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Entanglement of Moving and Non-Moving Two-Level Atoms

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Abstract

In this paper we study the dynamics of the atomic inversion, von Neumann entropy and entropy squeezing for moving and non-moving two-level atoms interacting with a Perelomov coherent state. The final state of the system using specific initial conditions is obtained. The effects of Perelomov and detuning parameters are examined in the absence and presence of the atomic motion. Important phenomena such as the collapse and revival are shown to be very sensitive to the variation of the Perelomov parameter in the presence of detuning parameter. The results show that the Perelomov parameter is very useful in generating a high amount of entanglement due to variation of the detuning parameter.

Keywords

Atomic Inversion, Von Neumann Entropy, Atomic Motion, Entropy Squeezing

1. Introduction

Quantum entanglement is one of the most outstanding features of quantum mechanics. Quantum entangled states, as a fundamental physical resource of quantum information processing [1], are widely used, in quantum computation and quantum communication [2], quantum cryptography [3] [4], quantum teleportation [5], etc. Also, there is a lot of attention that has been focused on information entropies as a measure of entanglement in quantum information such as von Neumann entropy [6], Linear entropy, Shannon information entropy [7] and atomic Wehrl entropy [8]. On the other hand, we find that the most important problems in quantum optics are the studies of different systems interaction such as field-atom, atom-atom and the field-field. These problems have been extensively considered in a huge number of papers; see for example [9]-[26]. This is due to the richness of many different phenomena that have been observed in the laboratory.

In fact, these different kinds of interactions have been classified from the point of view of Lie algebra depending on the nature of the interaction. For example, the Hamiltonian which represents the interaction between two fields is described in the form of the parametric frequency converter which is of $SU(2)$ Lie algebra type. While the Hamiltonian which represents the non-degenerate parametric amplifier is of $SU(1,1)$ Lie algebra type. In this context a system describes the interaction between $SU(2)$ and $SU(1,1)$ Lie algebra, in which a Hamiltonian of the following form was treated

$$H = \omega\hat{K}_z + \frac{\omega_0}{2}\hat{\sigma}_z + gf(z)\left[\hat{\sigma}_+\hat{K}_- + \hat{K}_+\hat{\sigma}_-\right], \tag{1}$$

where ω and ω_0 are the field and atomic frequencies, respectively, $\hat{\sigma}_z$ and $\hat{\sigma}_\pm$ are the atomic pseudospin operators that obey the commutation relation

$$\left[\hat{\sigma}_+, \hat{\sigma}_-\right] = \hat{\sigma}_z, \quad \left[\hat{\sigma}_z, \hat{\sigma}_\pm\right] = \pm 2\hat{\sigma}_\pm \tag{2}$$

while g is the atom field coupling constant, $f(z)$ denotes a shape function of cavity field mode.

We restrict our study to the atomic motion along the z-axis so that the z-dependence of the field-mode function should be taken into account. The atomic motion can be incorporated in the usual way, *i.e.*

$$f(z) \to f(vt) = p_1 + \sin\left(\frac{p_2\pi vt}{L}\right), \tag{3}$$

where v denotes the atomic motion velocity, p_1 and p_2 are the atomic motion parameters well, if we put $p_1 = 1$ and $p_2 = 0$, then the shape function takes the form

$$\Omega(t) = \int_0^t f(vt')\mathrm{d}t' = t \tag{4}$$

which means, there is no atomic motion inside the cavity, but if $p_1 = 0$ and $p_2 = p$, where p represents the number of half-wave lengths of the field mode inside a cavity of the length L, the shape function for a particular choice of the atomic motion velocity $v = \frac{gL}{\pi}$ will be

$$\Omega(t) = \int_0^t f(vt')\mathrm{d}t' = \frac{1}{pg}\left[1 - \cos(pgt)\right] \tag{5}$$

Over the last two decades much attention has been focused on the properties of the Jaynes-Cummings model JCM for a moving atom. The theoretical efforts have been stimulated by experimental progress in cavity QED. Besides the experimental drive, there also exists a theoretical motivation to include atomic motion effect to JCM because its dynamics becomes more interesting. A number of authors have treated the JCM in the presence of atomic motion by the use of analytic approximations [27] [28] [29] [30] and numerical calculations [31].

The solution in the presence of atomic motion is not only of theoretical interest, but also important from a practical point of view for cold atoms. The important demonstration of the quantum collapse and revival phenomena was observed in a one-atom maser by Rempe *et al.* [32]. Some research groups were unable to build experimental setups capable of enhancing the coupling of an atom with a single field mode, simultaneously suppressing other modes.

In this article, we consider the extension of the problem by considering the two-level interaction with $SU(1,1)$ quantum system. We focus on the effect of the Perelomov parameter, field-mode structure parameter and detuning parameter on the evolution of the atomic inversion, von Neumann entropy in the case of absence and presence of the atomic motion effect.

We organize the material of this paper as follows: in Section 2 we introduce our Hamiltonian model which represents the interaction between $SU(1,1)$ and $SU(2)$, Then we derive the effective two-level atom Hamiltonian model, and we use the evolution operator method to find an exact expression of the wave function at the time $t > 0$. We devote Section 3 to discuss the atomic inversion in order to see the change that would occur in its behavior during interaction. While in Section 4 we discuss the degree of entanglement for the atomic system via von Neumann entropy and entropy squeezing. Finally, we draw a summary in Section 5.

2. The System Hamiltonian

The Hamiltonian which describe the interaction between a single two-level atom and $SU(1,1)$ quantum system take the following form

$$H = \omega \hat{K}_z + \frac{\omega_0}{2}\hat{\sigma}_z + gf(z)\left[\hat{\sigma}_+\hat{K}_- + \hat{K}_+\hat{\sigma}_-\right], \tag{6}$$

while $\hat{K}_\pm$ and $\hat{K}_z$ satisfy the following commutation relation

$$\left[\hat{K}_z, \hat{K}_\pm\right] = \pm\hat{K}_\pm, \quad \left[\hat{K}_-, \hat{K}_+\right] = 2\hat{K}_z \tag{7}$$

The Heisenberg equation of motion for any operator $\hat{O}$ is given by

$$i\frac{\mathrm{d}\hat{O}}{\mathrm{d}t} = \left[\hat{O}, H\right], \quad (\hbar = 1) \tag{8}$$

thus, the equations of motion for $\hat{\sigma}_z$ and $\hat{K}_z$ are given by

$$\frac{\mathrm{d}\hat{\sigma}_z}{\mathrm{d}t} = -i\left[\hat{\sigma}_z, H\right] = 2igf(vt)\left(\hat{K}_+\hat{\sigma}_- - \hat{\sigma}_+\hat{K}_-\right). \tag{9}$$

$$\frac{\mathrm{d}\hat{K}_z}{\mathrm{d}t} = -i\left[\hat{K}_z, H\right] = igf(vt)\left(-K_+\hat{\sigma}_- + \hat{\sigma}_+\hat{K}_-\right). \tag{10}$$

$$\frac{\mathrm{d}\hat{K}_z}{\mathrm{d}t} + \frac{1}{2}\frac{\mathrm{d}\hat{\sigma}_z}{\mathrm{d}t} = 0 \tag{11}$$

from the above equation, we can see that $N = \hat{K}_z + \frac{1}{2}\hat{\sigma}_z$ is constant of motion, therefore, the Hamiltonian takes the following form

$$H = \omega N + H_I, \tag{12}$$

where

$$H_I = \frac{\Delta}{2}\hat{\sigma}_z + gf(vt)\left(\hat{\sigma}_+\hat{K}_- + \hat{K}_+\hat{\sigma}_-\right), \tag{13}$$

where $\Delta = \omega_0 - \omega$ is the detuning parameter. We note that, $[N, H_I] = 0$, therefore $[N, H] = [H, H_I] = 0$, *i.e.* N and H_I are the constants of motion. Where the time evolution operator is defined as

$$U(t) = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \tag{14}$$

where

$$\begin{aligned} u_{11} &= \cos(\hat{\mu}_1 t) - i\frac{\Delta}{2\mu_1}\sin(\hat{\mu}_1 t), \quad u_{12} = -i\frac{g}{\hat{\mu}_1}\sin(\hat{\mu}_1 t)\hat{K}_- \\ u_{21} &= -i\frac{g}{\hat{\mu}_2}\sin(\hat{\mu}_2 t)\hat{K}_+, \quad u_{22} = \cos(\hat{\mu}_2 t) + i\frac{\Delta}{2\hat{\mu}_2}\sin(\mu_2 t) \end{aligned} \tag{15}$$

and

$$\hat{\mu}_j^2 = \frac{\Delta^2}{4} + v_j, \quad j = 1, 2, \quad v_1 = g^2\hat{K}_-\hat{K}_+, \quad v_2 = g^2\hat{K}_+\hat{K}_- \tag{16}$$

Let us assume the initial state of the atom in excited state $|1\rangle$ and the cavity field mode is in the Perelomov coherent state [33] [34].

$$|\mu;k\rangle = \sum_{m=0}^{\infty} Q_m |m,k\rangle, \quad Q_m = \left(1-|\mu|^2\right)^k \left(\frac{\Gamma(2k+m)}{m!\Gamma(2k)}\right)^{\frac{1}{2}} \mu^m, \tag{17}$$

where, μ the Perelomov parameter and

$$\begin{aligned} \hat{K}_z|m,k\rangle &= (m+k)|m,k\rangle, \\ \hat{K}_+|m,k\rangle &= \sqrt{(m+1)(m+2k)}|m+1,k\rangle, \\ \hat{K}_-|m,k\rangle &= \sqrt{m(m+2k-1)}|m-1,k\rangle, \end{aligned} \tag{18}$$

Therefore, we can write the wave function at the time $t > 0$ in the form

$$|\psi(t)\rangle = |C(t)\rangle|1\rangle + |S(t)\rangle|2\rangle, \tag{19}$$

where,

$$|C(t)\rangle = \sum_{m=0}^{\infty} X_1(m,t)|m,k\rangle, \quad |S(t)\rangle = \sum_{m=0}^{\infty} X_2(m,t)|m+1,k\rangle, \tag{20}$$

here,

$$\begin{aligned} X_1(m,t) &= Q_m\left[\cos(\mu_1 t) - i\frac{\Delta}{2\mu_1}\sin(\mu_1 t)\right], \\ X_2(m,t) &= -iQ_m\frac{g\sqrt{(m+1)(m+2k)}}{\mu_1}\sin(\mu_1 t), \end{aligned} \tag{21}$$

where,

$$\mu_1 = \sqrt{\frac{\Delta^2}{4} + g^2(m+1)(m+2k)}, \quad \mu_2 = \sqrt{\frac{\Delta^2}{4} + g^2 m(m+2k-1)}. \tag{22}$$

The atomic reduced density operator for the system is given by

$$\rho_A(t) = Tr_f |\psi(t)\rangle\langle\psi(t)| = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \tag{23}$$

where

$$\begin{aligned} \rho_{11} &= \sum_{m=0}^{\infty} |X_1(m,t)|^2, \quad \rho_{22} = \sum_{m=0}^{\infty} |X_2(m,t)|^2 \\ \rho_{12} &= \sum_{m=0}^{\infty} X_1(m+1,t) X_2^*(m,t), \quad \rho_{21} = \rho_{12}^* \end{aligned} \tag{24}$$

3. Atomic Inversion

Atomic inversion can be considered as the simplest important quantity to be calculated. It is related to the difference between the probabilities of finding the atom in the upper state $|1\rangle$ and in the ground state $|2\rangle$. Using Equation (20), we can calculate the time evolution of the atomic inversion as

$$W(t) = \sum_{m=0}^{\infty} \left[|X_1(m,t)|^2 - |X_2(m,t)|^2 \right] \tag{25}$$

In Figure 1, we have plotted the atomic inversion against the time gt, for fixed value of the Bargmann index $k = 5$. This, in fact, would help us to examine the effect of the other involved parameters. For instance, we consider the case in which the Perelomov parameter $\mu = 0.5$ and the detuning $\Delta = 0$. In this case the function fluctuates around the value *zero* and exhibits periods of revivals, see Figure 1(a).

When we increase the value of μ to 0.7 a dramatic change can be seen in the function behavior. In this case the function fluctuates around the value *zero* and exhibits periods of collapses and revivals, however the revival periods start to spread out as time increases. In the meantime we observe a decrease in the

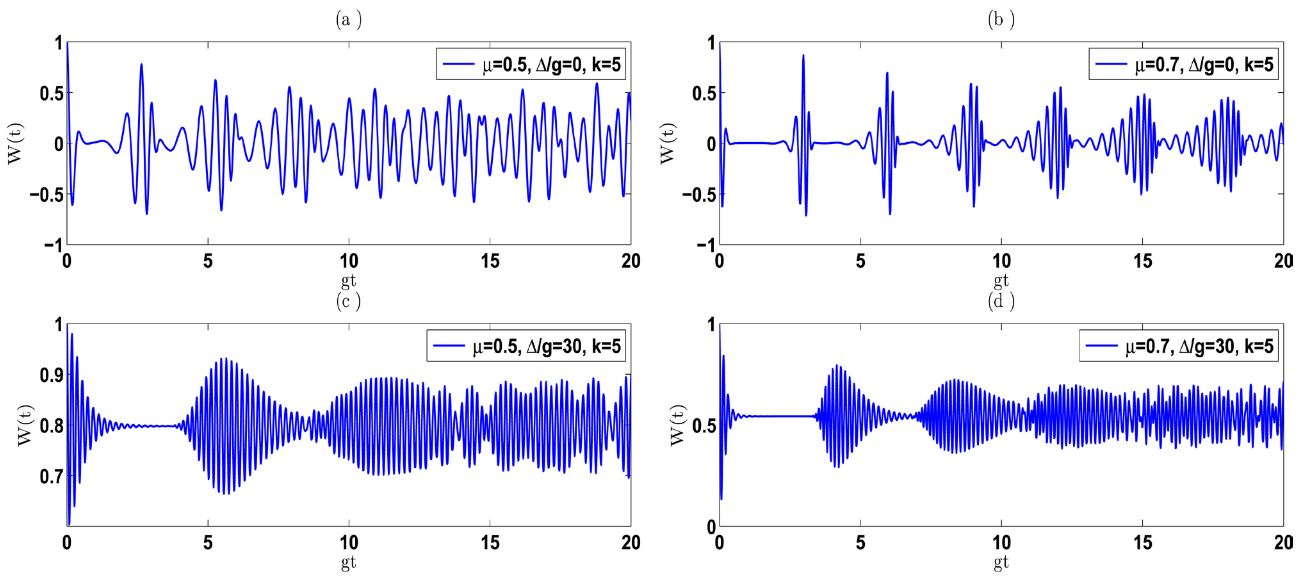

Figure 1. Effects of the detuning Δ and the Perelomov μ parameters on the evolution of the atomic inversion. The atom is initially in the excited state and the field mode in the Perelomov coherent state with the Bargmann index $k = 5$ and the atomic motion is neglected.

amplitude of each period of the revival (slow decay) and we observe somewhat a compress in the fluctuations during the revival periods see Figure 1(b). On the other hand, Figure 1(c) displays the case when the detuning is taken into account, where $\Delta/g = 30$ and $\mu = 0.5$ a dramatic change can be seen in the function behavior. In this case the function is shifted upwards and fluctuates around 0.8 and the atom most of the time in its upper state. When we increase the value of μ to 0.7 a more increment in the value of the function amplitude is observed. We also realize that the function is shifted downwards and fluctuates around 0.55 see Figure 1(d).

In order to discuss effects of atomic motion and field mode structure on the atomic inversion, we have plotted in Figure 2 the atomic inversion against the time gt, for fixed value of the Bargmann index $k = 5$ and absence of detuning parameter. In Figure 2(a) and Figure 2(b), the atomic motion is considered with $p = 1$ and different values of μ. One can see that as the values of μ decrease, the revival patterns also increase, while in Figure 2(a) and Figure 2(c), with $\mu = 0.5$ and different values of p. One can see that as the values of p increase, the revival patterns also increase. Also, we see that the $g\Omega(t)$ is a periodical function on the scale time gt with period $\dfrac{2\pi}{p}$.

4. Von Neumann Entropy (S_a)

In this section we study the evaluation of the von Neumann entropy, defined as

$$S = -Tr\{\rho \ln \rho\} \tag{26}$$

where ρ is the density operator for a given quantum system. For a pure state, $S = 0$, but when $S \neq 0$ the system is in a mixed state. The entropies of the atomic and the field systems can be defined through the corresponding reduced

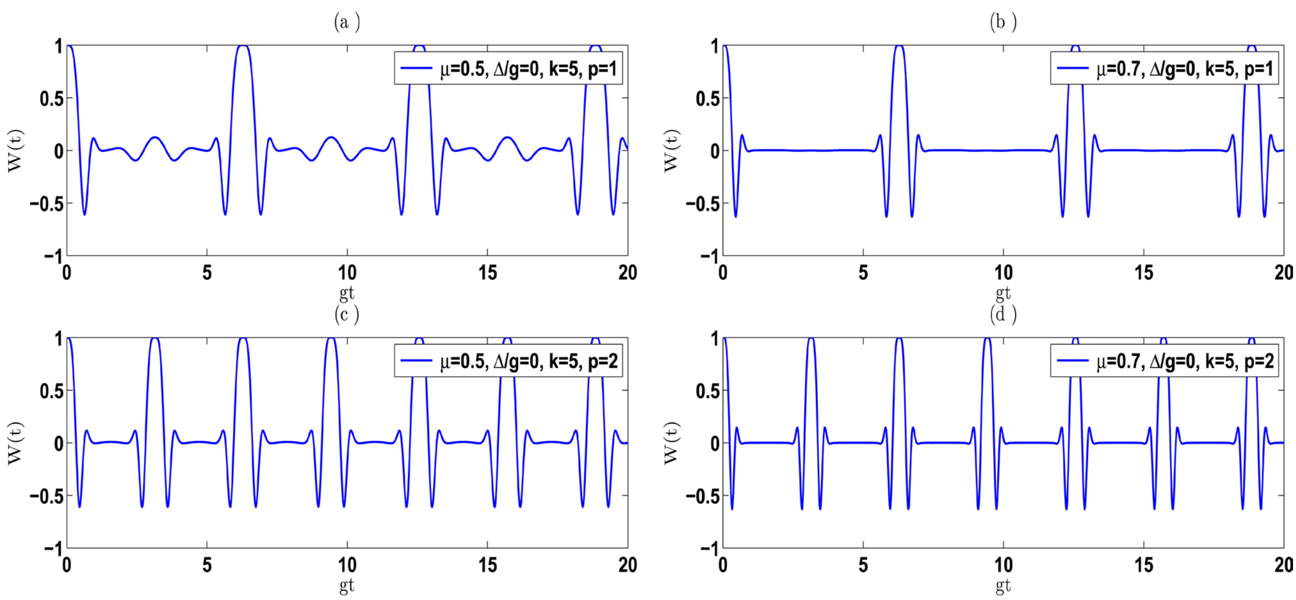

Figure 2. The same as Figure 1 but the atomic motion is taken into account and the absence of detuning parameter.

operators [35]. The time evolution of the field entropy (von Neumann entropy) carries information about the atom-field entanglement. For the system in which both the atom and the field starts from the decoupled pure states, the atomic and the field entropies are equal and may be expressed in terms of the eigenvalues $\lambda^{\pm}$ of the reduced field density operator through the relation.

$$\text{entropy} = -\left(\lambda^{(+)} \ln \lambda^{(+)} + \lambda^{(-)} \ln \lambda^{(-)}\right) \tag{27}$$

To discuss the entanglement of the atomic system with the field we plot **Figure 3** for the fixed value of $k=5$ and for different values of the other involved parameters. **Figure 3(a)** displays the evolution of the von Neumann entropy in the absence of the detuning parameter and the Perelomov parameter $\mu = 0.7$. We can see that the entanglement reaches the maximum (0.66) at the beginning and then gets back to its minimum (0.035) gradually and after that it returns back and oscillates irregular during the "revival" period. In the "collapse" period, the system tends to return the initial pure state. However, it is observed that as the detuning parameter $\Delta/g = 30$ is introduced the maximum value of the von Neumann entropy is decreased and the minimum value increased in contrast to the case of absence of the detuning parameter see **Figure 3(a)** & **Figure 3(c)**. On the other hand to show how the information entropy are affected by the increasing of the Perelomov parameter we set the Perelomov parameter $\mu = 0.97$ in **Figure 3(b)** and **Figure 3(d)** we can see that increase in the Perelomov parameter leads to an increase in the maximum value (0.69) and the minimum (0.572) entanglement and then the function S_a oscillates regularly around the maximum entanglement but it does not return the initial pure state anymore.

Figure 4 illustrate the dynamical properties of the von Neumann entropy

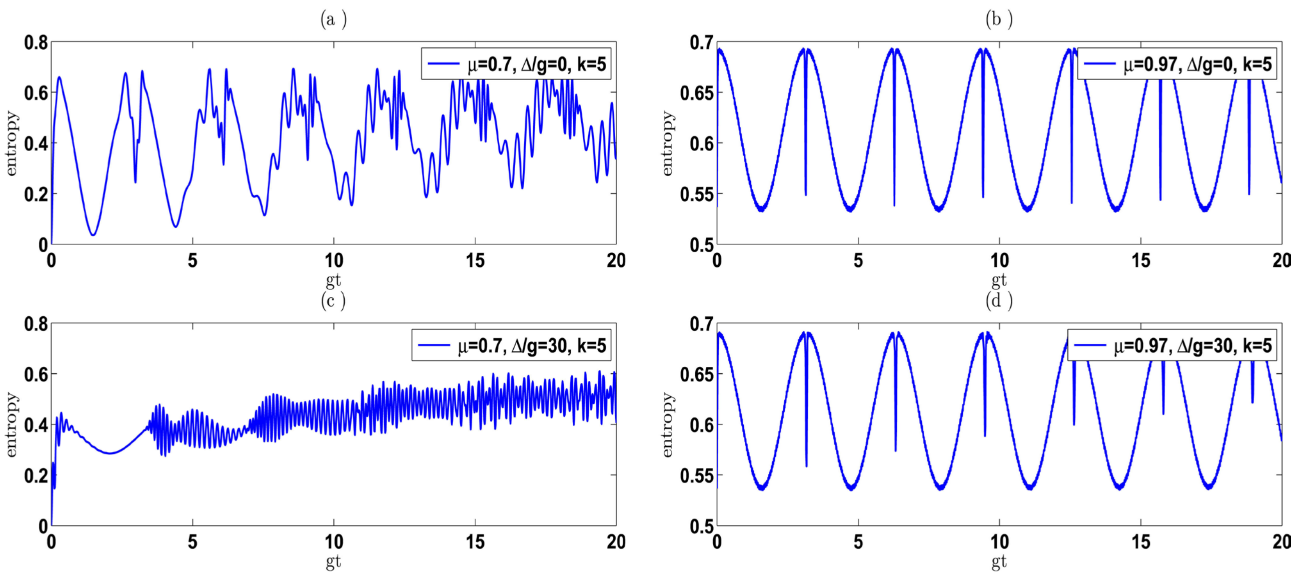

Figure 3. Effects of the detuning Δ and the Perelomov μ parameters on the evolution of the von Neumann entropy. The atom is initially in the excited state and the field mode in the Perelomov coherent state with the Bargmann index $k=5$ and the atomic motion is neglected.

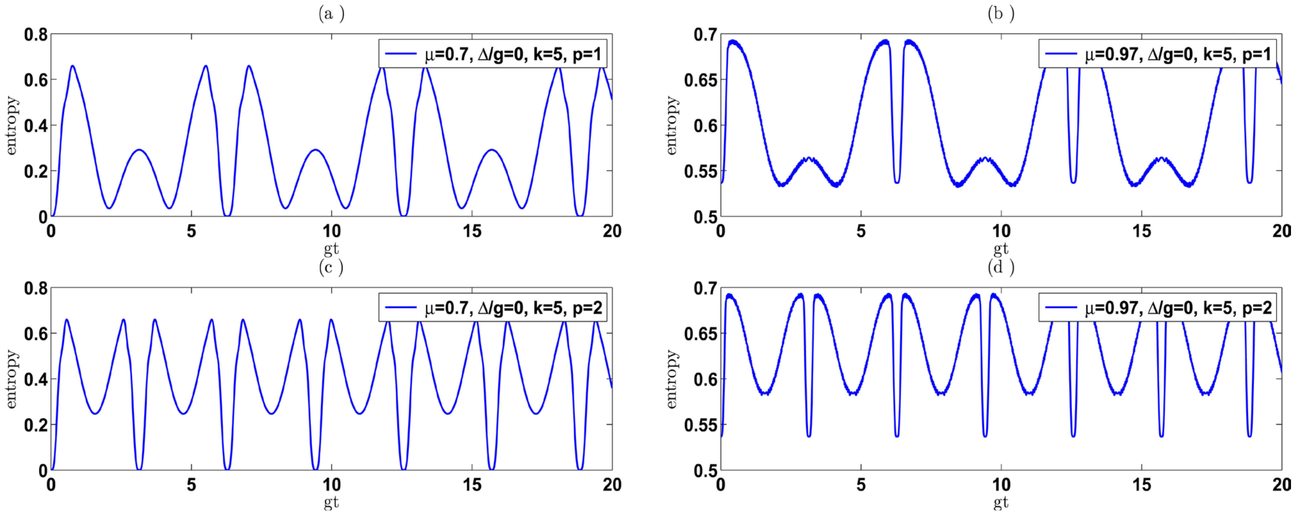

Figure 4. The same as **Figure 3** but the atomic motion is taken into account and the absence of detuning parameter.

when atomic motion is taken into account. From these figures, we can conclude that: 1) the atomic motion leads to the periodic evolution of the field entropy; 2) an increase of the parameter p leads to the shortening of the evolution periodicity of the von Neumann entropy.

Entropy Squeezing Properties

Now we study the squeezing of the atomic entropy, where we can express the entropy squeezing of the two-level atom by using the quantum information entropy theory. The information entropy of the Pauli operators $\langle\hat{\sigma}_\alpha\rangle(\alpha = x, y, z)$

$$\langle\hat{\sigma}_x\rangle = 2\,\mathrm{Re}(\rho_{12}), \quad \langle\hat{\sigma}_y\rangle = 2\,\mathrm{Im}(\rho_{12}), \quad \langle\hat{\sigma}_z\rangle = \rho_{11} - \rho_{22} \tag{28}$$

for a two-level atom system is

$$H(\hat{\sigma}_\alpha) = -\sum_{i=1}^{2} P_i(\hat{\sigma}_\alpha)\ln P_i(\hat{\sigma}_\alpha), \tag{29}$$

where $P_i(\hat{\sigma}_\alpha) = \langle\psi_{\alpha i}|\rho|\psi_{\alpha i}\rangle$ $(i = 1, 2)$, which are the probability distributions for two possible measurements of an operator $\hat{\sigma}_\alpha$. $H(\hat{\sigma}_x), H(y)$, and $H(z)$ satisfy

$$H(\hat{\sigma}_x) + H(\hat{\sigma}_y) \geq 2\ln 2 - H(\hat{\sigma}_z), \tag{30}$$

which may also be rewritten as

$$\delta H(\hat{\sigma}_x)\,\delta H(\hat{\sigma}_y) \geq \frac{4}{\delta H(\hat{\sigma}_z)}, \tag{31}$$

where $\delta H(\hat{\sigma}_\alpha) = \exp(H(\hat{\sigma}_\alpha))$. The squeezing of the atom is determined by using the EUR Equation (31) named entropy squeezing. The fluctuation in the component $(\hat{\sigma}_\alpha, \alpha = x, y)$ of the atomic dipole is said to be squeezed in entropy if the information entropy $H(\hat{\sigma}_\alpha)$ satisfies the following condition:

$$E(\hat{\sigma}_\alpha) = \delta H(\hat{\sigma}_\alpha) - \frac{2}{\sqrt{|\delta H(\hat{\sigma}_z)|}} < 0, \quad (\alpha = x, y). \tag{32}$$

By using $\rho_A(t)$, we can obtain the information entropies of the atomic operators $\hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$ as follows:

$$\begin{aligned} H(\hat{\sigma}_x) &= -(\xi_1 \ln \xi_1 + \xi_2 \ln \xi_2), \\ H(\hat{\sigma}_y) &= -(\xi_3 \ln \xi_3 + \xi_4 \ln \xi_4), \\ H(\hat{\sigma}_z) &= -(\rho_{22} \ln \rho_{22} + \rho_{11} \ln \rho_{11}), \end{aligned} \tag{33}$$

where

$$\begin{aligned} \xi_1 &= \frac{1}{2} + \mathrm{Re}(\rho_{12}), \quad \xi_2 = \frac{1}{2} - \mathrm{Re}(\rho_{12}), \\ \xi_3 &= \frac{1}{2} + \mathrm{Im}(\rho_{12}), \quad \xi_4 = \frac{1}{2} - \mathrm{Im}(\rho_{12}), \end{aligned} \tag{34}$$

We discuss the effects of the detuning parameter, the Perelomov parameter, the atomic motion and the field-mode structure on the properties of the entropy squeezing. The time evolution of the squeezing factors, $E(\hat{\sigma}_x), E(\hat{\sigma}_y)$ are shown in Figure 5 for the atom initially in the excited state and the field in a Perelomov coherent state with Bargmann index $k=5$ in the absence of the atomic motion. We see from Figure 5(b) that both $E(\hat{\sigma}_x)$ and $E(\hat{\sigma}_y)$ predict no squeezing in the variables $\hat{\sigma}_x$ and $\hat{\sigma}_y$ when the Perelomov parameter $\mu = 0.97$. A comparison of frame (a) in Figure 5 shows that, entropy squeezing in every period of the evolution of $E(\hat{\sigma}_y)$ while still no squeezing in the behavior of the entropy squeezing factor $E(\hat{\sigma}_x)$ when the Perelomov parameter $\mu = 0.7$. From these we can conclude the following an decrease in parameter μ results in not only the spread of revival periods but also squeezing of the evolution of $E(\hat{\sigma}_y)$. On the other hand Figure 5(c) shows the effect of the detuning parameter on the time evolution of the squeezing factors $E(\hat{\sigma}_x)$ and $E(\hat{\sigma}_y)$ with $\mu = 0.7$. We see that the squeezing in

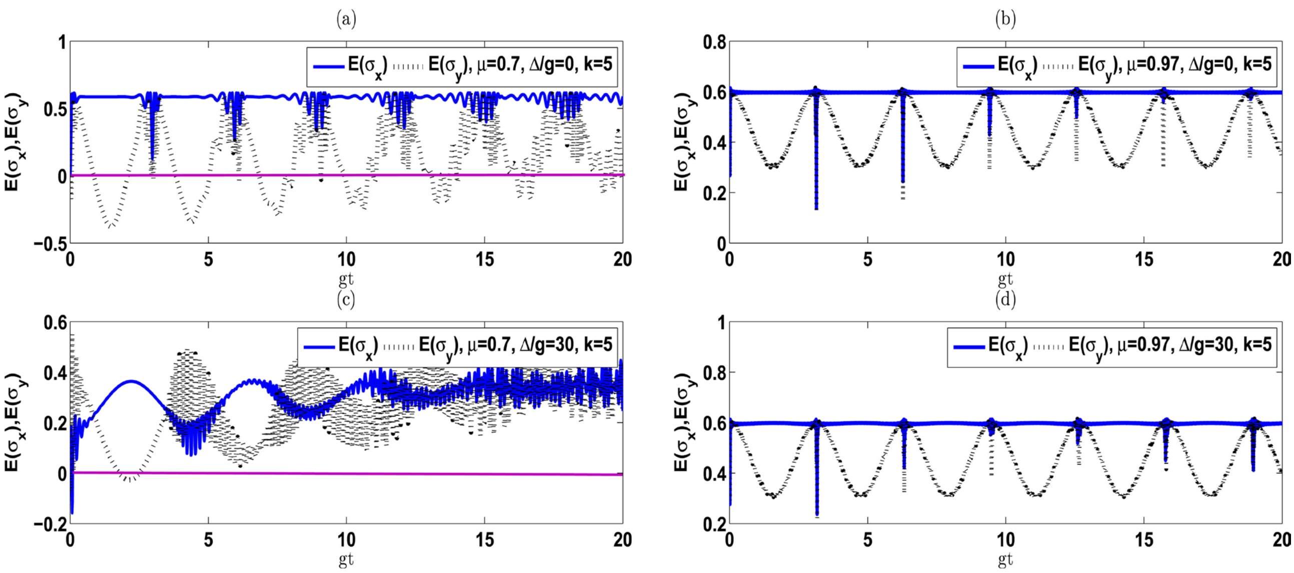

Figure 5. Effects of the detuning Δ and the Perelomov μ parameters on the evolution of the entropy squeezing factors $E(\hat{\sigma}_x)$ and $E(\hat{\sigma}_y)$. The atom is initially in the excited state and the field mode in the Perelomov coherent state with the Bargmann index $k=5$ and the atomic motion is neglected.

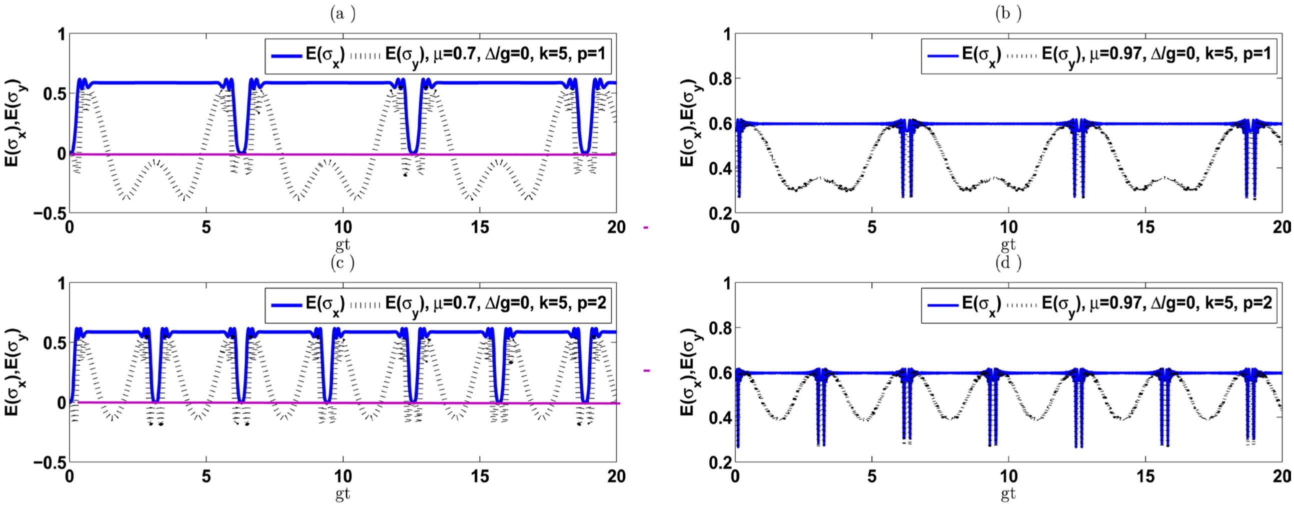

Figure 6. The same as **Figure 5** but the atomic motion is taken into account and the absence of detuning parameter.

$E(\hat{\sigma}_x)$ and $E(\hat{\sigma}_y)$ disappears quickly and will not reappear anymore. While in Figure 5(d) we observe the squeezing Completely disappeared when $\mu = 0.97$. From these we can conclude the following an increases in parameter μ leads to the disappearance of the squeezing in the absence and presence of the detuning parameter. Finally, to discuss the impact of the atomic motion and the field-mode structure, on the evolution of the entropy squeezing factors $E(\hat{\sigma}_x)$ and $E(\hat{\sigma}_y)$ we have plotted Figure 6 against the time t for fixed value of the Bargmann index $k = 5$ and $\Delta = 0$. This, in fact, would help us to examine the effect of the atomic motion and the field-mode structure. In Figure 6 we show the influence of the atomic motion and the field-mode structure parameter on the time evolution of $E(\hat{\sigma}_x)$. It does not show the squeezing in every period of the evolution of $E(\hat{\sigma}_x)$. Figure 6(a) and Figure 6(c) illustrate the time evolution of $E(\hat{\sigma}_y)$ when the atom is moving at a speed $v = gl/\pi$ and the field-mode structure parameters are $p = 1, p = 2$ respectively. In Figure 6(a) ($p = 1$), the range of entropy squeezing time increases more distinctly than that in Figure 5(a), but the value of maximal squeezing remains the same. From Figure 6(c) ($p = 2$) one can observe that the duration of entropy squeezing decreases and the degree of squeezing weakens as p increases. From these we can conclude that atomic motion causes the curve of the evolution of $E(\hat{\sigma}_y)$ to change. The period of the entropy squeezing and the duration of the information entropy squeezing are determined by the field-mode structure.

5. Summary

We have introduced the problem of the interaction between a two-level atom and a quantum system. We have used the Perelomov generalized coherent state as the initial state for the $SU(1,1)$ quantum system, while we considered the atom to be initial in the excited state. The wave function is calculated via the evolution operator and the result is used to obtain the atomic density operator. The effects of the different parameters such as detuning parameter, atomic motion and Perelomov parameter on the atomic inversion, von Neumann entropy and Entropy squeezing have been studied. It is shown that some new features such as: 1) the Perelomov parameter is very useful in generating a high amount of entanglement in the absence and presence of the detuning parameter; 2) the atomic inversion is sensitive to the variation of the Perelomov parameter in the presence of detuning parameter; 3) the atomic motion and field-mode structure parameter play an important role on the time evolution of the atomic inversion, entropy squeezing and von Neumann entropy.

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