

Method of Detection Abnormal Features in Ionosphere Critical Frequency Data on the Basis of Wavelet Transformation and Neural Networks Combination

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ABSTRACT

The research is focused on the development of automatic detection method of abnormal features, that occur in recorded time series of ionosphere critical frequency f_0F_2 during periods of high solar or seismic activity. The method is based on joint application of wavelet-transformation and neural networks. On the basis of wavelet transformation algorithms for the detection of features and estimation of their parameters were developed. Detection and analysis of characteristic components of time series are performed on the basis of joint application of wavelet transformation and neural networks. Method's approbation is performed on f_0F_2 data obtained at the observatory "Paratunka" (Paratunka settlement, Kamchatskiy Kray).

Keywords: Wavelet Transformation; Neural Networks; Critical Frequency of Ionosphere; Abnormalities; Earthquakes

1. Introduction

The subject of the investigations is recorded time series of ionospheric parameters, which include components with different internal structure and determined by density of atmosphere, its chemical compound and the spectral characteristics of solar radiation [1,2]. Ionosphere research is carried out by distant methods, one of which is vertical radio probing. Frequency of carrier radio impulse for which the given area of ionosphere becomes transparent, is called critical (f_0F_2) and it characterises electron concentration.

Against the regular changes caused by a daily and seasonal course, abnormal features with duration from some tens minutes till several hours [2-13] are observed in the f_0F_2 data. These anomalies have various structure and arise against powerful ionospheric disturbances which are caused by solar activity, in seismoactive areas they can arise during the seismic activity increase periods [2-13]. Complex structure of the ionospheric data makes traditional methods of the time series analysis inefficient for their analysis and abnormalities detection, because these methods are based on the procedure of

smoothing and lead the important information loss [2,5]. Main tools for abnormalities detection are based on the analysis of the average and median values that does not allow to find out internal dependences in the data and single abnormal features.

In connection with the wide variety of basis functions with compact carriers, wavelet-transformation is an effective tool for complex time series analysis [3,4,14-19]. Using the discrete wavelet-transformation construction, the algorithm allowing to allocate abnormal features and to define their parameters in f_0F_2 data in an automatic mode is offered in this paper.

For characteristic components of f_0F_2 time series detection and analysis this paper proposes the method based on joint application of wavelet-transformation with neural networks. Neural networks have well proved in complex nonlinear dependences reproduction [6,21-23]. The efficiency of this mathematical tool application for ionospheric data processing and analysis is demonstrated in [6,11,12,22,23]. These authors offer ways of f_0F_2 data analysis and prognosis on the basis of neural networks and show that in many respects their work result is defined by properties of training set. Experimental search

of suitable training set and neural network architecture is carried out in [22,23]. If modelled data are complex and noisy it is necessary to perform their preprocessing and to solve problems of uninformative and redundant data [5,6,13]. In [6,13] offered ways of joint application of wavelet-transformation and neural networks for uninformative data removal, developed algorithms of training set formation on the basis of a wavelet-filtration, and showed that the given approach allows to optimise process of network training and to increase length of data anticipation interval. This paper, where the method of f_0F_2 characteristic components detection and prognosis on the basis of wavelet-packages construction and neural networks joint application is developed, is continuation of these investigations.

In process of proposed method approbation, abnormal features in f_0F_2 data, arising during the periods of increased solar activity or caused by processes in lithosphere (seismic events of a power class with $k>12$ analysis) were detected.

2. Method description

Detection of abnormal features and their parameters definition on the basis of discrete wavelet-transformation. Formally complex time series can be presented as sum of different-scale components with various internal structure [5] $f(t) = \sum_j f_j(t)$, where j is scale.

As the f_j components structure is subject to change in random time moments, the most effective way for their description is application of approximation methods, based on decomposition of function on basis. Considering analyzed features local character, their different-scale and forms variety, the most suitable space for their representation is wavelet-space [5,13,14].

On the basis of discrete wavelet-transformation for f_j components the following representation in the form of the wavelet-scheme is obtained [14, 23]:

$$f_j(t) = \sum_n c_{j,n} \Psi_{j,n}(t), \quad (1)$$

where $\{\Psi_{j,n}\}_{(j,n) \in \mathbb{Z}^2}$ is orthonormalized basis of the $L^2(R)$ Lebege space, $\Psi_{j,n} = 2^{j/2} \Psi(2^j t - n)$, $f_j \in L^2(R)$. $\bar{c}_j = \{c_{j,n}\}_{n \in \mathbb{Z}}$ coefficients are result of mapping of f into the space with resolution j , $c_{j,n} = \langle f, \Psi_{j,n} \rangle$.

Without breaking general coherence, we will consider that an initial discrete time series belong to space with scale $j=0$. The importance of representation f as Equation (1) is defined by sorting and storing of

different-scale components of complex time series in various spaces W_j with resolution j :

$$W_{j=0} = \bigoplus_{j=-1}^{-J} W_j, \{\Psi_{j,n}\}_{n \in \mathbb{Z}} \text{ is basis of } W_j \text{ space.}$$

For the purpose of possibility to construct adaptive approximating wavelet-schemes, we will use nonlinear mappings [5, 14]:

$$f_M(t) = \sum_{(j,n) \in I_M} c_{j,n} \Psi_{j,n}(t), \quad (2)$$

where f_M is projection of f onto M vectors which indexes are contained in some set I_M . In this case f function approximation is carried out by M vectors depending on its structure.

The error of approximation (2) is the sum of the remained coefficients $\varepsilon[M] = \|f - f_M\|^2 = \sum_{(j,n) \notin I_M} |c_{j,n}|^2$.

Assuming that $e(t) = \sum_{(j,n) \in I_M} c_{j,n} \Psi_{j,n}(t)$ component is a consequence of the noise factor influence, we receive representation of random time series in wavelet-space:

$$f(t) = \sum_{(j,n) \in I_M} c_{j,n} \Psi_{j,n}(t) + e(t).$$

As a time series includes characteristic components and abnormal features, we will present it as follows:

$$f(t) = \sum_{(j,n) \in I_A} a_{j,n} \Psi_{j,n}(t) + \sum_{(j,n) \in I_D} d_{j,n} \Psi_{j,n}(t) + e(t) = f_1(t) + f_2(t) + e(t), \quad (3)$$

where $f_1(t) = \sum_{(j,n) \in I_A} a_{j,n} \Psi_{j,n}(t)$,

$$f_2(t) = \sum_{(j,n) \in I_D} d_{j,n} \Psi_{j,n}(t), \{a_{j,n}\}_{(j,n) \in I_A}$$

are set of approximating coefficients, describing characteristic features of data, $\{d_{j,n}\}_{(j,n) \in I_D}$ is set of the detailing coefficients describing abnormal features, $I_A \cup I_D = I_M$.

In [14,24] demonstrated that absence of amplitude coefficients decrease when $j \rightarrow 0$, characterises presence of local features in $f(t)$ and operation of their detection can be realized on the basis of requirement check $|d_{j,n}| \geq T$, when $j \rightarrow 0$, where T is some threshold value. Meantime, the least analyzed scale is limited by step of discrete time series sampling.

If wavelet Ψ has compact carrier equal to $[-C, C]$, then assemblage of (j, n) point pairs, such that some point ν is contained in $\Psi_{j,n}$ carrier, defines influence cone of point ν in scale-spatial plane [14]. As the

carrier $\Psi_{j,n}$ on the scale j is equal to

$$\left[n - C * 2^{-j}, n + C * 2^{-j} \right],$$

then influence cone of point ν on the scale j is defined by inequality:

$$|n - \nu| \leq C * 2^{-j}, j = -1, -2, \dots, -J.$$

Let's consider that function f in the neighbourhood of some point ν has abnormal feature of scale j , if in the neighbourhood of the point ν with the sizes defined by an influence cone, the condition is satisfied:

$$|d_{j,n}| \geq T_j, \quad (4)$$

where T_j is threshold value on scale j , time duration of abnormality is defined by the influence cone of point ν .

Operation of scale j abnormal features detection can be realized on the basis of threshold functions application

$$P_{T_j}(x) = \begin{cases} x, & \text{ecnu}|x| \geq T_j \\ 0, & \text{ecnu}|x| < T_j \end{cases}.$$

The sets of detailing components $\{d_{j,n}\}_{(j,n) \in I_D}$ allocated in such a way define the component $f_2(t)$ of model Equation (3).

Intensity of abnormality on scale j in point ν neighborhood we will define as $E_{f_{j,\nu}} = \max_n |d_{j,n}|$, where $n: |n - \nu| \leq C * 2^{-j}$.

Changes of intensity in time can be analyzed on the basis of value

$$E_{f(t)} = \sum_j |c_{j,n}|. \quad (5)$$

The construction of wavelet-packages [14, 24] assumes recursive splitting of space $W_j: W_j = \bigoplus_{i=1}^L W_{j_i}^{p_i}$. Space

$W_{j_i}^{p_i}$ admits orthonormalized basis

$$\Psi_{j_i}^{p_i} = \left\{ 2^{j_i/2} \Psi_{j_i}^{p_i} \left(2^{j_i} t - k \right) \right\}_{k \in \mathbb{Z}}.$$

Integration of corresponding bases of wavelet-packages $\left\{ 2^{j_i/2} \Psi_{j_i}^{p_i} \left(2^{j_i} t - k \right) \right\}_{k \in \mathbb{Z}, 1 \leq i \leq L}$ defines orthonormalized basis W_j , that allows to restore function completely.

Detection and analysis of characteristic components of time series on the basis of wavelet-packages and neural networks joint application.

The Neural network creates mapping $y: f \rightarrow f'$.

The set of weight coefficients of neuron input connections represents a column-vector [21]

$$U = [u_1, \dots, u_N]^T,$$

where N is length of network input vector.

If \hat{f}' is a real network output, and f' is a desired one, then $f' = y(f)$ is an unknown function, and a $\hat{f}' = G(f, U)$ is its approximation which is reproduced by neural network. Procedure of network training is reduced to minimisation of approximation mean-square error on parameter U .

Giving to the input of the trained neural network values of function f from an interval $[l - T + 1, l]$, network becomes capable to compute anticipated function values on time interval $[l + 1, l + \alpha]$, where l is a current discrete moment of time; α is length of anticipation interval. The decision error is defined as difference between desired f' and real \hat{f}' output values during the discrete time moment l .

The error vector is the vector where i element is

$$\varepsilon_i(l) = \hat{f}'_i(l) - f'_i(l), \quad (6)$$

where l is a current time moment, i is a current position on anticipation interval.

Algorithm of training and control sets formation:

1. An initial data array $\{f(k)\}_{k=1}^K$, where K is a sampling length, is divided on L blocks $Q: \{f(k)\}_{k=1}^K = \left\{ \{f(k)\}_{k=1}^Q, \{f(k)\}_{k=2}^{Q+1}, \dots, \{f(k)\}_{k=K-Q}^K \right\}$ long.

2. On the basis of wavelet-packages construction, for each block s we have representation f in the form of a linear combination different-scale components: $f^s = f_1^s + f_2^s + \dots + f_p^s$, where every component

$f_i^s = \sum_{(j_i, k) \in I_s^{p_i}} \beta_{j_i, k}^s \Psi_{j_i}^{p_i}, \Psi_{j_i}^{p_i} \in W_{j_i}^{p_i}$ in wavelet-space is

uniquely defined by coefficients sequence $\bar{\beta}_{j_i}^s = \left\{ \beta_{j_i, k}^s \right\}_{(j_i, k) \in I_s^{p_i}}, \beta_{j_i, k}^s = \left\langle f^s, \Psi_{j_i}^{p_i} \right\rangle$, $W_{j_i}^{p_i}$ are wavelet-package spaces.

3. Every detected component defines a subspace of time series features space. As $W_{j_i}^{p_i}$ are wavelet-packages

spaces, then is obtained: $\bigcap_{i=1}^L W_i^{p_i} = \{0\}, \bigcup_{i=1}^L W_i^{p_i} = V$.

Thus, for each unit s separation of data features in space is received **Figure 1**. Using the following sets of detected features

$$\left\{ f_{j_1, p_1}^s \right\}_{s=1, L}; \left\{ f_{j_1, p_2}^s \right\}_{s=1, L}; \dots; \left\{ f_{j_1, p_L}^s \right\}_{s=1, L}; \dots; \left\{ f_{j_1, p_1}^s, f_{j_1, p_2}^s \right\}_{s=1, L}; \dots; \left\{ f_{j_1, p_1}^s, f_{j_1, p_2}^s \right\}_{s=1, L};$$

$\{f_{j_1, p_1}^s, f_{j_1, p_2}^s, f_{j_1, p_3}^s\}_{s=1, L} ; \dots ; \{f_{j_1, p_1}^s, f_{j_1, p_2}^s, \dots, f_{j_1, p_l}^s\}_{s=1, L}$
 $;\dots ; \{f_{j_1, p_1}^s, f_{j_1, p_2}^s, \dots, f_{j_1, p_l}^s\}_{s=1, L}$ we form training and control sets for neural networks.

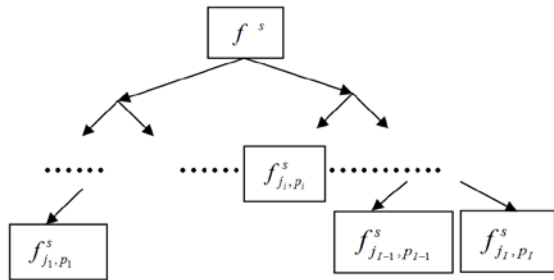


Figure 1. The scheme of data separation in wavelet-images space.

Algorithm of “the best” network construction:

Step 1: Carry out wavelet-restoration of a component f_{j_1, p_1}^s for each data unit s and form training set on the basis of combinations of the restored data from various units. Construct network 1 of variable structure [21] (variable structure network is a multilayer feed forward network, which architecture is defined by minimisation of decision error on training vectors set), train and test it.

Step 2: Carry out wavelet-restoration of components f_{j_1, p_l}^s for each data unit s and form training set on the basis of combinations of the restored data from various units. Construct a network 2 of variable structure, train and test it.

Etc.

Step r: Carry out wavelet-restoration of components $f_{j_1, p_1}^s, f_{j_1, p_2}^s, \dots, f_{j_1, p_l}^s$ for each data unit s and form training set on the basis of combinations of the restored data from various units. Construct network r of variable structure, train and test it.

On the basis of the analysis of results of received neural networks operation the “best” network is defined: “the best” is considered to be the network having the least decision error on test set

$$E_{M, \min} = \min_{\alpha} \left(\frac{1}{M} \sum_{l=1}^M \sum_{i=1}^f \varepsilon_i(l)^2 \right), \tag{7}$$

where α is a neural network number, $\alpha = \overline{1, r}$, M is a length of analyzed network output vector, f is a length of an anticipation interval.

The training data set is a set of observations containing features of studied process. On the basis of wavelet-packages construction, we have data features separation in space. During training and designing each network

learns a subset of input data features and approximates them. “The best” network is the network having the least decision error on test set. Therefore data subset used at training of the “best” network will contain the most typical features of studied process. In wavelet-space this subset is represented by set of coefficients $\{a_{j,n}\}_{(j,n) \in I_A}$, defining component $f_1(t)$ of time series model Equation (3).

If there is an abnormal feature in the data, then a change of their structure occurs. Therefore operation of abnormal features detection on the basis of a neural network can be constructed by processing and analysis of decision errors ε_i : if

$$E_Z = \frac{1}{Z} \sum_{l=1}^Z \sum_{i=1}^f \varepsilon_i(l)^2 \geq P, \tag{8}$$

where Z is an observation frame length, P is a beforehand specified threshold value, then within an analyzed time frame we have abnormality.

3. Results of experiments

In experiments f_0F2 data were used, received by automatic ionospheric station located in Paratunka settlement, Kamchatka peninsula. Data recording occurs once an hour. For experiments results of f_0F2 measurements dated 1979 - 2011 were taken. In the process of analysis, data of the Earth magnetic field (H-component) were used to define magnetospheric disturbances degree, characterising Solar activity. As basic functions the class of Daubechies orthogonal wavelets: db2, db3, db4 was used.

Following the results obtained in [3], for detection of abnormalities on the basis of operation Equation (4) were used the threshold values defined in the process of algorithm operation by formula:

$$T_j = \underset{n=1, V}{\text{med}} \left(|d_{j,n}| \right) + \theta * St_j, \quad \text{where}$$

$$St_j = \sqrt{\frac{1}{V-1} \sum_{n=1}^V \left(|d_{j,n}| - \bar{d}_{j,n} \right)^2}, \quad \bar{d}_{j,n} \text{ is the average}$$

value defined within the analyzed sliding time frame of length V , $V = 168$ readouts, med is a median defined within the analyzed sliding time frame of length V . The coefficient $\theta = 3$ has been defined statistically. The detected time-and-frequency intervals containing abnormal features, are shown on **Figure 2-5 (b)** by shades of grey colour. Ionospheric disturbances intensity changes in time were analyzed on the basis of value Equation (5), **Figure 2-5 (c)**.

On the basis of described above algorithms training and control sets for neural networks have been generated and “the best” network consisting of three layers that

allows to perform forecast of the f_0F2 data with anticipation step equal to 3 hours has been constructed. Detected on the basis of "the best" network characteristic component of f_0F2 time series looks like follows:

$$f_1(t) = \sum_{j_i, k} a_{j_i, k} \Psi_{j_i}^{p_i}, \Psi_{j_i}^{p_i} \in W_{j_i}^{p_i}, j_i = 3, p_i = 1, k \in Z.$$

The analysis of neural networks decision errors (Equations (6), (7)) has shown that Daubechies basis function of an order 3 provides the least f_0F2 data approximation error for the analyzed time periods. The analysis **Figure 2-5 (a)** shows that during the periods of increasing seismic activity, neural network error increase, characterising presence of abnormal features in the data is observed. The abnormalities detected on the basis of discrete wavelet-transformation (Equation (4), **Figure 2-5 (b)**) also prove this result. The detailed analysis of abnormalities shows that they are non-uniformly distributed both in time and on scales and characterised by various intensity (value $E_{f(t)}$, **Figure 2-5 (c)**). Comparison of the received results with the Earth magnetic field data **Figure 2-5 (d)** shows that analyzed lithospheric processes in most cases are observed against increased solar activity.

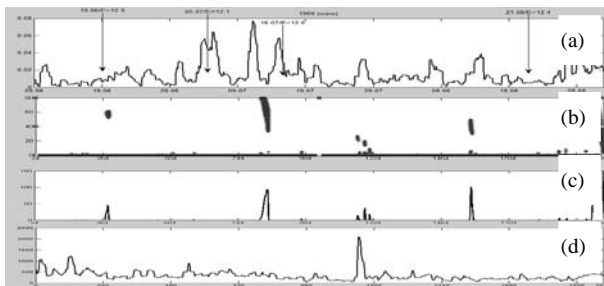


Figure 2. Results of f_0F2 data processing 1969: (a) a vector of a neural network error; (b) the time-and- frequency intervals containing abnormal features; (c) intensity of abnormalities; (d) H-component of the Earth magnetic field. Arrows note the moments of earthquakes occurrence.

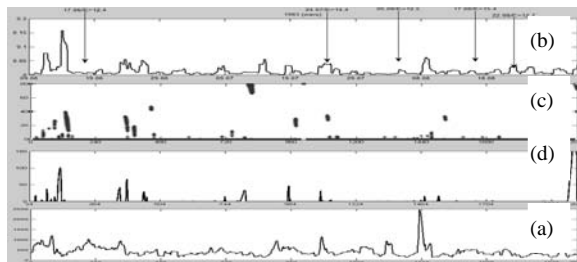


Figure 3. Results of f_0F2 data processing 1983: (a) a vector of a neural network error; (b) the time-and-frequency intervals containing abnormal features; (c) intensity of abnormalities; (d) H-component of the Earth magnetic field. Arrows note the moments of earthquakes occurrence.

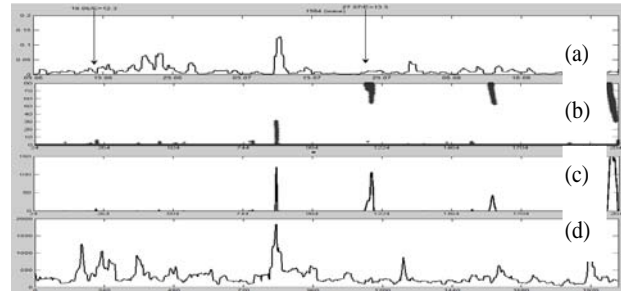


Figure 4. Results of f_0F2 data processing 1984: (a) a vector of a neural network error; (b) the time-and-frequency intervals containing abnormal features; (c) intensity of abnormalities; (d) H-component of the Earth magnetic field. Arrows note the moments of earthquakes occurrence.

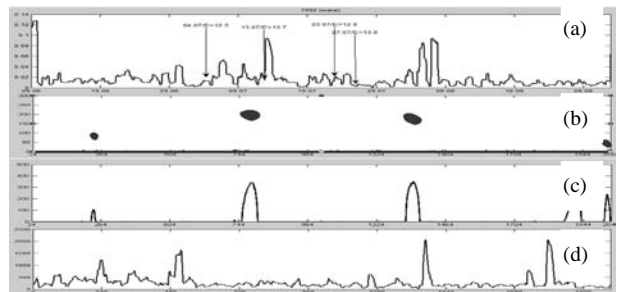


Figure 5. Results of f_0F2 data processing 1992: (a) a vector of a neural network error; (b) the time-and-frequency intervals containing abnormal features; (c) intensity of abnormalities; (d) H-component of the Earth magnetic field. Arrows note the moments of earthquakes occurrence.

4. Conclusions

On an example of the f_0F2 data for studying of time features of ionosphere parameters and detection of abnormalities arising during the periods of increased solar or seismic activity, the method based on combination of wavelet-transformation and neural networks is offered. Automatic algorithms of detection and analysis of characteristic components of f_0F2 series are developed.

Method application on the data received by automatic ionospheric station Paratunka settlement Kamchatka peninsula, has proved its efficiency and has allowed to detect the abnormal features arising during the periods of solar activity increasing and on the eve of catastrophic earthquakes on Kamchatka. The detected characteristic components of f_0F2 series have allowed to analyse ionospheric parameters variations during the summer period of time and their essential change during the periods of seismic and solar activity increasing. The detailed analysis of the allocated abnormal features has shown that during the periods of seismic or solar activity increasing in variations of f_0F2 series local different-scale periodicities having non-uniform distribution both on time and on scales arise.

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