

Determination of Fourier Components of Spatial Correlation Function of Dielectric Susceptibility of Random Medium

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Abstract

In this work, we present a new method of directly determining Fourier components of the spatial correlation function of the dielectric susceptibility of random medium. The method is based on the analysis of the ratio of the spectrum of the light scattered by the spatial correlation components of the dielectric susceptibility of tissue to the spectrum of light scattered by the randomly distributed scatterers which are independent on the value of the spectrum of the incident light and the direction of the observation. The results may find wide applications in areas such as in biomedical diagnosis.

Keywords

Spatial Correlation of Refractive Index, Spectrum of the Light Scattered, Dielectric Susceptibility, Fourier Components

1. Introduction

When light propagates through a random medium such tissue, it will be scattered due to the spatial variations of the refractive index of the tissue. The properties of the scattered light are dependent on both the properties of the incident light and the three-dimensional (3D) microstructures of the tissue. The relationships that exist between the properties of the scattered light and properties of the structure of the object under investigation can be explored to obtain information about cells and tissues.

In 1989, Wolf *et al.* investigated the scattering of polychromatic light by a medium whose dielectric susceptibility is a random function of position [1]. However, their concern is on the frequency shifts of the spectral density due to the

scattering process. They found that the spectrum of the scattered light is dependent on both the spectrum of the incident light and the Fourier components of the spatial correlation function of the dielectric susceptibility of the random medium. For light propagating through random medium, changes in coherence, polarization and spectrum of light have also been considered [2]-[12]. These results may be explored to derive some information about structures of tissues [13] [14] [15] [16].

Information about cell and tissue structures can also be obtained by combing measured properties of reflected and scattered light with theoretical models. In fact, it has been shown that the size distribution of the cell nuclei can be obtained by fitting measured light scattering spectra to the predictions of Mie theory [17]. In addition, by analyzing the amplitude and frequency of a fine structure component in backscattered light, the density and size distribution of epithelial cell nuclei can be extracted [18] [19]. Finally, it was also found that some Fourier components of the spatial correlation functions of refractive index fluctuations of living tissues can be measured by Fourier domain optical coherence tomography (FDOCT) [8] [13] [20].

Tissue has very complex structures and can be decomposed into different components from different points of view. It can be approximately represented by a structure consisting of an infinite number of particles with sizes within a large range in which the refractive index variations can be approximately generated by spherical particles with some size distribution (for example) [21]. The particle model of light scattering in tissue is appropriate in cases where the imaging resolution is much larger than the size of the scatterers [22]. At the microscopic level, there is no clear interface between constituents of tissue. A continuous model of refractive index was then proposed in which refractive index variation is represented by a continuous function of position within tissue [23]-[28].

It has been demonstrated experimentally that at the microscopic level, there is a degree of spatial correlation among tissue structures between any two points [2]-[28]. In this work, we present a new method of directly determining Fourier components of the spatial correlation function of the dielectric susceptibility of random medium. First, the ratio is derived of the spectrum of the light scattered by the spatial correlation components of the dielectric susceptibility of tissue to the spectrum of light scattered by the randomly distributed scatterers. It is found that the ratio of the spectrum of the scattered light induced by the spatial correlation of the refractive index fluctuations to the spectrum of the scattered light generated by the randomly distributed scatterers is independent on the value of the incident spectrum. There is also no ω^4 factor in the ratio. A new method is proposed for directly determining Fourier components of the spatial correlation function of the dielectric susceptibility of random medium. A model for characterizing the Fourier spectrum of the refractive index correlation function of tissue is used as an example to demonstrate the analysis. The results obtained may find wide applications in areas such as in biomedical diagnosis.

2. Theory

Within the accuracy of the first-order born approximation, the spectrum $S^{(\infty)}(rs, \omega)$ of the light scattered by deterministic medium and observed in the far zone can be expressed as Equation (1)

$$S^{(\infty)}(rs, \omega) = \frac{1}{r^2} \left(\frac{\omega}{c} \right)^4 \tilde{\eta}^* [k(s - s_0), \omega] \tilde{\eta} [k(s - s_0), \omega] S^{(i)}(\omega), \quad (1)$$

where $*$ denotes the complex conjugate, $k = \omega/c = 2\pi/\lambda$ is the wavenumber, ω is the frequency of the light, c is the speed of light in vacuo, λ is the wavelength, s is the unit vector in the direction of scattering, r is the distance from the observation point to the reference point, s_0 is a real unit vector in a direction, $S^{(i)}(\omega)$ is the spectrum of the incident light, and

$$\tilde{\eta} [k(s - s_0), \omega] = \int_{V(r')} \eta(\mathbf{r}', \omega) \exp[-jk(s - s_0) \cdot \mathbf{r}'] d^3r', \quad (2)$$

is the Fourier transform of the dielectric susceptibility

$\eta(\mathbf{r}, \omega) = [n^2(\mathbf{r}, \omega) - 1]/4\pi$, $n(\mathbf{r}, \omega)$ is the refractive index of the medium (see **Figure 1**). Equation (1) is valid when the amplitude of the scattered light field is small compared with the amplitude of the incident field. It can be seen from Equation (1) that in this case, by measuring the spectrum of the scattered light, one can directly obtain the Fourier components represented by the spatial frequency vector $\mathbf{K} = k(s - s_0)$ in \mathbf{K} space. The Fourier components available, of course, are determined by the range of the wavenumber of the incident light and the directions of the observation achievable.

Note that the basic principle of the Fourier domain optical coherence tomography (FDOCT) is based on the interferometric measurement of one dimension simplified version of Equation (1) [29] [30] [31] [32] [33].

For the random scattering medium, its refractive index and dielectric susceptibility fluctuate with position, which causes scattering of light. An ensemble average is then needed to find the expectation value of the spectrum in Equation (1). We have [1]

$$S_c^{(\infty)}(rs, \omega) = \frac{1}{r^2} \left(\frac{\omega}{c} \right)^4 \tilde{C}_\eta [-\mathbf{K}, \mathbf{K}, \omega] S^{(i)}(\omega), \quad (3)$$

where \tilde{C}_η is the Fourier transform of the spatial correlation function

$C_\eta(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = \langle \eta^*(\mathbf{r}'_1, \omega) \eta(\mathbf{r}'_2, \omega) \rangle$ of the dielectric susceptibility and is defined as

$$\begin{aligned} \tilde{C}_\eta(\mathbf{K}_1, \mathbf{K}_2, \omega) &= \langle \tilde{\eta}(\mathbf{K}_1, \omega) \tilde{\eta}^*(\mathbf{K}_2, \omega) \rangle \\ &= \int_V \int_V C_\eta(\mathbf{r}'_1, \mathbf{r}'_2, \omega) \exp[-j(\mathbf{K}_2 \cdot \mathbf{r}'_2 - \mathbf{K}_1 \cdot \mathbf{r}'_1)] d^3r'_1 d^3r'_2 \end{aligned} \quad (4)$$

Equation (3) shows that, for the random scattering medium, the spectrum of the scattered light observed in the direction s in the far zone is proportional to the product of a factor proportional to ω^4 and the Fourier component of the spatial correlation function of the dielectric susceptibility with the spatial frequency vector $\mathbf{K} = k(s - s_0)$ in \mathbf{K} space. On the other hand, Equation (1)

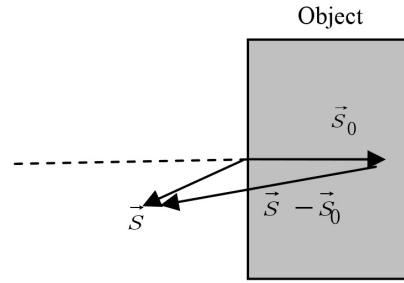


Figure 1. Illustrating the notations. The real unit vector s_0 denotes the direction of the incident light, the real unit vector s denotes the direction of the scattered light, and $\mathbf{K} = k(s - s_0)$ is the spatial frequency vector.

reveals that for deterministic medium, the spectrum observed in the direction s is proportional to the square of the Fourier component of the dielectric susceptibility itself.

Thus when there exist spatial correlated fluctuations in the dielectric susceptibility or refractive index of the medium, the Fourier component of the dielectric susceptibility is also random function. In this case, it is the spatial correlation of the dielectric susceptibility that determines the spectrum of the scattered light.

In the case when the dielectric susceptibility of the medium is completely uncorrelated, we have

$$C_{\eta}(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = I(\omega) \delta^{(3)}(\mathbf{r}'_2 - \mathbf{r}'_1) \quad \text{when } \mathbf{r}'_1 \in V, \mathbf{r}'_2 \in V \quad (5)$$

$$= 0 \quad \text{otherwise,}$$

where $\delta^{(3)}(\mathbf{r}')$ is the 3D Dirac delta function and $I(\omega)$ is a nonnegative function of frequency. In this case, from Equation (3) we have:

$$S_u^{(\infty)}(rs, \omega) = \frac{1}{r^2} \left(\frac{\omega}{c}\right)^4 I(\omega) S^{(i)}(\omega). \quad (6)$$

We can see from Equation (6) that for a given frequency ω of light, there is no observation direction dependence of the spectrum of the scattered light. Note that when the dielectric susceptibility of the medium is completely uncorrelated, from the measured spectrum of the scattered light, we can obtain no information about the spatial correlated structure of the scatterers, as expected. This fact has important implications in the biomedical imaging of tissues. For example, this fact shows that it is then possible to detect some forms of variations of tissue structures by analyzing properties of ratio of the spectrum of the light scattered by the spatial correlation of the fluctuations of the tissue refractive index and the scattered light arising from the deterministic change of the refractive index such as in the small interfaces between different tissue elements.

For the light scattering induced by the spatial correlation of the refractive index fluctuations, there is an intimate relationship between its spectrum and the Fourier components of the of the spatial correlation function of the dielectric susceptibility. Notice that it has been found experimentally that there exists only

finite spatial correlation length between the fluctuations of tissue refractive index at two points. For tissues, the spatial correlation length is found to equal or exceed $4 \mu\text{m}$ [16]. Thus, tissue elements with sizes larger than the spatial correlation may be regarded as being randomly distributed. In this case, there is no scattering due to the correlation of its refractive index.

If we calculate the ratio of Equation (3) to Equation (6) and denote it by $R_S^{(\infty)}$, we have

$$R_S^{(\infty)}(rs, \omega) = \frac{S_c^{(\infty)}(rs, \omega)}{S_u^{(\infty)}(rs, \omega)} = \frac{\tilde{C}_\eta[-\mathbf{K}, \mathbf{K}, \omega]}{I(\omega)}. \quad (7)$$

Equation (7) demonstrates that the ration $R_S^{(\infty)}$ is equal to the ratio of the Fourier component of the spatial correlation function of the dielectric susceptibility to a function of the frequency. From Equation (7) we can see that the ratio of the spectrum of the scattered light induced by the spatial correlation of the refractive index fluctuations to the spectrum of the scattered light generated by the randomly distributed scatterers is independent on the value of the incident spectrum. There is also no ω^4 factor in the ratio. Thus, the value of the ratio may reveal some inherent micro-structural characteristics of the scattering medium.

3. Method

Now we consider a specific case when incident light is monochromatic or with a very small bandwidth. In this case, $I(\omega)$ can be regarded as a constant $I(\omega_0)$, where ω_0 the average frequency of the light. For different observation direction, denoted by s , $R_S^{(\infty)}$ is equal to the ratio of the Fourier component of the spatial correlation function of the dielectric susceptibility with the spatial frequency vector $\mathbf{K} = k(s - s_0)$ to the constant $I(\omega_0)$. In this case, we have

$$\tilde{C}_\eta[-\mathbf{K}, \mathbf{K}, \omega_0] = R_S^{(\infty)}(rs, \omega_0)I(\omega_0). \quad (8)$$

Equation (8) shows that if we have the knowledge about $I(\omega_0)$, for a given value of ω_0 , we can directly obtain the values of $\tilde{C}_\eta[-\mathbf{K}, \mathbf{K}, \omega_0]$.

The spatial correlation function of the dielectric susceptibility is an important parameter for characterizing the effects of the random medium on the propagation properties of light within it. It is related to the refractive index correlation function. Consider the case of tissue, if we express refractive $n(\mathbf{r}, \omega)$ index in the form of $n(\mathbf{r}, \omega) = \langle n(\mathbf{r}, \omega) \rangle + \delta n(\mathbf{r}, \omega)$, where \mathbf{r} denotes the position within the medium, $\langle n(\mathbf{r}, \omega) \rangle$ is the mean of the refractive index, $\delta n(\mathbf{r}, \omega)$ is the varying part of refractive index with position, it has been shown that [4]

$$C_\eta(\mathbf{r}_1, \mathbf{r}_2, \omega) = (1/16\pi^2) \left(\langle n \rangle^4 - 2\langle n \rangle^2 + 1 + 4\langle n \rangle^2 C_n(\mathbf{r}_1, \mathbf{r}_2, \omega) \right), \quad (9)$$

where $C_n(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle \delta n(\mathbf{r}_1, \omega) \delta n(\mathbf{r}_2, \omega) \rangle$ is the refractive index correlation function. The Fourier transform of the spatial correlation function of the dielectric susceptibility can then be expressed as

$$\tilde{C}_\eta(K, \omega) = [1/(4\pi)^2] \langle n \rangle^2 \Phi(K), \tag{10}$$

where $\Phi(K)$ is the Fourier spectrum of the refractive index correlation function of tissue. For tissue samples, $\Phi(K)$ has been measured and is of the form [23]

$$\Phi(K) = 4\pi\sigma_n^2 L_0^2 (m-1) (1 + K^2 L_0^2)^{-m}, \tag{11}$$

where L_0 is the outer scale of tissue index inhomogeneities, $L_0 = l_c / [2(m-1)^{1/2}]$, m is a parameter ($1 < m < 2$), where σ_n is the standard deviation of the refractive index, l_c is the index correlation distance, and

$$K = |\mathbf{K}| = |\mathbf{k} - \mathbf{k}_0| = 2k \sin(\theta/2), \tag{12}$$

where θ is the angle between the unit vector s_0 along the incident direction and the unit vector s along the observation direction, respectively. **Figure 2** shows the normalized curve of the variation of the ration $R_S^{(\infty)}$ with the observation direction for the light backscattered from the human upper dermis [23].

The refractive index correlation function determines the light scattering characteristics in tissue as well as the changes of the coherence and polarization of the light beam propagating through tissue [2]-[8]. Equation (10) shows that the Fourier spectrum of the refractive index correlation function of tissue may be obtained by measuring the spectrum of the scattered light through relations (8) and (10). Here, it should be emphasize that the result obtained in this work is based on the first-order born approximation. In our analysis, it is also assumed that the scattering properties of the medium can be described by its refractive index $n(\mathbf{r}, \omega)$ which is a three-dimensional function of position \mathbf{r} .

A comparison of Equation (5) with Equation (9) shows that when dispersion effect of the tissue can be neglected, we have:

$$I(\omega_0) = \frac{\langle n(\omega_0) \rangle^2}{4\pi^2}. \tag{12}$$

Equation (12) demonstrates that the value of $I(\omega_0)$ can be obtained by measuring the mean of the refractive index and using the Equation (12). One possible method for measuring the ration $R_S^{(\infty)}$ is to use the fact that the spectrum of the light scattered by the spatially correlated structures of the dielectric susceptibility is a function of the angle of scattering and there is no observation direction dependence of the spectrum of the light scattered by completely uncorrelated components. So the angle varying part of the measured spectrum of the scattered light represents the contribution of the spatially correlated structures of the dielectric susceptibility and the angle independent part describes the contribution due to the completely uncorrelated components of the medium.

In general, the real medium, such as living tissue, is a complex system, in which light is scattered or reflected by various aspects of its structure. First of all, the layer structures of tissue may be regarded as its deterministic aspect. In this case, the spectrum of the scattered light is related to the Fourier transform of the

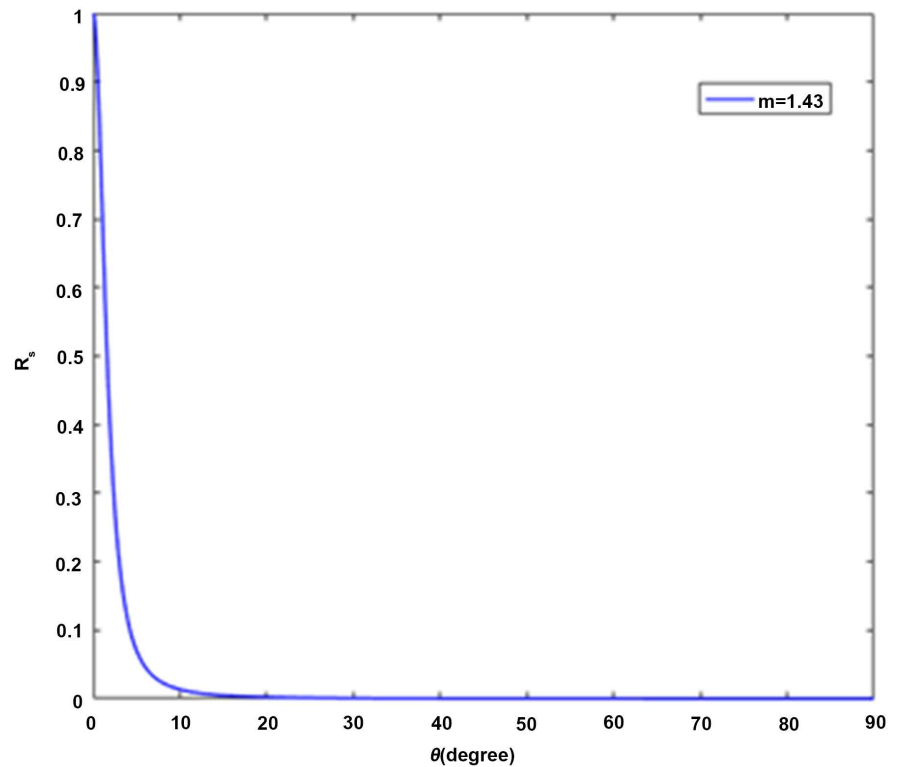


Figure 2. The normalized curve of variation of the ratio $R_s^{(\infty)}$ with the observation direction for some selected values of the parameters. $\langle n \rangle = 1.35$, $\sigma_n = 0.06$, $m = 1.43$, $L_0 = 4 \mu\text{m}$, $\lambda_i = 0.83 \mu\text{m}$.

dielectric susceptibility, as is also shown in the images obtained with FDOCT [14]. Second, for the microscopic structures that have no clear layer structure, the scattering of light may arise from the spatial fluctuations of the refractive index and the spectrum of the scattered light is related to the Fourier component of the spatial correlation function of the dielectric susceptibility, which reveals the microscopic morphological information about the cells or tissues.

In conclusion, we have demonstrated that the ratio of the spectrum of the scattered light induced by the spatial correlation of the dielectric susceptibility to the spectrum of the scattered light generated by the randomly distributed scatterers is independent on the value of the incident spectrum. There is also no ω^4 factor in the ratio. The results supply us a new method for direct determination of Fourier components of the spatial correlation function of the dielectric susceptibility of random medium by measuring the spectrum of the scattered light field, including atmospheric turbulence, oceanic turbulence, and biological tissue. Because the spatial correlation characteristics of the dielectric susceptibility of random medium is a manifestation of the underlying process that produces local correlations in the positions of growing cells in living tissue, the work has potential application in biomedical diagnosis.

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI,

MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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