotropic hardening. Poisson's ratio was set to  $\nu = 0.3$ , and Young's modulus and the work hardening curve were measured by a uniaxial tensile test.

The hardness was calculated from the load, displacement curve obtained by FEA. The hardness, *H*, is given as follows:

$$H = \frac{P}{A} \tag{2}$$

where *P* and *A* denote the indentation load and the contact area of the hardness impression, respectively. The contact area, *A*, is a function of the contact depth, *h<sub>c</sub>*, and is given by the following:

$$A = 24.5h_c^2 \tag{3}$$

where *h<sub>c</sub>* is given by the following Equation:

$$h_c = h_{max} - \varepsilon \frac{P_{max}}{S} \tag{4}$$

where *P<sub>max</sub>*, *h<sub>max</sub>*,  $\varepsilon$ , and *S* denote the peak load, the maximum indentation depth, the geometric constant and the contact stiffness, respectively. In the case of the conical indenter,  $\varepsilon = 0.726$  [15]. The FEA was done to demonstrate that the relationship between hardness and residual stress does not depend on the value of the yield stress.

## 2.2. Evaluation of Vickers Hardness and the Yield Stress Varied by Several Surface Treatments

In order to investigate the variation in Vickers hardness with respect to the variation in yield stress, Vickers hardness tests were conducted on several specimens with various yield stresses. The yield stress of these specimens was varied by peening and polishing, according to the conditions described in **Table 1**. In the cavitation peened specimens, processing time per unit length,  $t_p$ , was changed to vary amount of the treatment. In the laser peened specimens, laser energy,  $E$ , was also changed.

The yield stress of not-treated specimen (SR heat treatment) and rolling rate of 50% specimen were 0.2% proof stress determined by the stress-strain curve. The yield stresses of polished, cavitation peened (CP) and laser peened (LP) specimens were estimated by inverse analysis using indentation by a spherical indenter [8], since the tensile test cannot be used to evaluate the yield stress distributed in the sub-surface. The inverse analysis procedure used to estimate the yield stress is described below. This method has already constructed in the past report [8]. The response surface relating the material parameters to the indentation load-displacement curves was obtained using FEA of the indentation. Young's modulus,  $E$ , yield stress,  $\sigma_y$ , and the work hardening exponent,  $n$ , were defined as material parameters. The response surface enables the indentation curve of a material with arbitrary parameters to be estimated. Using inverse analysis based on a genetic algorithm, the material parameters were obtained by minimizing the error between the approximate values calculated by the response surface and the experimental values obtained by the indentation tests.

**Table 1. Treatment conditions for the specimen.**

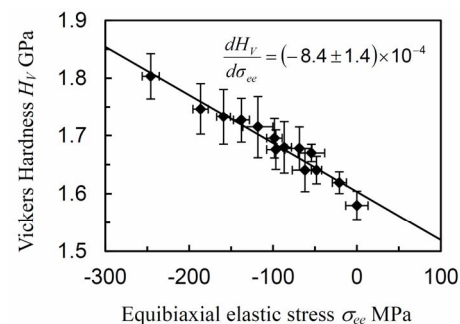
Evaluated by a tensile test		Evaluated by inverse analysis	
1	Not-treated	3	Polished
2	Rolling rate 50%	4	Cavitation peening using Cavitating jet in water [16] $t_p = 1$ s/mm
		5	Cavitation peening using Cavitating jet in water [16] $t_p = 4$ s/mm
6		6	Cavitation peening using Cavitating jet in water [16] $t_p = 16$ s/mm
		7	Cavitation peening using Cavitating jet in air [17] $t_p = 20$ s/mm
8		8	Laser peening conducted by Cincinnati $E = 6$ J
9		9	Laser peening conducted by Cincinnati $E = 10$ J
10		10	Laser peening conducted by Toshiba Co. Ltd. [18] $E = 20$ mJ

Additionally FEA of the indentation was conducted using the material parameters found. If the accuracy of the response surface was sufficiently high, the FEA result corresponded to the experimental indentation result. If not, the FEA result was added to the data set of the response surface to improve its accuracy. This updating process was repeated until the accuracy of the response surface was sufficiently high. The effect of residual stress was eliminated by estimating the response surface from the indentation load-displacement curve of a specimen with no residual stress. The validity of this method was verified through application to specimens whose yield stress was known. It had already been confirmed that the yield stress obtained by inverse analysis corresponded well to the yield stress measured by the tensile test [8].

## 3. Results and Discussion

### 3.1. Relationship between Vickers Hardness and the Compressive Stress

**Figure 3** shows the Vickers hardness,  $H_V$ , as a function of the equibiaxial elastic stress,  $\sigma_{ee}$ , obtained by a Vickers hardness test conducted on specimens with various residual stress. The compressive stress is presented as negative. It is confirmed that the maximum the equibiaxial elastic-compressive stress induced on the front sides of the specimens whose back sides had been exposed to the cavitating jet was 250 MPa, which is less than the yield stress of 300 MPa of the not-treated specimen, *i.e.*, within the range of elastic stress. The measured Vickers hardness increases linearly with increasing compressive stress. The line was approximated by the weighted least squares method. The regression coefficient obtained from **Figure 3** is  $(-8.4 \pm 1.4) \times 10^{-4}$ . The effect on Vickers hardness due to the compressive stress was obtained by multiplying the compressive stress by this gradient. This result can be applied to estimate the Vickers hardness of a material under compressive residual stress. According to past studies [11, 12], the effect of tensile residual stress on the Vickers hardness is greater than that of compressive residual stress. In the case of compressive residual stress,



**Figure 3. Relationship between residual stress and Vickers hardness of SUS316L.**

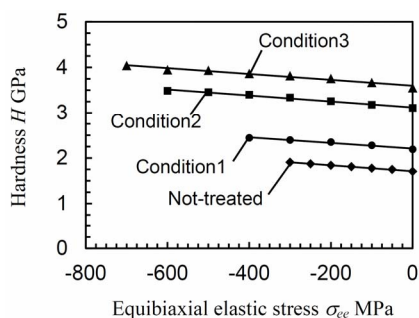
such as in a peened surface, the gradient of the regression line obtained from the present study should be used.

**Figure 4** shows the FEA results of indentation with a conical indenter. The applied compressive stress is lower than the elastic limit. The hardness increases proportionally with increasing compressive stress. The gradients of the regression lines of the not-treated specimen, and Condition 1 (10% rolled,  $\sigma_y = 453$  MPa), Condition 2 (30% rolled,  $\sigma_y = 734$  MPa) and Condition 3 (50% rolled,  $\sigma_y = 839$  MPa) specimens are  $(-6.54 \pm 0.13) \times 10^{-4}$ ,  $(-6.17 \pm 0.54) \times 10^{-4}$ ,  $(-6.54 \pm 0.35) \times 10^{-4}$  and  $(-6.43 \pm 0.64) \times 10^{-4}$ , respectively. These values are slightly smaller than the experimental data. The reason for the difference might be due to the different shape of the indenter. The equibiaxial elastic stress field affects indented area significantly. The tensile stress increases the indented area and the compressive stress does opposite. So, Suresh and Giannakopoulos proposed a method to estimate the residual stress by variation of the indented area due to that stress [19]. The compressive stress corresponds to a reactive force against the indenter. Therefore the relation between the hardness and compressive stress should be independent from yield stress and the gradient keeps constant even though yield stress changes.

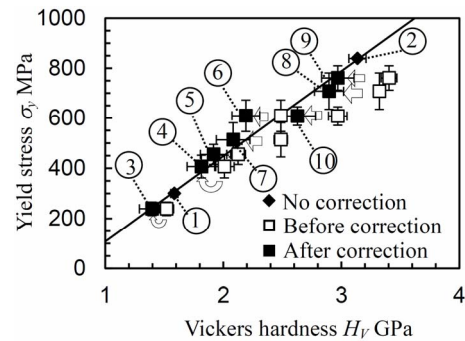
Since there is no significant difference in regression coefficient between each condition, the relationship between the compressive stress and the hardness of SUS316L is nearly the same in each case, even though the yield stress is different. So, the effect of compressive residual stress on the Vickers hardness of a specimen modified by peening can be easily taken into account.

### 3.2. Relationship between Vickers Hardness and the Yield Stress

**Figure 5** shows the yield stress,  $\sigma_y$ , obtained by a tensile test or inverse analysis with a spherical indenter as a function of Vickers hardness,  $H_V$ . The residual stress, and the Vickers hardness both before and after correction, and the yield stress of the treated specimens were shown in **Table 2**. The open symbols show the Vickers hardness before correction for residual stress and the closed sym-



**Figure 4. Relationship between residual stress and hardness for various yield stresses from FEA.**

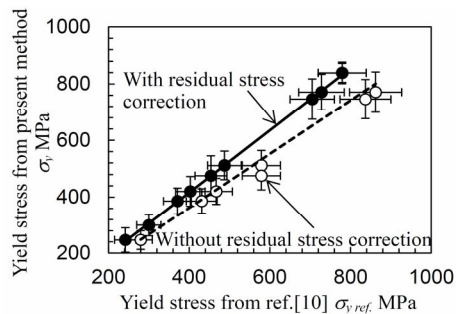


**Figure 5. Relationship between yield stress and Vickers hardness of SUS316L.**

bols show the Vickers hardness after correction for residual stress using the regression coefficient obtained from **Figure 3**. The residual stress was varied from 0 to  $-500$  MPa depending on the conditions of treatment. The yield stress of these specimens was estimated by inverse analysis. The absolute values of the residual stresses of the not-treated and rolling rate of 50% specimens could be ignored, and no correction was made to the Vickers hardness of the specimen, *i.e.*, specimen No. 1 and No. 2. The relationship between the yield stress,  $\sigma_y$ , and the Vickers hardness corrected for the effect of residual stress,  $H_V'$ , was found to be given by the linear approximation,  $\sigma_y = (332 \pm 18) \cdot H_V' - (218 \pm 30)$ .  $\sigma_y$  and  $H_V'$  are expressed in units of MPa and GPa, respectively. The value for without the residual stress correction deviates from the regression line obtained after the residual stress correction, along with increase in Vickers hardness as shown in **Figure 5**. The yield stress becomes higher, the compressive residual stress is also getting higher. Then, the effect of it on the Vickers hardness becomes larger. Therefore the residual stress correction is more important than the case of which compressive residual stress is relatively small. According to Busby *et al.* [10], the proportionally coefficient between yield stress and Vickers hardness of austenitic stainless steel is  $309 \pm 18$  MPa/GPa. The value obtained in the present experiment is reasonably close to the value in the previous report. This proportionally coefficient did not consider residual stress. In order to confirm the relevance of the proposed method taking account of residual stress, **Figure 6** plots the yield stress obtained from the proposed method as a function of that from the ref. [10] in the case of with and without correlation of residual stress. From the result shown in **Figure 6**, the gradients of these regression lines show 0.84 for without the correction and 1.04 for with the correction. When the residual stress correction is performed, the equation proposed in ref. [10] can also be effective in the peened surface. It was verified that the present method can be used to estimate the yield stress of SUS316L in which equibiaxial residual stress is present.

**Table 2. Residual stress, Vickers hardness and yield stress varied by several treatments.**

Specimen number	Treatment	Residual stress $\sigma_r$ MPa	Vickers hardness $H_V$ GPa	Vickers hardness $H_V'$ GPa	Yield Stress $\sigma_y$ MPa
1	Not-treated (SR treatment)	$7 \pm 9$	$1.58 \pm 0.03$	-	300
2	Rolling rate 50%	$-10 \pm 22$	$3.14 \pm 0.07$	-	839
3	Polishing	$-146 \pm 30$	$1.52 \pm 0.06$	$1.40 \pm 0.11$	$239 \pm 27$
4	Cavitation peening in water $t_p = 1$ s/mm	$-231 \pm 16$	$2.01 \pm 0.08$	$1.81 \pm 0.12$	$407 \pm 45$
5	Cavitation peening in water $t_p = 4$ s/mm	$-246 \pm 18$	$2.12 \pm 0.06$	$1.92 \pm 0.11$	$455 \pm 40$
6	Cavitation peening in water $t_p = 16$ s/mm	$-351 \pm 25$	$2.48 \pm 0.03$	$2.19 \pm 0.12$	$609 \pm 63$
7	Cavitation peening in air $t_p = 20$ s/mm	$-479 \pm 25$	$2.49 \pm 0.06$	$2.08 \pm 0.14$	$513 \pm 68$
8	Laser peening $E = 6$ J	$-506 \pm 7$	$3.32 \pm 0.03$	$2.90 \pm 0.12$	$708 \pm 63$
9	Laser peening $E = 10$ J	$-514 \pm 7$	$3.40 \pm 0.07$	$2.97 \pm 0.14$	$761 \pm 49$
10	Laser peening $E = 20$ mJ	$-405 \pm 40$	$2.97 \pm 0.08$	$2.63 \pm 0.15$	$608 \pm 37$

**Figure 6. Comparison of yield stress obtained from the present method with from ref. [10] with and without residual stress correction.**

#### 4. Conclusions

In conclusion, a straight-forward method for estimating the yield stress of SUS316L stainless steel taking account of the compressive residual stress was proposed in the present paper. The procedure for estimating the yield stress is as follows:

1) Measure the hardness  $H_V$  using a Vickers hardness tester and the residual stress  $\sigma_r$  by an X-ray diffraction method.

2) Obtain the corrected Vickers hardness  $H_V'$  from  $H_V$  and the compressive residual stress using  $H_V' = H_V + (8.4 \pm 1.4) \times 10^{-4} \sigma_r$ .

3) Estimate the yield stress using  $\sigma_y = (332 \pm 18) H_V' - (218 \pm 30)$ .

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