

Research on the Method of Calculating Node Injected Reactive Power Based on L Indicator

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Abstract

With the power grid load increasing, the problem of grid voltage stability is increasingly prominent, and the possibility of voltage instability is also growing. In order to improve the voltage stability, this paper analyzed how the voltage stability was influenced by different reactive power injection based on the simplified L-indicator of on-line voltage stability monitoring. According to the basic differential property of the simplified L-indicator, a general and normative analytical algorithm about reactive power optimization was deduced. The analytical algorithm can calculate the load node injected reactive power, and then the network can run in the optimal steady state on the basis of the calculation results. According to the simulation results of IEEE-14, IEEE-30, IEEE-57 and IEEE-118, the feasibility and effectiveness of the proposed algorithm to improve voltage stability and reduce the risk of grid collapse were verified.

Keywords

Voltage Stability; L-Indicator; Injected Reactive Power; Reactive Power Optimization; Fast Calculation

1. Introduction

With the extending of power grids and the increasing of electricity demands, the challenge for voltage stability is increasing. One of the challenges, the system voltage declining, can reduce the power grid transmission capacity and increase the power losses as well, and this is not conducive to the electrical equipment operation. What's worse, it may also lead to system voltage collapse and even cause more serious accidents of the whole power grids [1] [2]. In order to ensure the safety of power grid operation and avoid voltage instability incidents, it is necessary to do some research on how to monitor voltage stability on-line and improve voltage stability quickly [3]-[5].

Voltage stability evaluation indicator achieved by the voltage stability study is a significant progress, it is an effective method to measure the system voltage stability as well as the basis for the implementation of the voltage stability control [6]-[9]. The proposed L-indicator in article [10] [11], which can be used in on-line monitoring, has attracted much attention for its accuracy, linearity, quickness, etc., and it has been applied in the actual pow-

er grids. Considering that the value of reactance is far greater than that of resistance in actual power grids and the bus voltage phase is small, the L-indicator has been simplified and an L - Q sensitivity analysis method has been proposed in [12]. The L - Q sensitivity analysis method facilitates the quantitative analysis of voltage influence between different nodes. However, this method requires a large amount of computation, and its reaction speed to solve voltage instability still needs to be improved.

Considering the above problems, this paper analyzed how the voltage stability was influenced by different reactive power injection based on the simplified L-indicator of on-line voltage stability monitoring. According to the basic differential property of the simplified L-indicator, a general and normative analytical algorithm about reactive power optimization was deduced. The analytical algorithm can calculate the load node reactive power injection volume, and then the network can run in the optimal steady state on the basis of the calculation results. It can ensure the safety of power grid operation and avoid voltage instability incidents.

2. Simplified L-Indicator

Voltage stability local L-indicator was derived by Kessel, etc. according to the simplest two nodes system. Applying the L-indicator into common multi-node systems needs dividing the nodes into two categories. One is characterized by the behavior of the PQ-node which stands for a type consumer node and is defined as α_L . The other comprises the generator node which may be given by PV-node or by a slack node and is defined as α_G . For any load node j ($j \in \alpha_L$), the equation for local L-indicator (L_j) can be described as follows [10] [11]:

$$L_j = \frac{\left| \frac{S_j^+}{Y_{jj}^+ \cdot v_j^2} \right|}{\left| \frac{S_j + S_j^{corr}}{Y_{jj}^+ \cdot v_j^2} \right|} = \frac{\left| \frac{S_j}{Y_{jj}^+ \cdot v_j^2} + \frac{\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{Z_{ij}^* S_i}{V_i}}{v_j} \right|}{\left| 1 + \frac{\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{Z_{ij}^* S_i}{V_i}}{v_j} \right|} \quad (1)$$

where S_j is nodal power of load node j , S_j^{corr} is equivalent power which stems from other loads in the system. S_j^{corr} is given by

$$S_j^{corr} = \sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{Z_{ij}^* S_i}{V_i} \quad (2)$$

In Equation (1), Y_{jj}^+ is the node self-admittance and it equals to $1/Z_{jj}$. Y_{ji}^+ is conjugate value of self-admittance. α_L is the collection of load nodes in the system. V_i, V_j are the load node voltage vectors. v_j is the voltage amplitude value of load node, and v_j^2 is equal to $V_j^* V_j$. Z_{ij} is conjugate value of the mutual impedance between load node i and j . S_i is the equivalent load of the system for consumer node i . In order to ensure the voltage stability, each node j in the system must satisfy the condition $L_j \leq 1$. The global L -indicator of the system is defined as follows.

$$L = L_{\max} = \max_{j \in \alpha_L} (L_j) \quad (3)$$

Thereby, L ranges from 0 to 1.0. A smaller L means a more steady system. When L is close to 1.0, the system tends to critical stable state. In order to guarantee system stability, the value of L must be less than 1.0, and the difference between L value and 1.0 can be used as the voltage stability margin of the system. Equation (1) is a complex expression with complex operation, and the computation will increase sharply with the expansion of power grids. Because the value of reactance is far greater than that of resistance in actual power grids, and bus voltage phase is small, etc., the literature [12] has proposed a simplified L -indicator without considering the reactance and voltage phase. Based on Equation (1), Equation (4) can be obtained.

$$\left| \frac{\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{Z_{ij}^* S_i}{V_i}}{v_j} \right| \leq 1 \quad (4)$$

According to Equation (4), Equation (5) can be introduced.

$$L_j = \begin{cases} 1 + \frac{\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{Z_{ij}^* S_i}{V_i}}{v_j} & (\text{rejection}) \\ 1 - \frac{\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{Z_{ij}^* S_i}{V_i}}{v_j} = 1 - \frac{1}{v_j} \sqrt{f^2 + g^2} \end{cases} \quad (5)$$

According to the simplified method, Equation (6) can be obtained.

$$\begin{cases} f = \sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{f_i}{v_i} \\ f_i = Q_i X_{ji} \\ g = \sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{g_i}{v_i} \\ g_i = -P_i X_{ji} \end{cases} \quad (6)$$

where P_i and Q_i represent the active power injection and reactive power injection respectively. X_{ji} is the reactance between load node i and j . Therefore, the Equation (7) is obtained.

$$L_j = 1 - \frac{1}{v_j} \sqrt{f^2 + g^2} = 1 - \frac{1}{v_j} \sqrt{\left(\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{Q_i X_{ji}}{v_i}\right)^2 + \left(\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{-P_i X_{ji}}{v_i}\right)^2} \quad (7)$$

3. Optimal Reactive Power Compensation

The initial value of L -indicator is set in Equation (8).

$$L_{jori} = 1 - \frac{1}{v_j} \sqrt{\left(\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{Q_i X_{ji}}{v_i}\right)^2 + \left(\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{-P_i X_{ji}}{v_i}\right)^2} \quad (8)$$

When reactive power compensation is needed, the injected reactive power is set as ΔQ_i . Then the reactive power is updated to $Q_i + \Delta Q_i$, and the relationship between reactive power injection and L_j can be described as Equation (9).

$$L_j = 1 - \frac{1}{v_j} \sqrt{\left(\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{(Q_i + \Delta Q_i) X_{ji}}{v_i}\right)^2 + \left(\sum_{\substack{i \in \alpha_L \\ i \neq j}} \frac{-P_i X_{ji}}{v_i}\right)^2} \quad (9)$$

When the first order partial derivative function value of local indicator L_j is set to zero, the extreme value of L_j is obtained. As L_j is a convex function, the extreme value is the minimum. The power grid can run in optimum steady state with the minimum L_j . The first order partial derivative function is described as follows.

$$\frac{\partial L_j}{\partial \Delta Q} = \begin{bmatrix} \frac{\partial L_j}{\partial \Delta Q_1} \\ \vdots \\ \frac{\partial L_j}{\partial \Delta Q_m} \end{bmatrix} = \mathbf{0} \quad (10)$$

where m is the number of load nodes.

Because there are usually more than one node in the power grid, solving the extreme value needs to be carried out repeatedly. To simplify the computation and facilitate the understanding, the value of L_j corresponding to each node is added and the sum is set to L_{sum} , then multiple minima ($L_j, j = 1, 2, \dots, m$) can be converted into one minimum (L_{sum}) [13]. Its expression is as follows.

$$L_{sum} = L_1 + L_2 + \dots + L_m \tag{11}$$

$$L_{sum} = \sum_{j=1}^m \left[1 - \frac{1}{v_j} \sqrt{\left(\sum_{\substack{i \in a_L \\ i \neq j}} \frac{(Q_i + \Delta Q_i) X_{ji}}{v_i} \right)^2 + \left(\sum_{\substack{i \in a_L \\ i \neq j}} \frac{-P_i X_{ji}}{v_i} \right)^2} \right] \tag{12}$$

with

$$\mu = \sum_{\substack{i \in a_L \\ i \neq j}} \frac{(Q_i + \Delta Q_i) X_{ji}}{v_i}, v = \sum_{\substack{i \in a_L \\ i \neq j}} \frac{-P_i X_{ji}}{v_i} \tag{13}$$

Therefore, there is

$$\frac{\partial L_{sum}}{\partial \Delta Q} = \begin{bmatrix} \frac{\partial L_{sum}}{\partial \Delta Q_1} \\ \vdots \\ \frac{\partial L_{sum}}{\partial \Delta Q_m} \end{bmatrix} = \begin{bmatrix} \sum_{\substack{j \in a_L \\ j \neq 1}} -\frac{1}{v_j} \cdot \frac{X_{j1}}{v_1} \cdot \frac{\mu}{\sqrt{\mu^2 + v^2}} \\ \vdots \\ \sum_{\substack{j \in a_L \\ j \neq m}} -\frac{1}{v_j} \cdot \frac{X_{jm}}{v_m} \cdot \frac{\mu}{\sqrt{\mu^2 + v^2}} \end{bmatrix} \tag{14}$$

When the value of Equation (14) is equal to zero vector, it can be converted into matrix form as Equation (15).

$$D \cdot K \cdot Q + D \cdot K \cdot \Delta Q = 0 \tag{15}$$

$$D = \begin{bmatrix} 0 & \dots & X_{m1} \\ & \ddots & \\ X_{1m} & \dots & 0 \end{bmatrix} \tag{16}$$

$$K = \begin{bmatrix} 0 & \dots & \frac{X_{1m}}{v_m} \\ & \ddots & \\ \frac{X_{m1}}{v_1} & \dots & 0 \end{bmatrix} \tag{17}$$

Finally, get

$$\Delta Q = -Q \tag{18}$$

4. Simulation Results

According to the above derivation, the IEEE-14, IEEE-30, IEEE-57, and IEEE-118 systems have been tested on matpower platform, and the simulation results are shown as below.

In **Figure 1**, with the original load of IEEE-14 standard model as a benchmark and one fifth of the original load as the step length, the load changes from 0.2 times to 4.2 times of the original load. The curve L in **Figure 1** is the global L-indicator value without reactive power compensation, while L_c is global L-indicator value with reactive power compensation. L_s and L_{sc} in **Figure 1** are the summation of all local L-indicator $L_j (j = 1, 2, \dots, m)$ without and with reactive power compensation respectively. Contrasting curves L and L_c , as depicted in **Figure 1**, it can be seen that the effect of improving voltage stability is obvious, and the heavier the load, the more significant the effect is, which illustrate that the proposed method is conducive to ensure the safety of power

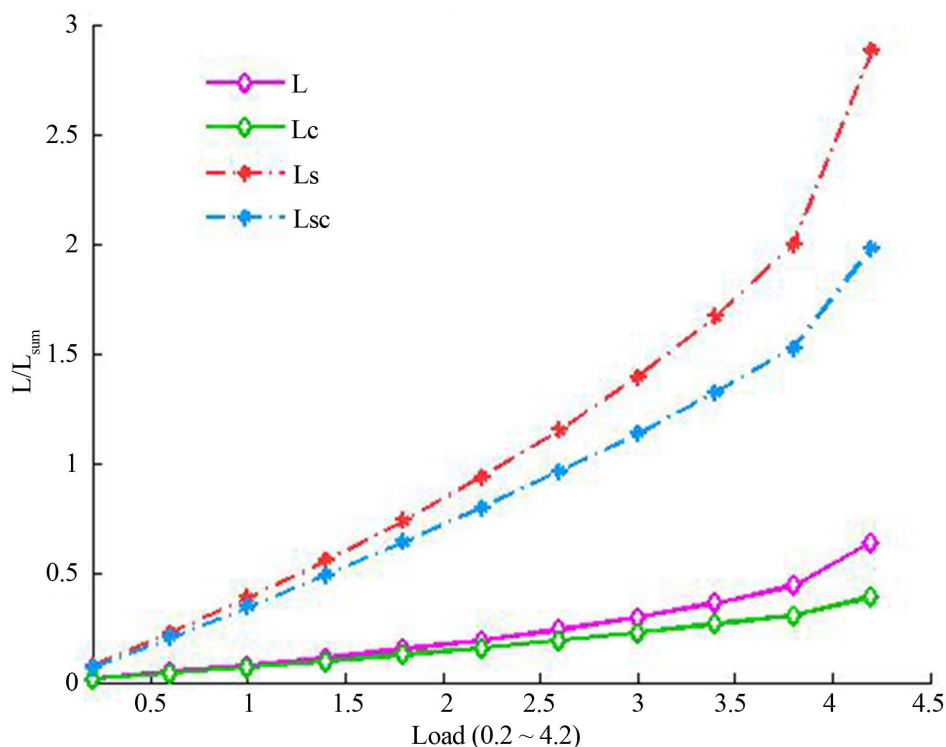


Figure 1. L/L_{sum} of IEEE-14 with load changes.

grid operation and reduce the risk of grid collapse, especially when the grid voltage stability is poorer. For example, corresponding to 3.8 of the horizontal axis, L -indicator value of L curve is close to 0.5, which means that the system is in so sensitive a state that a slight increase of load will augment the possibility of voltage instability, while L -indicator value of L curve is about 0.3 enough to avoid voltage instability. Comparing curves L and L_c to curves L_s and L_{sc} , what can be concluded is that the two groups of curves are the same-the shape and the trend. Therefore, L and L_c can be enlarged as L_s and L_{sc} , which is more convenient for observation.

In Figure 2, the IEEE-30 standard model has been tested. The load changes from 0.2 times to 4.0 times of the original load. The effect of improving voltage stability is more apparent, and load power margin is also improved.

According to Figures 3(a) and (b), the conclusions obtained are similar to the above. The simulation results of the IEEE-14, IEEE-30, IEEE-57 and IEEE-118 have illustrated the universal applicability of the proposed method.

However, the difference between the above models and the actual power grids can not be ignored, which makes the voltage drop proportion of model greater than that of the actual power grids. In actual power grids, the resistance is far less than the reactance and the voltage drop caused by the active load is almost negligible. If the proposed method were applied to one model more similar to the actual power grid, the results would be more ideal.

5. Conclusion

This paper analyzed how the voltage stability was influenced by different reactive power injection based on the simplified L -indicator of on-line voltage stability monitoring. According to the basic differential property of the simplified L -indicator, a general and normative analytical algorithm about reactive power optimization was deduced. The simulation results that the proposed method applied to all systems of IEEE-14, IEEE-30, IEEE-57 and IEEE-118 can improve voltage stability and reduce the risk of power grid collapse significantly and effectively, and the heavier load, the more significant effect, have verified that the proposed method could ensure the safety of power grids operation and avoid voltage instability incidents with its universal applicability.

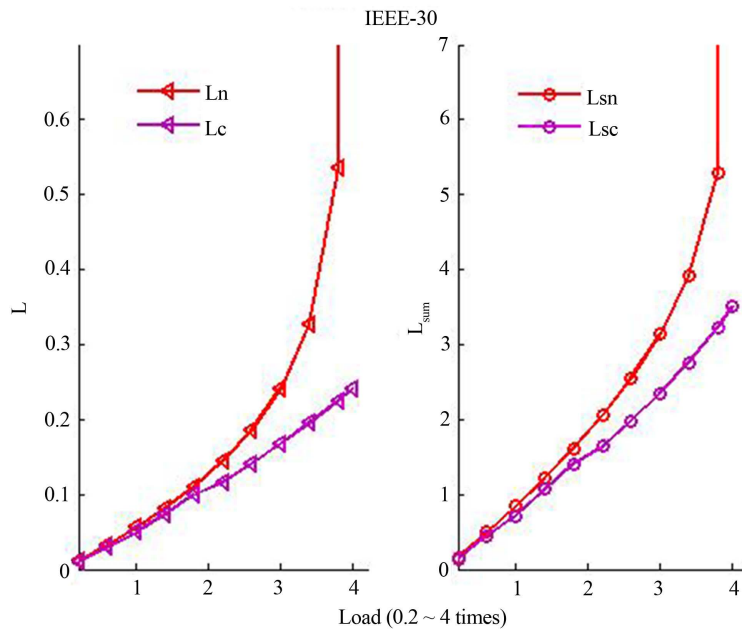


Figure 2. IEEE-30 L/L_{sum} with load change.

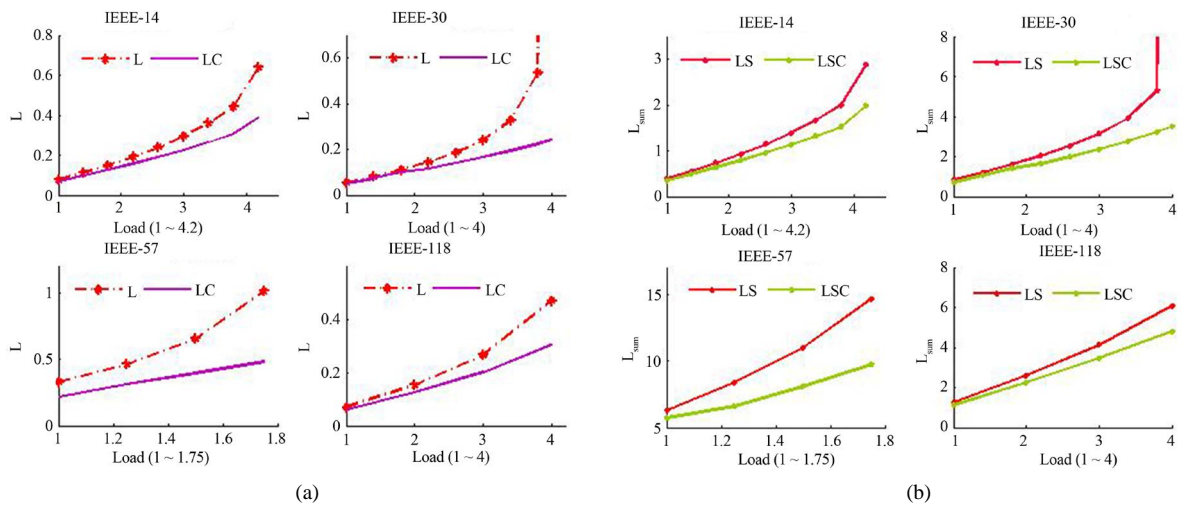


Figure 3. $L(a)/L_{sum}(b)$ curves of 4 systems corresponding to each load changes.

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