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On the Mechanism of Nucleon-Nucleon Attraction by Pion Exchange

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Abstract

A formula is derived for the central nucleon-nucleon potential, based on an analysis of the physical origin of the nucleon-nucleon attraction by pion exchange. The decrease of the dynamical mass of the interaction field, exchanged pion in this case, is the principal mechanism responsible for the nuclear attraction in a similar way that the decrease of the kinetic energy of the exchange electron in the diatomic molecule is directly responsible for the covalent molecular attraction. The minimum value of this central nucleon-nucleon potential and the position of the minimum are similar with the values reported in literature for a potential calculated by lattice QCD, which shares the features of the phenomenological nucleon-nucleon potentials. The Schrodinger equation with this central nucleon-nucleon potential was solved numerically for different values of the pion mass. The binding energy increases with the decrease of the pion mass. For masses higher than the real pion mass the nucleon-nucleon system is unbound. We discuss on the two pion exchange and hard core repulsion. The minimum value of the potential for two pion exchange is comparable with the minimum value of the CD Bonn potential. For a hard core radius of 0.5 fm the binding energy is equal to the deuteron binding energy.

Keywords

Pion Exchange, Dynamical Mass, Central Nucleon-Nucleon Potential, Bound State

1. Introduction

The effective degrees of freedom in the nuclear interaction at low energy, in particular in the nuclear bound state, are the nucleons and the pions. The pion exchange is the basic mechanism of nucleon-nucleon (NN) attraction at low energy [1]-[6].

As it is well known the current masses of quarks (antiquarks) u (ū) and d (đ) are very small; the nucleon and pion masses are mainly of dynamical origin ("kinetic" energy). The confinement of the quarks into nucleon and pion associate an energy, given by the Heisenberg uncertainty relation, which is just the dynamical mass.

The long range structure of the nucleon is given roughly by the pion [7] [8], which also gives the range of nuclear forces. In particular, the Compton wavelength of the pion $\lambda\!\!\!^{-}_{\pi} = \hbar / m_{\pi} c$, which has a value of 1.41 fm for the charged pion, gives the range r_N of the nucleon extension. The pion in nucleon can be represented by a degree of freedom of current mass $\cong$ 0, localized into a region of radius $r_N = \lambda\!\!\!^{-}_{\pi}$. This localization into the nucleon associates an energy (dynamical mass) $E \cong pc = \hbar c / \lambda\!\!\!^{-}_{\pi} = m_{\pi} c^2$, given by the uncertainty relation, which is just the mass of the pion.

At the formation of a nuclear bound state this dynamical mass decreases, which determines a mass defect and consequently a binding energy for the nucleon-nucleon state [9] [10]. Indeed, when two nucleons approach each other to form a bound state, in particular the deuteron, they put in common their pion degrees of freedom (pion exchange). This is equivalent with a slight de-localization of the pion degree of freedom from a region of radius $r_N \cong \lambda\!\!\!^{-}_{\pi}$ to a region of radius $r_N + \Delta(R) = \lambda\!\!\!^{-}_{\pi} + \Delta(R)$, where $\Delta(R)$ is direct proportional to the distance R between the two bound nucleons and is strongly dependent on the probability of the pion to penetrate the potential barrier between the two nucleons [10]. The dynamical mass gets:

$$E_{\Delta} \cong \frac{\hbar c}{\lambda\!\!\!^{-}_{\pi} + \Delta(R)} \tag{1}$$

and is lower than the initial one (that in the free nucleon). To form a bound state, the decrease of the dynamical mass of the pion degree of freedom:

$$\Delta E = \frac{\hbar c}{\lambda\!\!\!^{-}_{\pi}} - \frac{\hbar c}{\lambda\!\!\!^{-}_{\pi} + \Delta(R)} \tag{2}$$

must be larger than the kinetic energy acquired by the system of two nucleons due to their localization at a distance R each other:

This mechanism of nuclear binding has similarities with the mechanism of molecular binding of diatomic molecules [11]. In fact Heisenberg was the first who presented the attractive force between proton and neutron in analogy to that in the hydrogen molecular ion H_2^+, where the electron is the particle exchanged between the two protons [2].

2. Feynman Approach to the Molecular and Nuclear Exchange Interactions

Let's start with the physical interpretation of the mechanism of H_2^+ ion binding presented by Feynman in [11].

In the H_2^+ ion, since there are two protons, there is more space where the electron can have a low potential energy than in the case of hydrogen atom. The exchanged electron spreads out lowering its kinetic energy, in accord with uncertainty relation. This kinetic energy decrease is at the origin of the molecular attraction in covalent bond, in particular in the H_2^+ ion [11] [12] [13] [14].

For large distances between the two protons of the H_2^+ ion the electrostatic potential energy of the exchanged electron is nearly zero over most of the space between the protons and the electron moves nearly like a free particle in empty space but with a negative energy [11]:

$$\frac{p^2}{2m_e} = -W_H \tag{3}$$

where W_H is the binding energy (+13.6 eV) of the hydrogen atom. This means that p is an imaginary number:

$$p = i\sqrt{2m_e W_H} \tag{4}$$

The probability amplitude A for a particle of definite energy to get from one place to another a distance R away is proportional to [11]:

$$A \sim \frac{\mathrm{e}^{(i/\hbar)pR}}{R} \tag{5}$$

where p is the momentum corresponding to the definite energy. Replacing p one obtains that the amplitude of jumping of electron from one proton to the other, for large separation R between the two protons, will vary as [11]:

$$A \sim \frac{\mathrm{e}^{-\left(\sqrt{2m_e W_H}/\hbar\right)R}}{R} = \frac{\mathrm{e}^{-R/a_0}}{R} \tag{6}$$

where a_0 is the Bohr radius.

If the particle goes in one direction the amplitude is [11]:

$$A \sim \mathrm{e}^{-R/a_0} \tag{7}$$

One can note that this exponential function limits drastically the amplitude of electron exchange for large separation.

The nuclear interaction which takes place between a neutron and a proton by pion exchange is described by Feynman with similar arguments [11]. Since in the nuclear process the proton and the neutron have almost equal masses, the exchanged pion will have zero total energy. But for a pion of mass m_π:

$$E^2 = p^2c^2 + m_\pi^2 c^4 \tag{8}$$

where E and p are the total energy and the momentum of the pion.

Since the exchanged pion have practically zero total energy the momentum is again imaginary [11]:

$$p = im_\pi c \tag{9}$$

This means the amplitude for the pion to jump from one nucleon to another is for large R:

$$A \sim \mathrm{e}^{-(m_\pi c/\hbar)R} = \mathrm{e}^{-R/\lambda\!\!\bar{}_\pi} \tag{10}$$

The exponential function is typical for a Yukawa potential or exponential potential, and again it limits drastically the exchange for large *R*.

3. The Central Nucleon-Nucleon Potential Due to Pion Exchange

In fact, the exponential factor is well known from the tunneling of a potential barrier of width R by a particle with an energy much lower than the barrier height. The probability of transmission is the (absolute) square of the amplitude [11], this means in our case:

$$P \sim \mathrm{e}^{-2R/\lambda\!\!\bar{}_\pi} \tag{11}$$

The increase $\Delta(R)$ of the radius of localization region of the pion degree of freedom, which appears in formulae (1) and (2), is strongly limited by this exponential function (11), *i.e.* the probability for the exchanged pion to penetrate (tunnel) the potential barrier between the two nucleons. From a physical point of view one expects that this probability of transmission is 1 for a barrier width $R \to 0$. Therefore $\Delta(R)$, which is proportional both to the distance *R* between the nucleons and to the probability of transmission of the exchanged pion, is equal to:

$$\Delta(R) = R\mathrm{e}^{-2R/\lambda\!\!\bar{}_\pi} \tag{12}$$

By replacing (12) in formula (2), we obtain the decrease of the dynamical mass of the pion degree of freedom, which is at the origin of nuclear attraction. In fact with sign minus this decrease is just the NN potential due to pion exchange [10]:

$$V(R) = -\Delta E = \frac{\hbar c}{\lambda\!\!\bar{}_\pi + R\mathrm{e}^{-2R/\lambda\!\!\bar{}_\pi}} - \frac{\hbar c}{\lambda\!\!\bar{}_\pi} = -\frac{\hbar c}{\lambda\!\!\bar{}_\pi} * \frac{R\,\mathrm{e}^{-2R/\lambda\!\!\bar{}_\pi}}{\lambda\!\!\bar{}_\pi + R\,\mathrm{e}^{-2R/\lambda\!\!\bar{}_\pi}} \tag{13}$$

where $\hbar c/\lambda\!\!\bar{}_\pi = m_\pi c^2$.

The NN potential *V(R)* as a function of inter-nucleon distance *R* is shown in **Figure 1** for a charged pion exchange between the two nucleons $(\lambda\!\!\bar{}_\pi = 1.41\ \mathrm{fm})$. This potential has some similarities with that obtained by lattice QCD [15] [16], which shares the features of the phenomenological NN potentials: an attractive well at intermediate and larger distances and a hard core repulsion at small range, with the maximum depth of the potential in the intermediate range [1] [2] [3] [4] [5]. The minimum value of the potential (−22 MeV) and the position of the minimum (0.7 fm) in **Figure 1**, are comparable with the values shown in **Figure 3** from [15]: about −25 MeV and 0.7 fm. The fall of the potential towards zero value in **Figure 1** for small *R* ($R < 0.6$ fm) is compatible with the beginning of the hard core repulsion region [1] [2] [3] [4] [5] [15] which gets dominant at short range.

The potential in **Figure 1** is a little larger, in particular the fall towards higher values of *R* is slower than in the case of potential obtained in [15]. But the results in [15] were obtained for a pion mass (530 MeV) higher than the real pion mass.

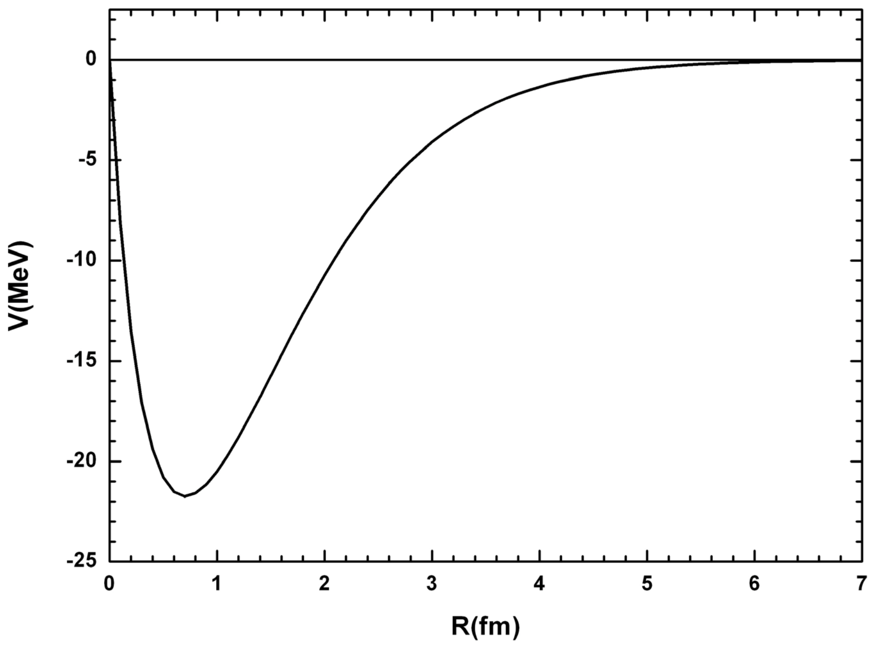

Figure 1. The NN potential in the case of charged pion exchange.

For a lower pion mass (360 MeV) the range of the potential gets wider also [15].

We solved numerically the Schrodinger equation for the potential $V(R)$ given in relation (13), for different values of the pion mass m_π:

$$-\frac{\hbar^2}{2\mu}\Delta\psi(R)+V(R)\psi(R)=E\psi(R) \tag{14}$$

where μ is the reduced mass of the two nucleons.

From a numerical point of view, the Schrodinger equation in central potential has been solved using the substitution: $\psi(R)=\phi(R)/R$ which, in turn, gives a standard Sturm-Liouville eigenvalue problem with a constant coefficient for the second order derivative:

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2\phi(R)}{\mathrm{d}R^2}+V(R)\phi(R)=E\phi(R) \tag{15}$$

We look for the ground state, this means zero centrifugal energy ($l = 0$).

The spatial dimension has been truncated as $R\in[0,20]\,\text{fm}$ with a standard discretization in equal intervals of $\Delta R=10^{-3}$ fm. Such large radial extension is needed in order to resolve properly the states lying closely to the continuum ($E<\simeq 0$). The eigenvalue problem is solved by means of a finite difference method with the boundary values $\phi(0)=\phi'(0)=0$ and imposing an exponential decay at large radius. The resulting Hamiltonian is diagonalized and the eigenvalues (energy) are obtained.

In **Figure 2** is given the dependence of the total energy of the two nucleons in function of the exchanged pion mass m_π. For the real pion mass corresponds about −0.1 MeV, this means a very small binding energy.

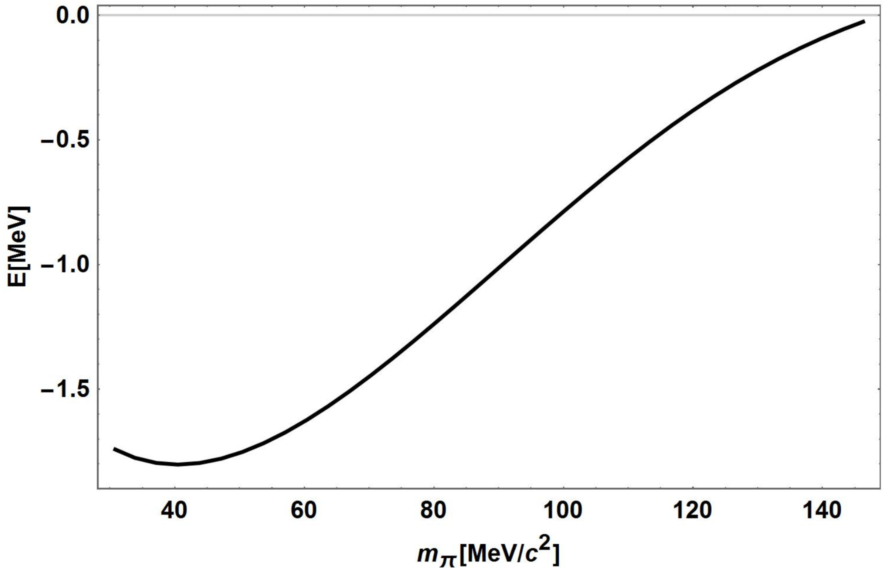

Figure 2. The total energy of the two nucleons as a function of the pion mass m_π.

The total energy of the two nucleons decreases, this means the binding energy increases with the decrease of the pion mass. For masses higher than the real pion mass the total energy gets positive (unbound state). This underline the central role of the pion as main player in the production of nuclear attraction, as largely accepted in literature [1]-[6].

4. The Two Pion Exchange and the Hard Core Repulsion

The maximum molecular attraction is realized in the diatomic molecular bond by exchange of two electrons [11]. This is the case of the hydrogen molecule H_2 in which the two hydrogen atoms put in common (exchange) their electrons.

Similarly, the maximum of NN attraction is given by the exchange of two pions. Each nucleon puts in common (exchanges) a pion degree of freedom with the other nucleon. This means two pion degrees of freedom are slightly de-localized. The NN potential in this case is practically two times the potential given by relation (13). The minimum value of the potential gets −44 MeV, which is comparable with the minimum value (−50 MeV) of the CD Bonn potential [2].

If one adds to this two pion exchange potential $2\ V(R)$, where $V(R)$ is given by relation (13), a hard core repulsion at $r_0 \le 0.6\ \text{fm}$, a typical value for phenomenological nucleon-nucleon potentials [1] [2] [3] [4] [5], it results the NN potential shown in **Figure 3**.

The Schrodinger equation for this two pion exchange potential $2V(R)$ with hard core repulsion was solved numerically for different values of the hard core radius r_0. In **Figure 4** it is shown the total energy of the two nucleons in function of the hard core radius.

For a value of the hard core repulsion radius equal to 0.5 fm, the binding energy is equal to the deuteron binding energy. For this hard core radius

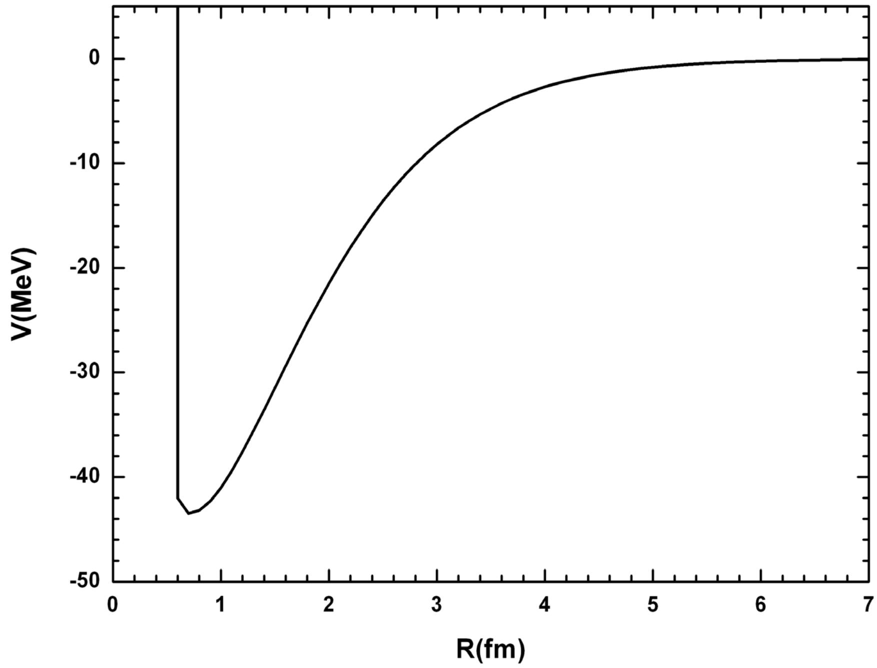

Figure 3. The NN potential in the case of two pion exchange and hard core repulsion at 0.6 fm.

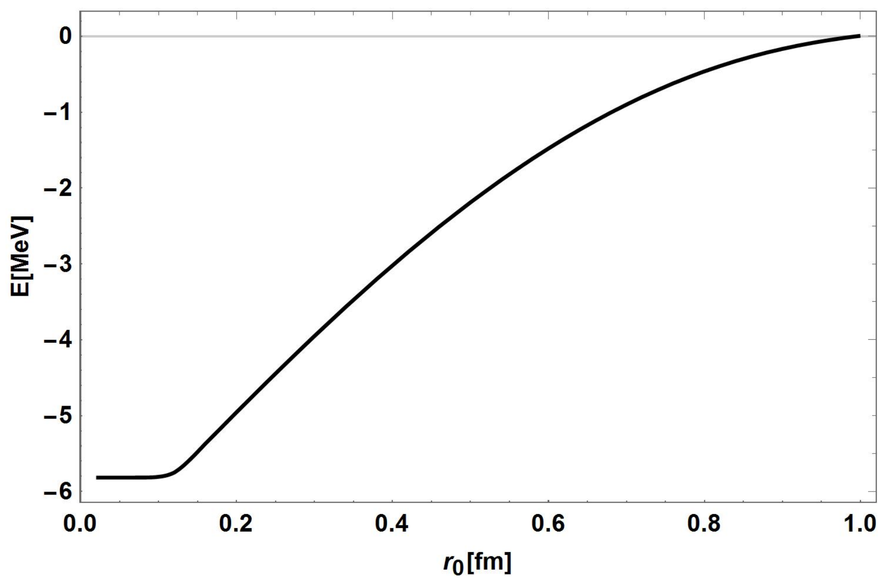

Figure 4. The total energy of the two nucleons as a function of hard core repulsion radius r_0 for the two pion exchange potential.

($r_0 = 0.5\ \text{fm}$) in **Figure 5** is given the dependence of the total energy of the two nucleons bound by two pion exchange in function of the pion mass m_π.

With pion mass increase, for pion masses higher than about 60 MeV, the total energy increases, *i.e.* the binding energy decreases, and gets zero at about 190 MeV, a result comparable with that obtained in [4] for deuteron binding energy. In [17] the binding energy becomes zero at about 300 MeV.

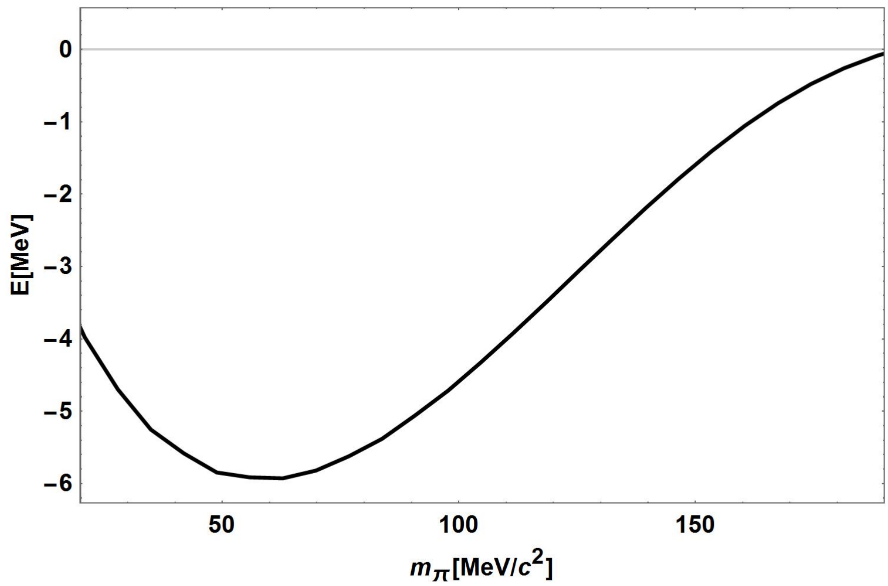

Figure 5. The total energy of the two nucleons as a function of the pion mass m_π, for the two pion exchange potential and a hard core repulsion radius $r_0 = 0.5$ fm.

5. Discussion and Conclusions

The pion exchange is at the origin of the nuclear attraction, in a similar way that the electron exchange is at the origin of attraction in the molecular covalent bond. The decrease of the kinetic energy (in fact dynamical mass decrease from a relativistic point of view) of the exchange electron in the H_2^+ ion, directly responsible for the formation of the molecular bound state, is replaced by the decrease of the dynamical mass of the pion degree of freedom in the case of the nuclear attraction by pion exchange. The slight de-localization of the pion degree of freedom, which is at the origin of this dynamical mass decrease, is drastically limited by an exponential function, which represents the probability for the pion to penetrate the potential barrier between the two nucleons. A similar exponential function exists in the case of molecular bond.

The analytical formula of the central nuclear potential (13) derived for the NN interaction by pion exchange does not contain any unknown parameter. The minimum value of the NN potential and the position of the minimum are similar with the values reported in literature for the central NN potential obtained by lattice QCD, which shares the features of the phenomenological NN potentials. A very small binding energy (0.1 MeV) was obtained by solving numerically the Schrodinger equation. The binding energy increases for pion masses lower than the real pion mass. For masses higher than the real pion mass the nucleon-nucleon system is unbound.

The fall of the potential (13) towards zero value for small R (**Figure 1**) is compatible with the beginning of the well known hard core repulsion region which is dominant at short range. On the other hand the Yukawa potential, derived in analogy with the coulomb attraction (virtual photon exchange), gets infinite attractive for $R \to 0$ due to factor $-1/R$ [1] [2].

Since the maximum value of $R\,\mathrm{e}^{-2R/ƛ_\pi}$ is 0.26 fm (for $R = ƛ_\pi/2$), this means substantially smaller than $ƛ_\pi = 1.41\ \mathrm{fm}$, relation (13) can be written in a good approximation as:

$$V(R) \approx -m_\pi c^2 \frac{R}{ƛ_\pi} \mathrm{e}^{-2R/ƛ_\pi} \tag{16}$$

The NN nuclear potential is proportional to the mass of the interaction field. It is also proportional to the ratio $R/ƛ_\pi$, which is directly related to the slight de-localization of the pion degree of freedom and its (dynamical) mass decrease. This de-localization is drastically limited by the exponential function, which is similar to an exponential potential except the factor 2. Due to this exponential function the width of potential (16) gets larger for pion masses lower than the real pion mass and this explains the increase of the binding energy with the pion mass decrease (**Figure 2**). For too small pion masses this dependence reverses because the depth of potential (16) becomes too low.

In the case of two pion exchange the minimum of the potential gets comparable with the minimum value of the CD Bonn potential. The potential in this case is in a good approximation:

$$V_{2\pi}(R) \approx -2m_\pi c^2 \frac{R}{ƛ_\pi} \mathrm{e}^{-2R/ƛ_\pi} \tag{17}$$

The dependence of the type $x\mathrm{e}^{-x}$ of the nuclear potential, where $x = 2R/ƛ_\pi$, is similar with the attractive part of the Rydberg potential used to describe the molecular covalent bonding [18].

A hard core repulsion was added to this two pion exchange potential and the Schrodinger equation was solved numerically for different values of the hard core repulsion radius. For a radius of 0.5 fm the binding energy is equal to the deuteron binding energy.

Let's compare the strength of the nuclear potential $V_{2\pi}(R)$ from relation (17) with the coulombian potential $V_C = \frac{q^2}{R}$. The ratio of the two potentials for $R = ƛ_\pi$ is:

$$V_{2\pi}(ƛ_\pi)/V_C(ƛ_\pi) = 2\frac{\hbar c}{q^2}\mathrm{e}^{-2} = 2\frac{1}{\alpha}\mathrm{e}^{-2} \tag{18}$$

where α is the e.m coupling constant, a typical result for the relative strength of the nuclear interaction to the e.m. interaction.

If we analyze the mechanism of NN interaction at quark level, we could say that by pion exchange between two nucleons some quark degrees of freedom are implicitly exchanged and in consequence are slightly de-localized. The confinement region of a quark slightly increases and accordingly its dynamical mass decreases. This suggests to interpret the nuclear interaction as a residual strong interaction.

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