

Additive Fault Tolerant Control Applied to Delayed Singularly Perturbed System

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ABSTRACT

The additive fault tolerant control (FTC) for delayed system is studied in this work. To design the additive control, two steps are necessary; the first one is the estimation of the sensor fault amplitude using a Luenberger observer with delay, and the second one consists to generate the additive fault tolerant control law and to add it to the nominal control of delayed system. The additive control law must be in function of fault term, then, in the absence of fault the expression of additive control equal to zero. The generation of nominal control law consist to determinate the state feedback gain by using the Lambert W method. Around all these control tools, we propose an extension of the additive FTC to delayed singularly perturbed systems (SPS). So, this extension consists to decompose the delayed SPS in two parts: delayed slow subsystem (delayed SS) and fast subsystem (FS) without time delay. Next, we consider that the delayed SPS is affected at its steady-state, and we apply the principal of FTC to the delayed SS and finally we combine them with the feedback gain control of FS by using the principal of composite control.

Keywords: Additive Fault Tolerant Control; Composite Control; Lambert W Function; Order Reduction; Sensor Fault; Singularly Perturbed System

1. Introduction

Several physical processes are on one hand high order and on the other hand are complex what returns their analysis and especially their control, with the aim of certain objectives, very delicate. However, knowing that these systems possess variables evolving in various speeds (temperature, pressure, intensity, voltage...) it turned out interesting to separate their dynamics [1-6] with the aim of the implementation of singularly perturbation technique. Indeed, this one allows, in the case of two time scales, [7] to describe the behavior of global studied process by those of two their subsystem (slow and fast) obtained by a temporal decomposition.

The interest of this decomposition is to define a reduced order model which allows in several control problems to found a solution simpler to implement for some approximations.

Like for all the types of systems who can contain a time-delay in its dynamic or in their control, the singularly perturbed systems "SPS" can also contain a delay. This problem was studied in several references such as [8-12], etc.

Moreover the presence of fault in delayed SPS complicate their designs for this reason it's necessary to

elaborate a fault tolerant control FTC.

This paper is the extension of the additive fault tolerant control designed for the SPS in [13] to the delayed SPS case and it's the extension of additive FTC in [14, 15] to the delayed systems and specially to the delayed singularly perturbed system.

2. Delayed Singularly Perturbed System

Consider the standard linear singularly perturbed system with small state time-delay described by [16,9]:

Delayed SPS

$$\begin{cases} \dot{x}_1(t) = A_{11} x_1(t) + A_{12} x_2(t) \\ \quad + A_{13} x_1(t-d) + B_1 u(t) \\ \mu \dot{x}_2(t) = A_{21} x_1(t) + A_{22} x_2(t) \\ \quad + A_{23} x_1(t-d) + B_2 u(t), \\ y(t) = C_1 x_1(t) + C_2 x_2(t) \\ x_1(t) = \varphi(t), \quad -d \leq t \leq 0, \quad x_2(0) = x_{20} \end{cases} \quad (1)$$

where: $x_1(t) \in \mathfrak{R}^{n_s}$, $x_2(t) \in \mathfrak{R}^{n_f}$ are the states vector, $u(t) \in \mathfrak{R}^r$ the control input, d the positive time-delay, μ the small positive perturbation parameter A_{ij} , B_i , $i = 1, 2$; $j = 1, 2, 3$, constant matrices of appropriate dimensions,

and $\varphi(t)$ the continuously differentiable initial function.

The slow subsystem (SS) and fast subsystem (FS) are obtained by considering that the fast variable in $x_2(t)$ reached there established regime, which corresponds to the assumption $\mu = 0$ [2,17]. This leads then to the following reduced models:

Delayed SS

$$\begin{cases} \dot{x}_s(t) = A_0 x_s(t) \\ \quad + A_3 x_s(t-d) + B_0 u_s(t) \\ y_s(t) = C_0 x_s(t) + D_0 u_s(t) \\ x_s(t) = \varphi(t), \quad -d \leq t \leq 0 \end{cases} \quad (2)$$

with:

$$\begin{cases} A_0 = A_{11} - A_{12} A_{22}^{-1} A_{21} \\ A_3 = A_{13} - A_{12} A_{22}^{-1} A_{23} \\ B_0 = B_1 - A_{12} A_{22}^{-1} B_2 \\ C_0 = C_1 - C_2 A_{22}^{-1} A_{21} \\ D_0 = -C_2 A_{22}^{-1} B_2 \end{cases} \quad (3)$$

and FS

$$\begin{cases} \mu \dot{x}_f(t) = A_{22} x_f(t) + B_2 u_f(t) \\ y_f(t) = C_2 x_f(t) \\ x_f(0) = x_{20} - A_{22}^{-1} (A_{21} + A_{23}) \varphi(0) \end{cases} \quad (4)$$

3. Principle of Additive Fault Tolerant Control

The accommodation principle of fault is based on the addition of an additive control u_{ad} to the nominal control law u . To design this control law, we need two steps: estimation and compensation of the fault.

In our work we studied only the sensor fault case which appears as bias. In this section we will consider that the delayed SPS is affected at its steady-state, so that it is equivalent to consider that the fault affects only the delayed slow subsystem and by consequence we will design only the reduced additive FTC of slow subsystem and the additive control of delayed slow subsystem will be applied to delayed SPS to compensate its sensor fault. The first step consists to design nominal composite control for the delayed SPS, secondly the sensor fault amplitude will be estimated by using the delayed slow subsystem, and finally we determinate the expression of reduced additive FTC.

3.1. Composite Nominal Control

This approach was developed for the SPS without time-delay in [1,7,13]. The results will be extended it to the

delayed SPS. Its principle consists to determinate the state feedback gain of slow subsystem and of fast subsystem, the last ones will be regrouped to find a global gain of SPS.

We remark that the delayed SPS is decomposed in delayed slow subsystem and fast subsystem without time-delay, for this reason we will design the state feedback gain of slow subsystem by using the Lambert-W function and the state feedback gain of fast subsystem will be determinate by using the classical pole placement method.

3.1.1. Determination of Reduced Nominal Control of Delayed Slow Subsystem

The Gain of slow subsystem control is determined by using the Lambert-W approach.

Definition of Lambert W function:

Let's $x \in \mathbb{C}$ be a solution, to determinate, of the equation $x e^x = y$ for $y \in \mathbb{C}$, This type of equation can be resolve by using the Lambert W function W_k such that [18,19]: $x = W_k(y)$.

With: k branches of Lambert W function and $k \in [-\infty, +\infty]$, $k = 0$ principal branch [18,19].

Consider the delayed system:

$$\dot{x}(t) = Ax(t) + A_d x(t-d) \quad (5)$$

This can be represented by the delayed differential equation (DDE):

$$\dot{x}(t) - Ax(t) - A_d x(t-d) = 0 \quad (6)$$

Let's $x(t) = e^{St} x_0$ be a solution of (6) with S : matrix, with appropriate dimension, to determinate.

Replacing the expression of $x(t)$ in (6) we find:

$$(S - A - A_d e^{S(-d)}) e^{St} x_0 = 0 \quad (7)$$

By consequence we found:

$$(S - A - A_d e^{S(-d)}) = 0 \quad (8)$$

Then:

$$S - A = A_d e^{-dS} \quad (9)$$

Multiplying the Equation (9) by $(d e^{dS} e^{-dA})$ we find:

$$d(S - A) \left(e^{(S-A)d} \right) = A_d d e^{-Ad} \quad (10)$$

In this step we can use Lambert W function with $x = d(S - A)$ and $y = A_d d e^{-Ad}$, by consequence the expression of S is:

$$S = \frac{1}{d} W \left(A_d d e^{-Ad} \right) + A \quad (11)$$

Equation (11) represents the characteristic equation for the general time-delayed system [20]. The roots of Equation (6) are the eigenvalues of the matrix S and can be used to describe the stability of the DDE (6), which

represents the stability of the general system.

State feedback gain of slow subsystem by using Lambert W function

Consider the feedback control of delayed slow subsystem:

$$u_s(t) = K_s x_s(t) \quad (12)$$

To find K_s we place the desired eigenvalue as follow:

$$\begin{aligned} \operatorname{Re} \left(S_0 = \frac{1}{d} W_0 \left(a_d d e^{-(a+bK_s)d} \right) + (a+bK_s) \right) \\ = \operatorname{Re}(\lambda_{desired}) \end{aligned} \quad (13)$$

We consider, also, that the slow subsystem is augmented by an integrator to ensure a null static error. So, the expression (12) of Reduced will be:

$$u_{Rs}(t) = K_s x_s(t) + \varepsilon_s(t) \quad (14)$$

where:
$$\begin{cases} K_s: \text{Lambert } W \text{ gain,} \\ \varepsilon_s(t) = \int (y_{sr}(t) - y_s(t)), \\ y_{sr}(t): \text{reference input of delayed SS,} \\ y_s(t): \text{output of delayed SS.} \end{cases}$$

The expression of delayed slow subsystem in closed loop is:

$$\begin{cases} \dot{X}_s(t) = A_{0f} X_s(t) + A_{3f} x_s(t-d) + G y_{sr}(t) \\ y_s(t) = C_{0f} X_s(t) \end{cases} \quad (15)$$

with:
$$X_s(t) = \begin{bmatrix} x_s(t) \\ \varepsilon_s(t) \end{bmatrix}, A_{0f} = \begin{bmatrix} (A_0 + B_0 K_s) & B_0 \\ -C_0 & 0 \end{bmatrix},$$

$$A_{3f} = \begin{bmatrix} A_3 \\ 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{0f} = [1 \quad 0]$$

3.1.2. Gain State Feedback of Fast Subsystem

The Gain of fast subsystem control is determined by using the classical placement pole method.

$$u_f(t) = K_f x_f(t) \quad (16)$$

3.1.3. Composite State Feedback Control of SPS

The composite nominal control of SPS is determinate by regrouping the two gains of delayed slow subsystem and fast subsystem.

View on the composite control of SPS without time-delay

Consider the form of slow subsystem and fast subsystem control represented by the two Equations (12) and (16), the composite control can be determinate by using the following equation [1,7]:

$$u_s + u_f = K_s x_s + K_f x_f \quad (17)$$

However, the composite control law must be expressed using the system states x_1 and x_2 , this can be obtained by

replacing x_s by x_1 and x_f by $x_2 - x_{2s}$, with: $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and

x_{2s} is the slow part of x_2 .

By consequence, the composite control law is:

$$u = K_s x_1 + K_f \left[x_2 + A_{22}^{-1} (A_{21} x_1 + B_2 K_s x_1) \right] \quad (18)$$

$$u = K_1 x_1 + K_f x_2 = \begin{bmatrix} K_1 & K_f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = K_g x \quad (19)$$

$$K_1 = (I_f + K_f A_{22}^{-1} B_2) K_s + K_f A_{22}^{-1} A_{21} \quad (20)$$

Composite control of delayed SPS

The composite control gain of delayed SPS is determinate by analogy of non-delayed case of SPS, and as shown in **Figure 1** the composite nominal control $u_{Cn}(t)$ of delayed SPS is:

$$u_{Cn}(t) = K_g x(t) + \varepsilon(t) \quad (21)$$

with:
$$\begin{cases} K_g: \text{global gain,} \\ \varepsilon(t) = \int (y_r(t) - y(t)), \\ y_r(t): \text{reference input of SPS,} \\ y(t): \text{output of SPS.} \end{cases}$$

3.2. Additive Control Law

We suppose that the SPS is affected by a sensor fault in its steady-state. Given that the SPS output in steady-state is approximated by the one of slow subsystem; we can estimate the fault amplitude affecting the SPS by using the delayed slow subsystem.

The main goal in this part is the determination of reduced additive control by using the delayed slow subsystem in order to compensate the sensor fault affecting the global delayed SPS.

Figure 2 represents the principle of Fault tolerant control of delayed SPS.

3.2.1. Fault Amplitude Estimation

This step is realized by using the delayed Luenberger observer represented by the following equation:

$$\begin{cases} \dot{\hat{X}}_s(t) = A_{0f} \hat{X}_s(t) + A_{3f} \hat{x}_s(t-d) \\ \quad + G y_{sr}(t) + L(y_s(t) - \hat{y}_s(t)) \\ \hat{y}_s(t) = C_{0f} \hat{X}_s(t) \end{cases} \quad (22)$$

with:

$$\begin{cases} \hat{X}_s(t) = \begin{bmatrix} \hat{x}_s(t) \\ \hat{\varepsilon}_s(t) \end{bmatrix}: \text{augmented estimated state of SS} \\ \hat{x}_s(t-d): \text{delayed estimated state of SS} \\ L: \text{gain of observer} \\ \hat{y}_s(t): \text{estimated output of SS} \end{cases}$$

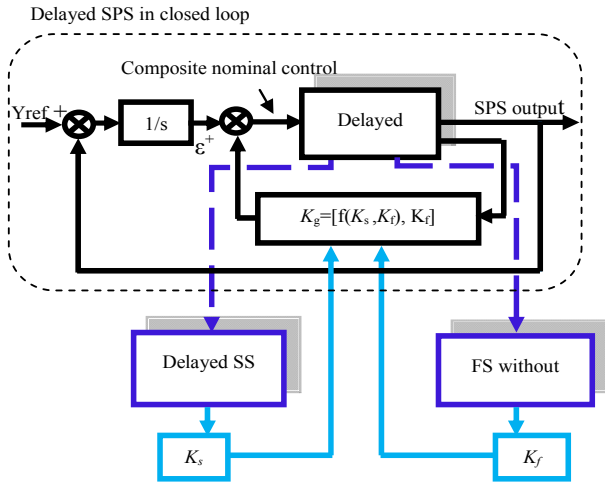


Figure 1. Principle of composite nominal control of delayed SPS.

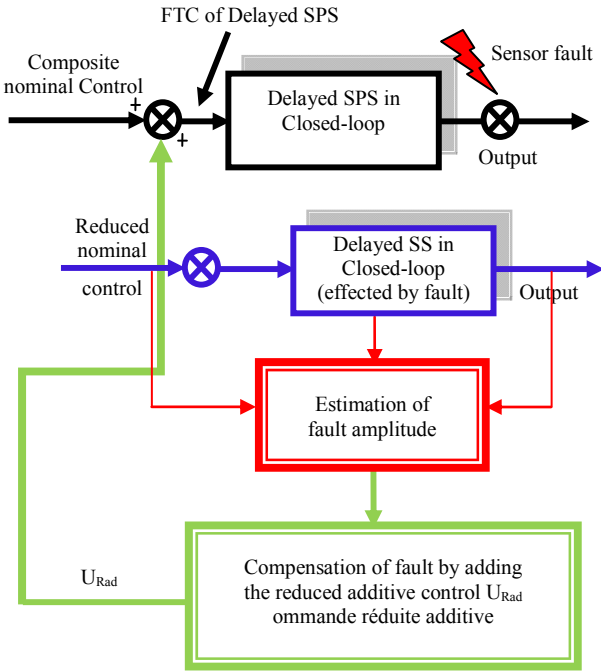


Figure 2. Principle of fault tolerant control of delayed SPS.

The dynamic of observation error is:

$$\dot{e}(t) = \dot{X}_s(t) - \dot{\hat{X}}_s(t) \quad (23)$$

The output of delayed slow subsystem affected by sensor fault is:

$$y_s(t) = C_{0f} X_s(t) + F_s f_s(t) \quad (24)$$

F_s : matrix of fault direction

$f_s(t)$: signal of sensor fault

Using the Equations (15), (22) and (24) the expression of $\dot{e}(t)$ becomes:

$$\dot{e}(t) = M e(t) + A_{3f} \tilde{x}_s(t-d) - L F_s f_s(t) \quad (25)$$

with:
$$\begin{cases} M = A_{0f} - L C_{0f} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \\ \tilde{x}_s(t-d) = x_s(t-d) - \hat{x}_s(t-d) \end{cases}$$

In steady-state the estimation error becomes null ($\dot{e}(t) = 0$) then via (25) we can find the expression of fault amplitude:

$$L F_s f_s(t) = M e(t) + A_{3f} \tilde{x}_s(t-d) \quad (26)$$

Knowing that F_s is scalar and L is the observer gain of augmented system ($\dim(L) = 2 \times 1$ and $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$) then ($L F_s$) is not invertible and by consequence we decompose the Equation (26).

So, the expression of fault is:

$$\hat{f}_s(t) = (L_1 F_s)^{-1} \times [M_{11} \tilde{x}_s(t) + M_{12} \tilde{e}_s(t) + A_3 \tilde{x}_s(t-d)] \quad (27)$$

with: $\tilde{e}_s(t) = \varepsilon_s(t) - \hat{\varepsilon}_s(t)$

3.2.2. Determination of Reduced Additive Control of Delayed Slow Subsystem

The compensation of sensor fault effect on the closed-loop system can be achieved by adding a new control law to the nominal one [14,15]:

$$u_{FTs}(t) = K_s x_s(t) + \varepsilon_s(t) + u_{Rad}(t) \quad (28)$$

The output and the integrator are affected with the fault such that:

$$\begin{cases} y_s(t) = C_0 x_s(t) + F_s f_s(t) \\ \varepsilon_s(t) = \varepsilon_s(t) + f_{\varepsilon_s}(t) \\ f_{\varepsilon_s}(t) = \int (-F_s f_s(t)) dt \end{cases} \quad (29)$$

If $C_0 = 1$ and by using Equations (28) and (29), the control law becomes the follow expression:

$$u_{FTs}(t) = K_s x_s(t) + K_s F_s \hat{f}_s(t) + \varepsilon_s(t) + f_{\varepsilon_s}(t) + u_{Rad}(t) \quad (30)$$

The effect of sensor fault can be compensated by using the additive control as follow:

$$u_{Rad}(t) = -K_s F_s \hat{f}_s(t) - f_{\varepsilon_s}(t) \quad (31)$$

3.2.3. Determination of Global Additive Control of Delayed SPS

We can deduce the expression of global additive control of delayed SPS by adding the reduced additive control (31) to the composite nominal control defined by expression (21):

$$u_{FTG}(t) = u_{Cn}(t) + u_{Rad}(t) \quad (32)$$

4. Simulation Example

Consider the delayed SPS:

$$\text{delayed SPS} \left\{ \begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \mu \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ &+ \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1(t-0.5) \\ &+ \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \\ y(t) &= [1 \quad 0]x(t) \end{aligned} \right.$$

If μ tends toward zero we find the delayed slow subsystem and the fast subsystem:

$$\text{delayed SS} \left\{ \begin{aligned} \dot{x}_s(t) &= -2x_s(t) + x_s(t-0.5) + 2u_s(t) \\ y_s(t) &= x_s(t) \\ x_s(0) &= 0 \end{aligned} \right.$$

$$\text{FS} \left\{ \begin{aligned} \mu \dot{x}_f(t) &= -1x_f(t) + u_f(t) \\ x_f(0) &= 0 \end{aligned} \right.$$

By using the Equation (13), “lambert” function in MATLAB and by placing the desired pole equal (- 2.1), we find $K_s = -1.5$;

So, the reduced control is $u_{Rs}(t) = -1.5x_s(t) + \varepsilon_s(t)$.

The gain $K_f = -0.1$ is determined by using expression (16).

By regrouping the two gains and using the expression (20) and (21) we find the following composite nominal control of SPS: $u_{Cn}(t) = [-1.8 \quad -0.1]x(t) + \varepsilon(t)$

The gain of observer is determined by placing the poles of $A_{0f} - LC_{0f}$ such as the eigenvalues of $A_{0f} - LC_{0f}$ equals 40 times the eigenvalues of A_{0f} , so

$$L = 10^3 \begin{bmatrix} 0.19 \\ 1.6 \end{bmatrix}.$$

So, the fault tolerant control of delayed SPS as show the expression (32) is:

$$u_{FTG}(t) = [-1.8 \quad -0.1]x(t) + \varepsilon(t) + 1.5\hat{f}_s(t) - \hat{f}_{\varepsilon_s}(t)$$

To simulate the effect of the fault tolerant control we generate a sensor fault as bias at time $t = 40$ sec with amplitude equal 0.5;

The following **Figures 3(a)** and **(b)** show the effect of sensor fault respectively in the control and in the output, of delayed SPS and delayed slow subsystem.

The estimation of the sensor fault using the delayed observer is shown in the **Figure 4**:

Figure 4 shows that the observer can estimate the am-

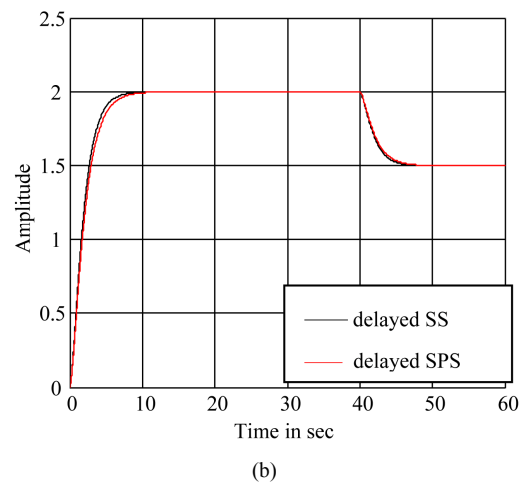
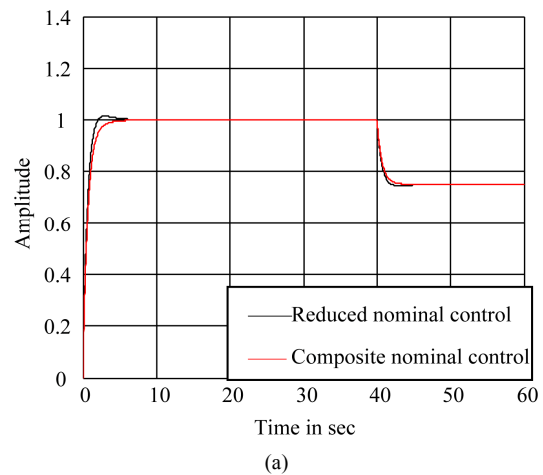


Figure 3. (a) Time evolution of controls in the occurrence of sensor fault; (b) Time evolution of outputs in the occurrence of sensor fault.

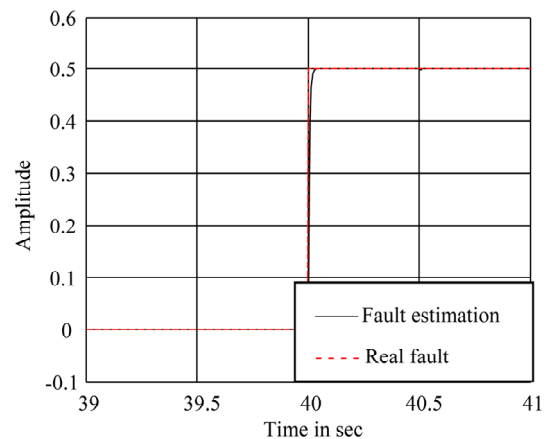


Figure 4. Sensor fault estimation.

plitude of sensor fault.

The following **Figures 5(a)** and **(b)** show the ability of the fault tolerant control to compensate the effect of sensor fault in the delayed SPS.

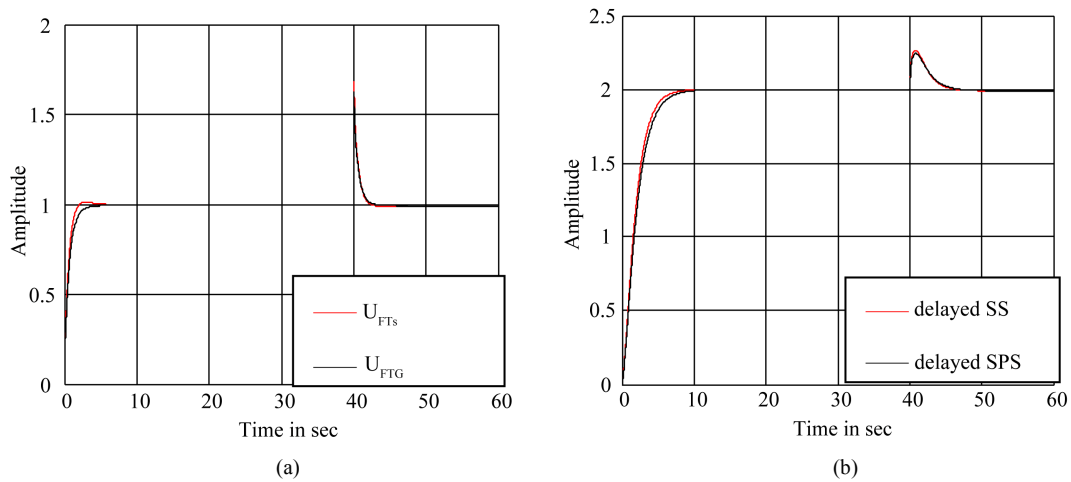


Figure 5. (a) Time evolution of fault tolerant controls; (b) Time evolution of outputs after compensation of sensor fault.

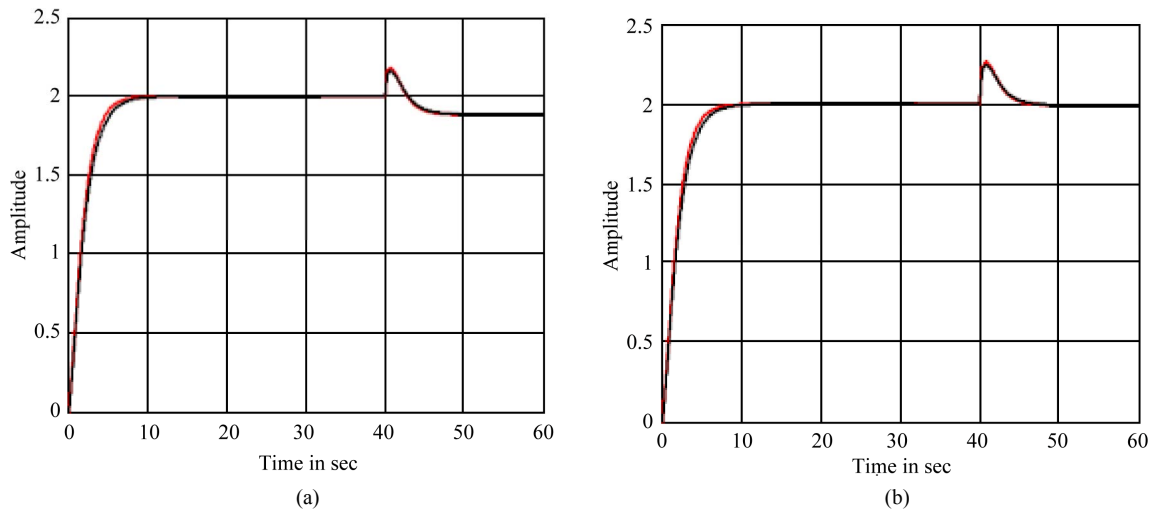


Figure 6. (a) The outputs of delayed SPS and delayed SS for L_1 ; (b) The outputs of delayed SPS and delayed SS for L_2 .

It's clear in the **Figure 5** that the reduced additive control added to the composite nominal control can compensate the effect of sensor fault in the delayed SPS output.

Effect of observer gain value on the fault compensation

In this paragraph we discuss the effect of observer gain value in the fault compensation error. **Figures 6(a)** and **(b)** show the delayed SPS and the delayed SS output for different gain values. **Figures 7(a)** and **(b)** represent the control signal for different gain values.

Case (a) for gain $L_1 = \begin{bmatrix} 9.91 \\ 8 \end{bmatrix}$.

Case (b) for gain $L_2 = 10^3 \begin{bmatrix} 0.19 \\ 1.6 \end{bmatrix}$

It's clear that for the case (b) the error value is equal to zero, but in case (a) it's equal to 25% from the amplitude

of fault (equal 0.5).

For the case (b) the deviation of control signal at the occurrence of fault (40 sec) is more important than the case (a).

So, more the observer gain value is important, more the compensation error in output of delayed SPS and delayed SS is better but the deviation of control signal is than more important.

5. Conclusions

In this work we studied the fault tolerant control of delayed SPS by using the reduced fault tolerant control based on the delayed slow subsystem. This slow-fast decomposition approach simplifies the design of complex systems control with sensor failure.

We notice an important deviation of the control signal at time of fault occurrence. So it will be interesting to study the fault tolerant control with constraints on the

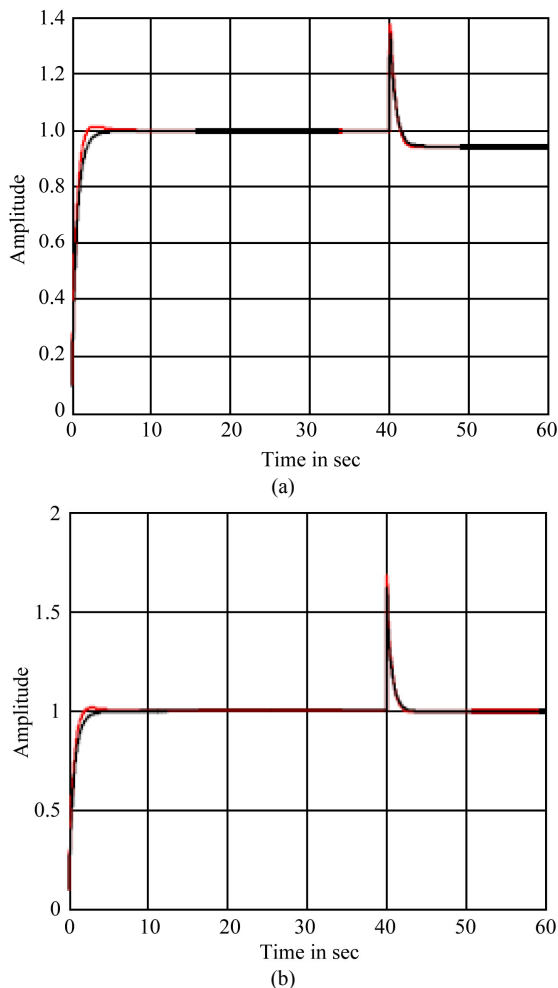


Figure 7. (a) The controls of delayed SPS and delayed SS for L_1 ; (b) The controls of delayed SPS and delayed SS for L_2 .

control signal.

Also, this result can be extended to nonlinear delayed process with other type of faults.

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