

A Literature Review of Stochastic Programming and Unit Commitment

Hang Dai, Ni Zhang, Wencong Su

Department of Electrical and Computer Engineering, University of Michigan, Dearborn, USA
Email: daih@umich.edu, niz@umich.edu, wencong@umich.edu

Received January 2015

Abstract

The study of unit commitment (UC) aims to find reasonable schedules for generators to optimize power systems' operation. Many papers have been published that solve UC through different methods. Articles that systematically summarize UC problems' progress in order to update researchers interested in this field are needed. Because of its promising performance, stochastic programming (SP) has become increasingly researched. Most papers, however, present SP's UC solving approaches differently, which masks their relationships and makes it hard for new researchers to quickly obtain a general idea. Therefore, this paper tries to give a structured bibliographic survey of SP's applications in UC problems.

Keywords

Unit Commitment, Stochastic Programming, Review

1. Introduction

Unit commitment (UC) refers to the task of finding an optimal schedule and a production level for power systems' each generating unit over a given period of time while satisfying device and operating constraints [1]. Motivated by the immense benefit an optimal schedule can provide, scholars have delved into solving the UC problem over the past half century [2].

In existing literatures, UC is defined both narrowly and broadly. Narrowly speaking, UC only serves to pinpoint the commitment status (on/off) of generators for a defined period. Then, economic dispatch (ED) is performed to specify the production level every few minutes [3] [4]. Broadly speaking, UC entails determining both the commitment status as well as the production level [1] [5]. In this paper, the broad definition is examined. The classifications of UC can be different. With respect to security, UC can be classified into two categories: traditional UC where network constraints are ignored and security-constrained UC where network constraints like line outage and transmission line capacity are considered [6]. From the market operation's perspective, UC can be divided by either scheduling in a vertically integrated environment where minimizing cost is the objective or in a deregulated environment where maximizing benefit is the goal [7]. With regard to the treatment of future events, UC can be categorized into deterministic and stochastic UC.

Two types of uncertainties exist in power systems' operation: departures from forecasts and unreliable equipment

[1]. Traditionally, forecast errors mainly result from load variations. With an increased penetration of renewable energy, the intermittency and volatility of renewable generation lead to noticeable generation forecast errors. To achieve minimum cost while satisfying power balance, power systems first determine power plants' commitment statuses and output capacities based on forecasts and constraints. Then, re-dispatch is performed in real-time to adjust the difference between the actual demand and scheduled output [8]. UC is thus a multi-stage decision process similar to SP's solving procedure. It is therefore natural, as well as reasonable, to implement SP to solve UC problems.

The first trial of applying SP to solve UC was done in 1977 [9]. However, due to the lack of computational capability, the results were unsatisfying. In contrast, recent increases of computational ability have caused investigations in this field to boom. Numerous papers have formed and modified SP-based methods to solve UC under different considerations. Regardless of which methods are used, the developing process remains relatively vague. Therefore, part of this paper's contribution is to give a systematic review of SP-based UC formulations.

The rest of the paper is organized as follows. Section 2 presents several existing synopsis articles on UC problems over the past years. Section 3 outlines several commonly used SP-based UC formulations. Section 4 discusses some commonly encountered issues in formulating and solving UC by SP. Section 5 concludes the paper and proposes some future work.

2. Synopsis Articles of Unit Commitment

The UC problem dates back to the 1940s [10]. Given the astronomical volume of papers in this field, several review articles have been presented. One representative review of UC in traditional power systems is given by [11]. The author reviews more than 150 published articles concerning UC in the past 35 years. Several mathematical methods for solving UC problems are outlined. It also points out UC's different considerations in both monopolized and deregulated markets. In addition, it urges that a hybrid model combining different methods' advantages should be implemented. Similar review articles can be found in [12]-[14].

With an increased penetration of renewable energy and the implementation of energy storage devices, power systems' operation strategies are significantly modified. Some articles have given reviews concerning UC in such novel power systems infrastructure [15] [16].

However, most of these review papers are either a general exposition of UC's mathematical solution methods or UC's applications in some specific areas. There are not any papers that give a general survey and structured explanation of stochastic programming's applications in UC. Given the situation, [17] presents a literature review of solving UC by stochastic optimization methods. Stochastic programming, robust optimization and stochastic dynamic programming are all outlined in the paper. This paper gives a particular review of stochastic programming's applications in UC. In addition, this paper generalizes several commonly encountered issues in formulating SP-based UC problems.

3. Stochastic Programming's Application in Unit Commitment

Power systems throughout the world have either a vertically integrated structure or a deregulated structure. In a vertically integrated environment, customers of generation companies (GENCOs) are set and guaranteed [18]. The goal of UC for such power systems is minimizing costs associated with scheduling, load loss, etc. To encourage competition and improve power supply quality, the electricity market in several countries have transformed into deregulated market where GENCOs bid to sell their generation and independent system operators (ISOs) establish the day-ahead and real-time market to guarantee the reliable operation of power systems [19]. In such a scheme, UC's goal is to maximize profit or social welfare [4].

Uncertainty in SP is almost always represented by scenarios, which are a finite number of possible realizations of the uncertain quantity. When scenario-based SP is implemented to solve the UC problem, power systems' operating constraints are enforced for all scenarios and the objective function is normally the expected cost over these representative scenarios. Since the scenario-based modeling method assigns probabilities of occurrence to each scenario, it is biased more towards the most likely forecasted conditions and therefore justifies its reasonability [20]. This section discusses SP's applications in UC for vertically integrated structure in subsections 3.1 to 3.4 and for deregulated structure in subsection 3.5. As can be seen later, UC formulations for both cases are largely the same with only minor alterations.

3.1. Basic Two-Stage Stochastic Programming Formulation for Unit Commitment

“Basic” in this section is used to differentiate these UC formulations from those that have security or risk considerations. These basic UC formulations serve as the basis for SP’s implementation in UC problems.

Two-stage SP is a commonly used approach for capturing uncertainty, and its general form is shown below [21]:

$$\min c^T x + E[Q(x, \xi)], \quad s.t. Ax = b, \quad x \geq 0$$

where

$$Q(x, \xi) = \min \{qy(\xi) \mid Wy(\xi) = h - Tx, y(\xi) \geq 0\}$$

Here, x and $y(\xi)$ denote the first-stage and second-stage decision variables respectively. The ξ in the second-stage is a random vector and $E[Q(x, \xi)]$ is used to return the cost associated with this random vector’s consequences to the objective function.

According to [22], power systems’ short-term operation has two stages. In the first stage, units are selected to meet the expected load during each hour based on generators’ operating costs and constraints. In the second stage, power outputs of committed units are decided to meet the actual load demand. The similarity between power systems’ real operation process and the two-stage SP’s formulation naturally leads us to employ the two-stage SP for UC. In practice, random variables in the second stage are represented by various scenarios. Reference [1] uses scenarios to model power generation and load demand, while [23] models the wind power output.

3.2. Basic Multi-Stage Stochastic Programming Formulation for Unit Commitment

Uncertainty in each scenario of the two-stage SP is in some sense treated only once. Contingencies, however, are very likely to evolve in power systems’ operational process. Therefore, some papers propose to use multi-stage models where unfolding uncertainties’ dynamics over time are captured and decisions are adjusted dynamically. Early trials of the multistage SP can be found in [5] [24]-[26].

Multi-stage SP employs scenario trees to model uncertainty. The number of branches in each node is determined by real circumstances or the assumption of the number of instances at each time period. A path from the original (root) node to one end (leaf) node is equivalent to a scenario. The scenario tree can model demand and unit failure uncertainties, as in [24] [27], and fuel and electricity uncertainties, as in [26].

3.3. Security Constrained Unit Commitment

In order to simplify computations, early SP-based UC formulations ignore some constraints like network congestion. Actual systems’ operation, however, must consider additional factors such as emission, fuel and transmission constraints. With the improvement of computational ability, formulations including these additional constraints have gradually evolved since the 1980s and such UC formulations are called security constraint UC (SCUC) [28] [29]. “Security” mainly refers to transmission network’s viability, and it is the essential factor of differentiating SCUC from traditional UC [6]. However, the definition criterion is sometimes not taken rigorously. In [30]’s SCUC model, only transmission constraint is added to the UC problem. However, in [31]’s SCUC model, fuel, emission and energy constraints are all considered.

3.4. Chance Constrained Two-Stage Stochastic Programming for Unit Commitment

The three models discussed above are used to model random variables like renewable generation, uncertain load and component outages with constraints enforced in all scenarios. However, constraints in these models are strict; they are required to be met in each stage of each scenario. Such stringent restrictions may lead to solutions that are too conservative and therefore result in undesirable consequences such as large amounts of wind spillage [20]. To handle this problem, some papers recourse to chance constraint programming, which was first proposed in [32]. In chance constraint programming, solutions are permitted to violate constraints to some extent within a certain confidence level and the optimal solution is achieved in a probabilistic sense. A common form of chance constraint is shown as follows:

$$\Pr\{g_j(x, \xi) \leq \alpha, j = 1, 2, \dots, k\} > \beta,$$

where $g_j(x, \xi)$ is the constraint restricting a certain variable. The formulation represents the probability that $g_j(x, \xi)$ holds and β is the confidence level. In contrast, constraints in previous UC formulations are similar to the following:

$$g_j(x, \xi) \leq \alpha, \quad j = 1, 2, \dots, k,$$

which demands the constraint be satisfied categorically.

One representative paper of applying chance constrained programming to solve the UC problem is [33], where the requirement that generation strictly meets the demand for each hour is replaced by that the condition is satisfied at a predetermined probability level.

However, chance constraint itself cannot model uncertainties. In many papers employing chance constraints, uncertain variables are usually captured by other methods instead of SP. For example, [33] uses a correlation matrix to model uncertain loads and the chance constraint is only used to “relax” the power balance requirement. A similar approach is observed in [34] where probabilistic variables are modeled by a Markov process transition matrix. Since scenario-based SP is suitable for capturing uncertainty, it may be favorable if we combine SP with chance constraint; *i.e.*, employ SP’s scenarios to model randomness and use chance constraint programming to represent constraints. Such combined models are first presented in [35], where wind power uncertainty is captured by a number of scenarios and wind spillage is restricted by a certain probability from the chance constraint. Such models are labeled as chance-constraint two-stage SP. Additionally, the same authors propose a refined CCTS model in [36] to define the probability that certain amounts of wind power bidding into the market can be accepted.

3.4. Stochastic Programming Formulations for Unit Commitment in Deregulated Structure

Power industries throughout the world have experienced a significant transformation since the 1980s to cater for the needs of higher power production and delivery efficiency. The gradual privatization of generation, transmission, and redistribution have led power systems change into a deregulated market [37]. In such an electricity market, interactions between GENCOs and ISOs are vital for the performance of the system. The UC schedules have an indirect influence on electricity price and a direct influence on cost, and the goal of UC schedules changes to maximize profit or social welfare.

For deregulated markets, UC schedules can be classified as pool-schedule and self-schedule [4]. In pool-schedule environments, ISOs make UC decisions based on both GENCOs’ biddings and other considerations similarly employed in vertically integrated markets. In self-schedule environments, GENCOs make their own UC decisions before submitting bids to ISOs [38]. For ISO, its goal is to maximize the whole system’s social welfare, which is defined as the sum of producer surplus and consumer surplus. For GENCO, its objective is to maximize its own profit. In such an environment, GENCOs are no longer bound to serve the given demand in the open electricity market [39]. As pointed out in [7], redefining the UC problem for deregulated environment involves three alterations compared to the formulation in vertically integrated environment: 1) changing the demand constraints from an equality to inequality; 2) changing the objective function from cost minimization to profit maximization; 3) changing the reserve power and transmission losses to per contract form. Some more detailed considerations can be found in [39]. Normally, GENCO’s bidding depends on its interpretation of three uncertainties: load, generation and market price. In [36], a CCTS-based wind power generation company’s bidding strategy is proposed to maximize the company’s profit. Considering the fact that merely maximizing payoff may expose GENCOs to undesirable risks in the market, [38] defines a metric called expected downside risk as an explicit constraint in a multi-stage SP formulation to obtain GENCOs’ bidding strategy. From the above explanations, it can be seen that SP’s applications in UC for deregulated structure are largely the same as those in vertically integrated structure. **Table 1** provides an overview of literatures involving SP-based formulations for solving UC.

4. Specific Issues

4.1. Risk Considerations

Similar to the chance constraint discussed in Section 3.4, some other measures are utilized to avoid costly solu-

Table 1. Summary of SP-based formulations in UC.

Electricity Market Types	SP-based Formulations	Literature References
Vertically Integrated Market	Basic two-stage SP UC	[1] [24]
	Basic multi-stage SP UC	[5] [25]-[28]
	Security-constrained UC (SCUC)	[6] [21] [29] [30] [32] [52] [58]
Deregulated Market	Chance constrained two-stage UC (CCTS)	[36] [37] [57]
		[7] [37] [39] [41] [58]

tions while keeping the risk level within an acceptable domain. These include expected load not served (ELNS), lost of load probability (LOLP), value at risk (VaR) and conditional value at risk (CVaR). They form what is called risk-averse UC decisions. A detailed mathematical exposition can be found in [17].

LOLP is the most widely used system-wide risk measure metric. In fact, the chance constraint is the probabilistic constraint restricting LOLP, and it is equivalent to bound a θ -level VaR of the loss of load [41]. However, VaR can be hard to compute due to its lack of subadditivity. Therefore, a more coherent and conservative risk measure based on continuous variables called CVaR is proposed. More detailed discussions between VaR and CVaR can be found in [42] [43].

ELNS is evaluated by taking the expectation of total net load minus the total dispatch. In [44] [45], it is argued that ELNS is better than LOLP since it accounts for both the probability of outages as well as the corresponding average load lost. More detailed discussions between ELNS and LOLP can be found in [46] [47]. **Table 2** lists a quick overview of risk-averse UC.

4.2. Explicit vs. Implicit Reserve Setting

Operating reserves are important for power systems to respond to contingencies like load peaks, generator failures, scheduled outages, regulation and local area protection [48]. It includes spinning reserves and non-spinning reserves. Here, only spinning reserves are considered.

Normally, there are two ways of specifying a system's reserve. One is through deterministic criterion and set reserve equals some fraction of peak load [49] or the capacity of the largest online generator [50]. In this way, reserve is set explicitly. Another is using probabilistic criterion. Discussion of different probabilistic reserve setting methods can be found in [45]. As noted in [51], SP is regarded as a representative and most commonly used probabilistic reserve setting method. In probabilistic methods, reserve is set implicitly. The rationale that SP-based methods can set reserve is that uncertainty is explicitly considered in the SP and the system's reserve needs are already taken into account by different scenarios. Therefore, reserve is implicitly set. However, limited number of scenarios in SP may miss out some contingencies. Given this possibility, [1] combines deterministic reserve with the reserve set by SP and verifies that the combined model can give more robust solutions in the case of generation and load uncertainties. Reference [23] follows this direction and also shows the satisfying performance of the combined reserve setting model in the case of wind power forecasting errors. **Table 3** lists a quick overview of literatures with reserve considerations in solving UC problems.

4.3. Perfect, Deterministic and Stochastic Cases

Perfect case is the one where uncertain variable can be precisely forecasted and is the same as the actual situation. It is also called realized case [23]. In SP's formulation, this means there is only one scenario exists in each stage. The deterministic case has only one scenario as well. But variables' values in deterministic case equal the expectation value of uncertainties in different scenarios and are therefore different from what will actually occur. Stochastic case is the situation where uncertainties are captured by the scenarios discussed previously.

5. Conclusion and Future Work

While numerous literatures on UC are focusing on improving mathematical computation methods to quickly solve the formulated objective functions, it has been pointed out that UC problems can be inherently simplified by improving the modeling quality. Reference [51] argues that in countries like P.R. China where vertically

Table 2. Summary of risk-averse UC.

Risk Considerations	Literature References
VaR and CVaR	[42]-[44]
LOLP and ELNS	[45]-[48]

Table 3. Reserve considerations in UC.

Reserve Considerations	secnerefeR erutaretiL
Deterministic reserve models	[50] [51]
Probabilistic reserve models	[45]-[47] [52] [55]
Combined reserve models	[1] [24]

integrated environment dominates, the maximum load can be as high as 70% of the total installed generation capacity. This means many generators are kept generating continuously and therefore they can be expelled from consideration when solving UC. Such reduction of generators' amounts will significantly alleviate UC problem's computational burden. Also, [53] suggests that relaxing fast-start unit as continuous variables and modeling the aggregate generation outage as load increment can speed calculation noticeably.

Until now, several parameters in UC formulations are unsatisfactorily assumed. The spinning reserve requirement given by UC is sensitive to the value of lost load (VOLL) in [54]. On the one hand, this indicates the lack of robustness of its UC formulation. On the other, this reveals the importance and necessity of a reasonable estimation of VOLL. Similarly, the change of wind spillage cost is shown to have a significant impact on reserve, generation and demand scheduling in [55]. Some of the paper's unreasonable scheduling may result from the arbitrary assignment of wind spillage cost. As noted in [56], wind spillage cost represents the cost of opportunity to produce using the spilled wind energy. One possible future work is linking the wind spillage cost with wind farm's construction and maintenance fees as well as policies regarding wind power utilization. To compare the performance of different UC formulations, several metrics need to be employed. The most commonly used metric is expected total cost because it includes the impacts of both service (cost of generation) and reliability (cost of load shedding). However, expected total cost may be of less interest for other parties like GENCOs. In addition, it fails to take other vital issues like UC models' convergence and computation time into account. As shown in [56], the SCUC model in the day-ahead market of Texas's ERCOT electricity market once failed to give any feasible solution for almost 12 hours. This indicates the importance of UC models' convergence. To evaluate the conflicting impact of WPG, a new metric considering WPG's dual effects of fuel reduction and reserve cost increase is proposed in [55]. Similarly, [57] defines a fuzzy membership function as the system's security level to resolve WPG's conflicting impact. Nevertheless, these metrics are only evaluated individually and limited work has been done to further test their performance. More reasonable matrices for evaluating different UC formulations are therefore demanded.

Acknowledgements

This work is supported by the new faculty start-up fund at University of Michigan-Dearborn and the China Scholarship Council.

References

- [1] Ruiz, P., Philbrick, C.R., Zak, E., Cheung, K.W. and Sauer, P.W. (2009) Uncertainty Management in the Unit Commitment Problem. *IEEE Transactions on Power Systems*, **24**, 642-651. <http://dx.doi.org/10.1109/TPWRS.2008.2012180>
- [2] Baldick, R. (1995) The Generalized Unit Commitment Problem. *IEEE Transaction on Power System*, **10**, 465-475. <http://dx.doi.org/10.1109/59.373972>
- [3] Wright, B. (2013) A Review of Unit Commitment. www.ee.columbia.edu/~lavaei/Projects/Brittany_Wright.pdf
- [4] Lelic, A. Unit Commitment and Dispatch. www.iso-ne.com/support/training/courses/wem101/07_unit_commitment_dispatch.pdf

- [5] Takriti, S., Birge, J.R. and Long, E. (1996) A Stochastic Model for the Unit Commitment Problem. *IEEE Transaction on Power Systems*, **11**, 1497-1508. <http://dx.doi.org/10.1109/59.535691>
- [6] Pinto, H., Magnago, F., Brignone, S., Alsaç, O. and Stott, B. (2006) Security Constrained Unit Commitment: Network Modeling and Solution Issues. *Proceedings of IEEE PSCE Conference*, Atlanta, 29 October 2006-1 November 2006, 1759-1766. <http://dx.doi.org/10.1109/PSCE.2006.296179>
- [7] Richter Jr., C.W. and Sheble, G.B. (2000) A Profit-Based Unit Commitment for the Competitive Environment. *IEEE Transactions on Power Systems*, **15**, 715-721. <http://dx.doi.org/10.1109/59.867164>
- [8] Wollenberg, B. and Wood, A. (1996) *Power Generation, Operation and Control*. 2nd Edition, Wiley, New York.
- [9] Wiebking, R. (1977) Stochastische Modelle Zur Optimalen Lastverteilung in Einem Kraftwerksverbund. *Zeitschrift Für Oper. Res.*, **21**, B197-B217.
- [10] Li, C., Johnson, R.B. and Svoboda, A.J. (1997) A New Unit Commitment Method. *IEEE Transaction on Power System*, **12**, 113-119. <http://dx.doi.org/10.1109/59.574930>
- [11] Padhy, N.P. (2004) Unit Commitment: A Bibliographical Survey. *IEEE Transaction on Power System*, **19**, 1196-1205. <http://dx.doi.org/10.1109/TPWRS.2003.821611>
- [12] Chen, H. and Wang, X. (1999) A Survey of Optimization Based Methods for Unit Commitment. *Automation of Electric Power Systems*, **23**, 51-56.
- [13] Sheble, G. and Fahd, G. (1994) Unit Commitment Literature Synopsis. *IEEE Transactions on Power Systems*, **9**, 128-135. <http://dx.doi.org/10.1109/59.317549>
- [14] Cohen, A.I. and Sherkat, V.R. (1987) Optimization-Based Methods for Operations Scheduling. *Proceedings of the IEEE*, **75**, 1574-1590. <http://dx.doi.org/10.1109/PROC.1987.13928>
- [15] Ren, B. and Jiang, C. (2009) A Review on the Economic Dispatch and Risk Management Considering Wind Power in the Power Market. *Renewable and Sustainable Energy Reviews*, **13**, 2169-2174. <http://dx.doi.org/10.1016/j.rser.2009.01.013>
- [16] Peng, M., Liu, L. and Jiang, C. (2012) A Review on the Economic Dispatch and Risk Management of the Large-Scale Plug-In Electric Vehicles (PHEVs)-Penetrated Power Systems. *Renewable and Sustainable Energy Reviews*, **16**, 1508-1515. <http://dx.doi.org/10.1016/j.rser.2011.12.009>
- [17] Zheng, Q.P., Wang, J. and Liu, A.L. (2014) Stochastic Optimization for Unit Commitment—A Review. *IEEE Transaction on Power System*, No. 99, 1-12. <http://dx.doi.org/10.1109/TPWRS.2014.2355204>
- [18] Steven Stoft. *Power System Economics Designing Markets for Electricity*.
- [19] Retail Electricity Competition in Arizona. 2013. https://www.azsba.org/wp-content/uploads/2013/08/Deregulation_Overview_Peterson_OBrien.pdf
- [20] Bouffard, F. and Galiana, F.D. (2008) Stochastic Security for Operations Planning with Significant Wind Power Generation. *IEEE Transaction on Power System*, **23**, 306-316. <http://dx.doi.org/10.1109/TPWRS.2008.919318>
- [21] Birge, J.R. and Louveaux, F. (1997) *Introduction to Stochastic Programming*. Springer-Verlag, New York.
- [22] Lajda, P. (1981) Short-Term Operation Planning in Electric Power Systems. *The Journal of the Operational Research Society*, **32**, 675-682. <http://dx.doi.org/10.1057/jors.1981.134>
- [23] Wang, J., et al. (2011) Wind Power Forecasting Uncertainty and Unit Commitment. *Applied Energy*, **88**, 4014-4023. <http://dx.doi.org/10.1016/j.apenergy.2011.04.011>
- [24] Carpentier, P., Cohen, G. and Culioli, J.C. (1996) Stochastic Optimization of Unit Commitment: A New Decomposition Framework. *IEEE Transactions on Power Systems*, **11**, 1067-1073. <http://dx.doi.org/10.1109/59.496196>
- [25] Birge, J.R., Takriti, S. and Long, E. Intelligent Unified Control of Unit Commitment and Generation Allocation. Final Report of EPRI Grant RP8030-13.
- [26] Takriti, S., Krasenbrink, B. and Wu, L.S.-Y. (2000) Incorporating Fuel Constraints and Electricity Spot Prices into the Stochastic Unit Commitment Problem. *Operations Research*, **48**, 268-280. <http://dx.doi.org/10.1287/opre.48.2.268.12379>
- [27] Tuohy, A., Meibom, P., Denny, E. and O'Malley, M. (2009) Unit Commitment for Systems with Significant Wind Penetration. *IEEE Transaction on Power System*, **24**, 592-601. <http://dx.doi.org/10.1109/TPWRS.2009.2016470>
- [28] Piekutowski, M. and Rose, I.A. (1985) A Linear Programming Method for Unit Commitment Incorporating Generator Configurations, Reserve and Flow Constraints. *IEEE Transaction on Power Apparatus and Systems*, **PAS-104**, 3510-3516. <http://dx.doi.org/10.1109/TPAS.1985.318903>
- [29] Ruzic, S. and Rajakovic, N. (1991) A New Approach for Solving Extended Unit Commitment Problem. *IEEE Transaction on Power System*, **6**, 269-277. <http://dx.doi.org/10.1109/59.131072>

- [30] Wang, J., Shahidehpour, M. and Li, Z. (2008) Security-Constrained Unit Commitment with Volatile Wind Power Generation. *IEEE Transaction on Power System*, **23**, 1319-1327. <http://dx.doi.org/10.1109/TPWRS.2008.926719>
- [31] Wu, L., Shahidehpour, M. and Li, T. (2007) Stochastic Security-Constrained Unit Commitment. *IEEE Transaction on Power System*, **22**, 800-811. <http://dx.doi.org/10.1109/TPWRS.2007.894843>
- [32] Charnes, A. and Cooper, W.W. (1959) Chance-Constrained Programming. *Management Science*, **6**, 73-79. <http://dx.doi.org/10.1287/mnsc.6.1.73>
- [33] Ozturk, U., Mazumdar, M. and Norman, B. (2004) A Solution to the Stochastic Unit Commitment Problem Using Chance Constrained Programming. *IEEE Transaction on Power System*, **19**, 1589-1598. <http://dx.doi.org/10.1109/TPWRS.2004.831651>
- [34] Guo, L., Liu, W., Jiao, B., Hong, B. and Wang, C. (2014) Multi-Objective Stochastic Optimal Planning Method for Stand-Alone Microgrid System. *IET Generation Transmission Distribution*, **8**, 1263-1273. <http://dx.doi.org/10.1049/iet-gtd.2013.0541>
- [35] Wang, Q., Guan, Y. and Wang, J. (2012) A Chance-Constrained Two-Stage Stochastic Program for Unit Commitment with Uncertain Wind Power Output. *IEEE Transaction on Power System*, **27**, 206-215. <http://dx.doi.org/10.1109/TPWRS.2011.2159522>
- [36] Wang, Q., Wang, J. and Guan, Y. (2013) Stochastic Unit Commitment with Uncertain Demand Response. *IEEE Transaction on Power System*, **28**, 562-563. <http://dx.doi.org/10.1109/TPWRS.2012.2202201>
- [37] Kirschen, D. and Strbac, G. (2004) Fundamentals of Power System Economics. John Wiley & Sons. <http://dx.doi.org/10.1002/0470020598>
- [38] Li, T., Shahidehpour, M. and Li, Z. (2007) Risk-Constrained Bidding Strategy with Stochastic Unit Commitment. *IEEE Transaction on Power System*, **22**, 449-458. <http://dx.doi.org/10.1109/TPWRS.2006.887894>
- [39] Saravanan, B., Das, S., Sikri, S. and Kothar, D.P. (2013) A Solution to the Unit Commitment Problem—A Review. *Frontiers in Energy*, **7**, 223-236. <http://dx.doi.org/10.1007/s11708-013-0240-3>
- [40] Padhy, N.P. (2003) Unit Commitment Problem Under Deregulated Environment—A Review. *IEEE Power Engineering Society General Meeting*, 13-17 July 2003, Vol. 2, 1088-1094.
- [41] Huang, Y., Zheng, Q.P. and Wang, J. (2014) Two-Stage Stochastic Unit Commitment Model Including Non-Generation Resources with Conditional Value-At-Risk Constraints. *Electric Power System Research*, **116**, 427-438. <http://dx.doi.org/10.1016/j.epsr.2014.07.010>
- [42] Rockafellar, R.T. and Uryasev, S. (2000) Optimization of Conditional Value-at-Risk. *Journal of Risk*, **2**, 21-42.
- [43] Sarykalin, S., Serraino, G. and Uryasev, S. (2008) Value-at-Risk vs. Conditional Value-at-Risk in Risk Management and Optimization. In: *Tutorials in Operations Research*, INFORMS, Hanover, 270-294.
- [44] Bouffard, F. and Galiana, F. (2004) An Electricity Market with a Probabilistic Spinning Reserve Criterion. *IEEE Transaction on Power System*, **19**, 300-307. <http://dx.doi.org/10.1109/TPWRS.2003.818587>
- [45] Ahmadi-Khatir, A., Bozorg, M. and Cherkaoui, R. (2013) Probabilistic Spinning Reserve Provision Model in Multi-Control Zone Power System. *IEEE Transactions on Power Systems*, **28**, 2819-2829. <http://dx.doi.org/10.1109/TPWRS.2013.2243923>
- [46] Ortega-Vazquez, M.A. and Kirschen, D.S. (2007) Optimizing the Spinning Reserve Requirements Using a Cost/Benefit Analysis. *IEEE Transactions on Power Systems*, **22**, 24-33. <http://dx.doi.org/10.1109/TPWRS.2006.888951>
- [47] Bouffard, F., Galiana, F.D. and Conejo, A.J. (2005) Market-Clearing with Stochastic Security-Part I: Formulation. *IEEE Transactions on Power Systems*, **20**, 1818-1826. <http://dx.doi.org/10.1109/TPWRS.2005.857016>
- [48] North American Electric Reliability Corp. (NERC) Std. (2006) Glossary of Terms Used in Reliability Standards. <http://www.nerc.com>
- [49] Red Eléctrica de España. (1998) Operación del Sistema Eléctrico, Procedimientos de operación. Red Eléctrica de España, Madrid, Spain. <http://www.ree.es/es>
- [50] Independent Electricity Operator (2002) Market Rules Independent Electricity Operator, Toronto, ON. <http://www.theimo.com/imoweb/manuals/marketdocs.asp>
- [51] Bertsimas, D., Litvinov, E., Sun, X.A., Zhao, J. and Zheng, T. (2013) Adaptive Robust Optimization for the Security Constrained Unit Commitment Problem. *IEEE Transactions on Power Systems*, **28**, 52-63. <http://dx.doi.org/10.1109/TPWRS.2012.2205021>
- [52] Xia, Q., Zhong, H. and Kang, C. (2013) Review and Prospects of the Security Constrained Unit Commitment Theory and Applications. *Proceedings of the CSEE*, **33**, 94-103.
- [53] Ruiz, P., Philbrick, C. and Sauer, P. (2010) Modeling Approaches for Computational Cost Reduction in Stochastic Unit Commitment Formulations. *IEEE Transaction on Power System*, **25**, 588-589. <http://dx.doi.org/10.1109/TPWRS.2009.2036462>

- [54] Ortega-Vazquez, M.A. and Kirschen, D.S. (2009) Estimating the Spinning Reserve Requirements in Systems with Significant Wind Power Generation Penetration. *IEEE Transactions on Power Systems*, **24**, 114-124. <http://dx.doi.org/10.1109/TPWRS.2008.2004745>
- [55] Morales, J., Conejo, A. and Perez-Ruiz, J. (2013) Economic Valuation of Reserves in Power Systems with High Penetration of Wind Power. *IEEE Transaction on Power System*, **24**, 900-910. <http://dx.doi.org/10.1109/TPWRS.2009.2016598>
- [56] Pozo, D. and Contreras, J. (2013) A Chance-Constrained Unit Commitment with an N-K Security Criterion and Significant Wind Generation. *IEEE Transactions on Power System*, **28**, 2842-2851; Cohen, A.I. and Yu, C. (2010) Unit Commitment in Energy Markets: Recent Experience and Future Directions. *FERC Technical Conference on Unit Commitment Software* Washington DC, FERC, 1-18.
- [57] Wang, L. and Singh, C. (2006) Tradeoff Between Risk and Cost in Economic Dispatch Including Wind Power Penetration Using Particle Swarm Optimization. *International Conference on Power System Technology*, Chongqing, 22-26 October 2006, 1-7. <http://dx.doi.org/10.1109/ICPST.2006.321416>