

How a Realistic Linear $R_h = ct$ Model of Cosmology Could Present the Illusion of Late Cosmic Acceleration

Eugene Terry Tatum¹, U. V. S. Seshavatharam²

¹760 Campbell Ln. Ste., Bowling Green, KY, USA

²Honorary Faculty, I-SERVE, Hyderabad, India

Email: ett@twc.com, seshavatharam.uvs@gmail.com

How to cite this paper: Tatum, E.T. and Seshavatharam, U.V.S. (2018) How a Realistic Linear $R_h = ct$ Model of Cosmology Could Present the Illusion of Late Cosmic Acceleration. *Journal of Modern Physics*, 9, 1397-1403.

<https://doi.org/10.4236/jmp.2018.97084>

Received: May 9, 2018

Accepted: June 5, 2018

Published: June 8, 2018

Copyright © 2018 by authors and Scientific Research Publishing Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

Realistic FLRW cosmic coasting models which contain matter now appear to be a reasonable alternative in explaining the accumulated Supernova Cosmology Project data since 1998. In sharp contrast to the unrealistic original classic Milne universe, which was entirely devoid of matter, these modified Milne-type models containing matter, often referred to as realistic linear $R_h = ct$ models, have rapidly become the primary competition with standard cosmology. This paper compares the expected relative luminosity distances and relative angular diameter distances for given magnitudes of redshift within these two competing models. A simple ratio formula is derived, which explains how *expected* luminosity distances and angular diameter distances for given magnitudes of redshift within a realistic Milne-type cosmic expansion could create the illusion (for standard model proponents) of cosmic acceleration where none exists.

Keywords

Dark Energy Survey, Cosmology Theory, Cosmic Coasting, Cosmic Flatness, Type Ia Supernovae, CMBR, Flat Space Cosmology, Milne Universe Theories

1. Introduction and Background

While final observations of the Dark Energy Survey (DES) are eagerly awaited, there is a vigorous debate in the scientific community as to whether cosmic acceleration is a reality or an illusion. Some very recent papers [1] [2] [3] [4] [5] present compelling statistical analysis of the accumulated data which clearly shows that *cosmic acceleration is not yet proven*. In particular, it appears that

constant velocity cosmic expansion (cosmic “coasting”) could produce very similar observations [6]-[14].

Until very recently, it was believed that *only* a classic Milne (“empty universe”) model could produce cosmic coasting. Unfortunately, Milne’s original model [15] was unrealistic, being entirely devoid of matter throughout its expansion history. Initially after the 1998 Type Ia supernovae discovery of dark energy [16] [17] [18], there appeared to be no realistic cosmic models containing matter which could compete with the current widely-accepted standard Friedmann-Lemaitre-Robertson-Walker (FLRW) Lambda Cold Dark Matter (LCDM) model (hereafter referred to as the “standard model”). Fortunately, within the last few years, a great deal of progress has been made in developing realistic FLRW cosmic coasting models which contain matter. *These $R_b = ct$ models have rapidly become the primary competition with the standard model.*

As detailed in Dam, *et al.* (2017), the luminosity distance (d_L) redshift relation in the standard model is given exactly by

$$d_L = \frac{(1+z)c}{H_0 \sqrt{|\Omega_{k0}|}} \left(\text{sinn} \int_{1/(1+z)}^1 \frac{\sqrt{|\Omega_{k0}|} dy}{H(y)} \right),$$

$$H(y) \equiv \sqrt{\Omega_{R0} + \Omega_{M0}y + \Omega_{k0}y^2 + \Omega_{\Lambda0}y^4}, \tag{1}$$

$$\text{sinn}(x) \equiv \begin{cases} \sinh(x), & \Omega_{k0} > 0 \\ x, & \Omega_{k0} = 0, \\ \sin(x), & \Omega_{k0} < 0 \end{cases}$$

In the standard model this equation simplifies to $d_L = (1+z) \left(\frac{c}{H_0} \right)$ for the extended period of cosmic expansion history in which space is essentially flat. Judging from recent Cosmic Microwave Background Radiation (CMBR) observations and current estimates of the Omega cosmic density parameter [19], this period of flat space cosmic expansion appears to have been present since at least the recombination epoch.

As also detailed in Dam, *et al.* (2017), luminosity distance in the realistic Milne-type model with linear expansion (*i.e.*, with $a(t)$ proportional to t) can be compared to luminosity distance in relation (1). Notably, *the realistic Milne-type model can have any matter content, so long as the luminosity distance is exactly*

$$d_L = z \left(1 + \frac{z}{2} \right) \left(\frac{c}{H_0} \right) \tag{2}$$

In the following presentation, the above two mathematical expressions of luminosity distance observed within these two distinctly different cosmological models will be compared and the important implications discussed.

2. Ratio of Standard Model Luminosity Distance to Realistic Milne-Type Model Luminosity Distance

To see how the luminosity distances within both models compare to one another

er, they can be expressed as a ratio in the redshift terms z and s , respectively. These redshift terms are related by $s = z + 1$.

Based upon relations (1) and (2), the ratio of the standard model luminosity distance to the realistic Milne-type model luminosity distance, expressed as a function of redshift term z , is given by

$$\frac{\text{LCDM } d_L}{\text{Milne-type } d_L} = (1+z) / z \left(1 + \left(\frac{z}{2} \right) \right) \quad (3)$$

With substitution of redshift term s for redshift term z , this ratio of the standard model luminosity distance to the realistic Milne-type model luminosity distance is also given by

$$\frac{\text{LCDM } d_L}{\text{Milne-type } d_L} = \frac{2s}{(s^2 - 1)} \quad (4)$$

Using relation (4), one can easily see that the luminosity distances in the two models are equal *only* when quadratic equation $s^2 - 2s - 1 = 0$ is solved for its positive value of $s = (\sqrt{2} + 1)$. Thus, the luminosity distances in the two models are also equal when $z = \sqrt{2}$. At this magnitude of redshift is the luminosity distance point of intersection for these models (See **Figure 1** below).

3. Discussion

Since the recent publications of FLRW linear coasting cosmology models, it has become especially important to compare such modified Milne-type models to the standard model. In this paper, particular attention is paid to how luminosity distances and angular diameter distances compare between these two models. One can easily see from relations (3) and (4) that, *beyond the redshift value at the point of intersection, the linear model luminosity distance value is always greater than the luminosity distance expected within the standard model. It is also evident that the difference between luminosity distances within these two*

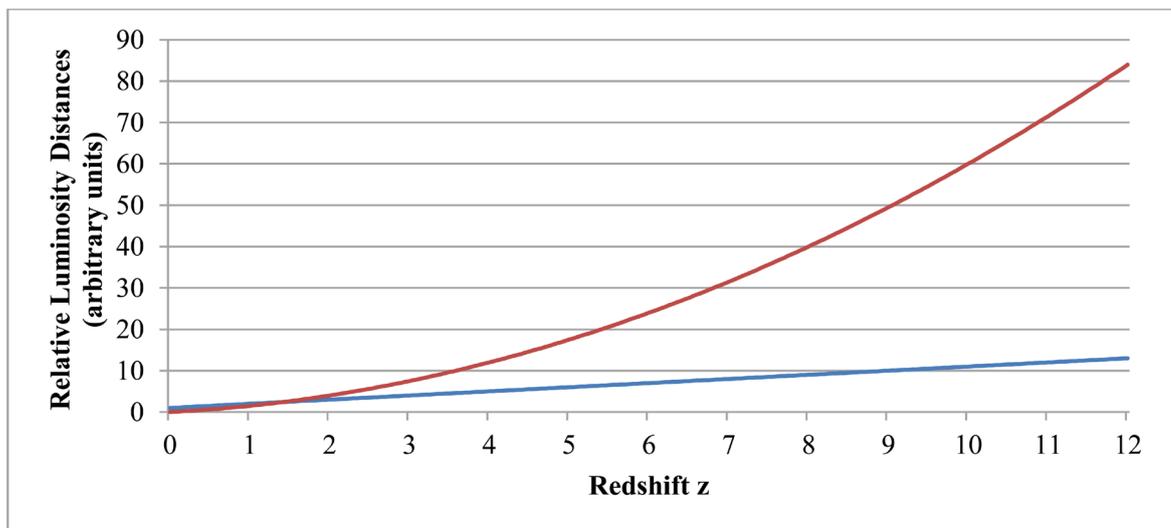


Figure 1. Relative luminosity distances vs. redshift z for standard (blue) and Milne (red) models.

models becomes ever larger as cosmological redshifts increase. The relative luminosity distances (in arbitrary units) are shown in **Figure 1**.

In FLRW models, the following relations (5) and (6) apply for luminosity distance $d_L(z)$ and angular diameter distance $d_A(z)$, respectively:

Luminosity distance:

$$d_L(z) = (1+z)d_M(z) \tag{5}$$

Angular diameter distance:

$$d_A(z) = \frac{d_M(z)}{(1+z)} \tag{6}$$

In these relations, $d_M(z)$ is the transverse co-moving distance which, during the extremely flat ($\Omega_K = 0$) cosmic expansion period extending from at least the recombination epoch to the present, equates to the co-moving distance [*i.e.*, $d_M(z) = d_C(z)$].

From relations (5) and (6) it is obvious that:

$$\frac{d_L(z)}{d_A(z)} = (1+z)^2 = s^2 \tag{7}$$

Therefore, *in a comparison of any two FLRW models, the relative angular diameter distances pertaining to any given cosmological redshift maintain exactly the same relationship (i.e., ratio) as do the relative luminosity distances.* This becomes the basis for **Figure 2** below:

For any FLRW model, linear or otherwise, to compete with the standard model, three major considerations must be addressed:

- 1) What is the capability of expected future observations to discriminate between the two models?
- 2) Given that the Type Ia supernovae data are just one piece of the cosmological data at present, does the competing model fall within other existing observational

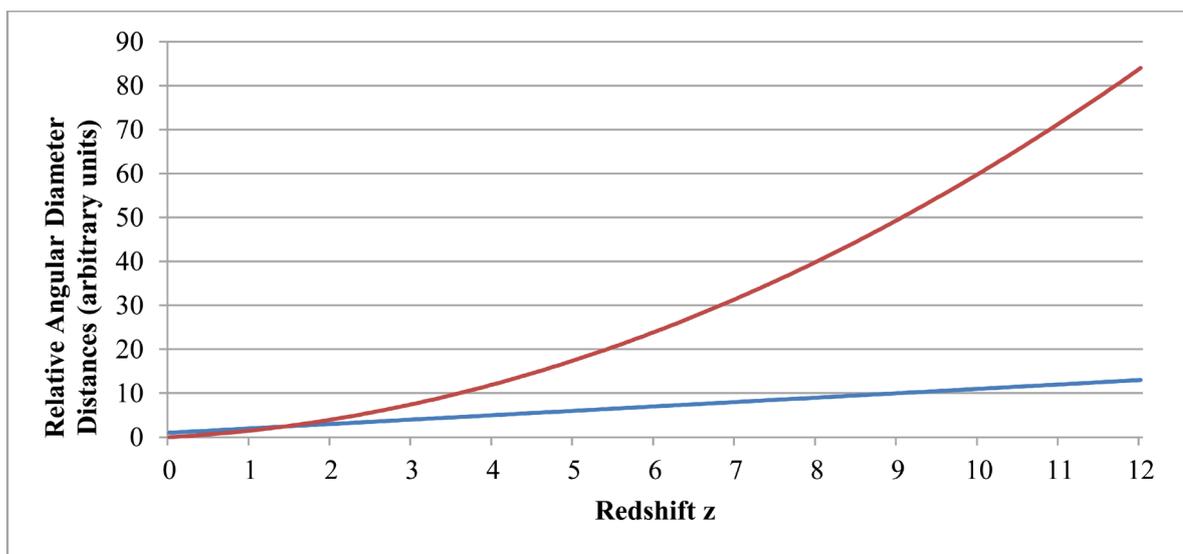


Figure 2. Relative angular diameter distances vs. redshift z for standard (blue) and Milne (red) models.

constraints, including those of baryonic acoustic oscillations (BAO) and the cosmic microwave background (CMB)?

3) Do the Planck constraints on Ω_m possibly fit with the competing model in question?

In realistic linear $R_b = ct$ models these considerations can be addressed as follows:

1) It may not yet be possible to propose a definitive observational study between these two competing models. Nevertheless, the *theoretical* analysis in the present paper shows the importance of not yet being completely dogmatic about one particular model to the exclusion of all others, particularly when the nature of the non-matter energy currently known as “dark energy” is completely mysterious to us. Perhaps, someday, we’ll be left only with Occam’s razor to choose which of the two models makes the most sense to us

2) With respect to the question of observational constraints and realistic linear (*i.e.*, flat) models, the following open source graph from the Supernova Cosmology Project [20] is presented as **Figure 3**:

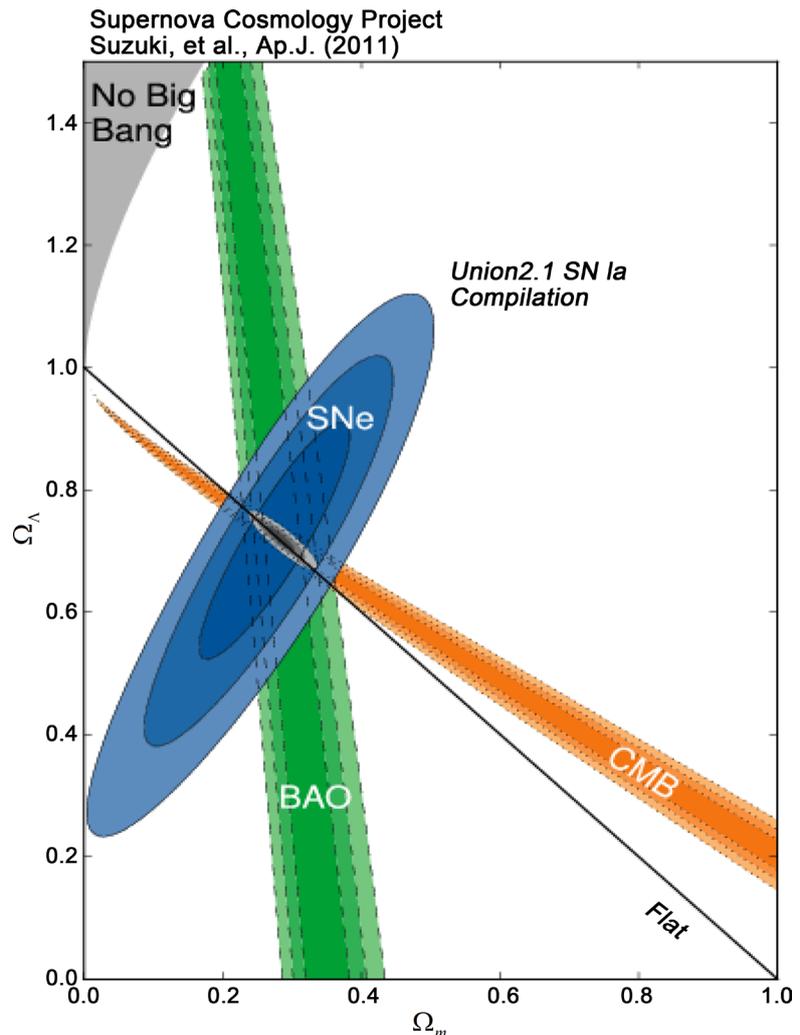


Figure 3. Accumulated Supernova, BAO and CMB Data with highlighted constraints.

It is obvious, judging from the intersections of the “flat” universe model line, that *realistic* linear flat space cosmology models would fit within the highly-constrained zone of intersection, and cannot be dismissed out of hand by the accumulated data so far. This conclusion is also strongly supported by the data analysis of Tutusaus, *et al.* (2017).

3) Given the current constraints on Ω_m presented in **Figure 3**, for a realistic linear $R_h = ct$ model (which is flat and non-accelerating by definition), the non-matter proportion of energy density (Ω_Λ) would likely be latent and stored within the cosmic vacuum, perhaps as virtual particles. This is speculation, of course, at the present time. Within a linear cosmic coasting model there is no *net* force acting upon the cosmic expansion. Obviously, whatever the nature of dark energy is, it requires further explanation within *both* accelerating and non-accelerating FLRW models.

4. Conclusions

The significance of the relative luminosity distance and relative angular diameter distance comparisons between these two competing models is paramount. If an observer of distant Type Ia supernovae expects particular luminosity distances, or angular diameter distances, corresponding to particular redshifts and, instead, sees greater-than-expected luminosity distances (*i.e.*, unexpected “dimming” of the supernovae) or greater-than-expected angular diameter distances, this can easily be misinterpreted by a standard model proponent as indicative of cosmic acceleration. *Entirely predictable supernova luminosity distances within a realistic Milne-type universe containing matter, as opposed to a standard model universe, could thus be one possible explanation for the Type Ia supernovae observations since 1998. Obviously, cosmic acceleration would not then be required to explain these observations.*

An interesting feature of this luminosity distance and angular diameter distance comparison is the point of intersection at $s = (\sqrt{2} + 1)$ and $z = \sqrt{2}$. It is expected that the DES study will ultimately show what would appear to standard model proponents as gradually accelerating cosmic expansion beyond this point of intersection at roughly 6 billion years of cosmic age. However, when the DES study is completed, one must be very careful to still consider the possibility of a linear $R_h = ct$ cosmic expansion presenting the illusion of late cosmic acceleration (as revealed mathematically herein) if the DES observations are interpreted (possibly incorrectly) within the context of standard cosmology.

Acknowledgements

Dr. Tatum thanks Dr. Rudolph Schild of the Harvard Center for Astrophysics for his past support and encouragement. Author Seshavatharam UVS is indebted to professors Brahmashri M. Nagaphani Sarma, Chairman, Shri K.V. Krishna Murthy, founder Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT

(CSIR), Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

References

- [1] Tutusaus, I., *et al.* (2017) *Astronomy & Astrophysics*, **602**, A73. arXiv:1706.05036v1 [astro-ph.CO].
- [2] Dam, L.H., *et al.* (2017) *Monthly Notices of the Royal Astronomical Society*, **472**, 835-851. <https://doi.org/10.1093/mnras/stx1858>
- [3] Nielsen, J.T., *et al.* (2015) *Scientific Reports*, **6**, Article Number: 35596. <https://doi.org/10.1038/srep35596>
- [4] Wei, J.-J., *et al.* (2015) *Astronomical Journal*, **149**, 102.
- [5] Melia, F. (2012) *Astronomical Journal*, **144**, arXiv:1206.6289 [astro-ph.CO]. <https://doi.org/10.1088/0004-6256/144/4/110>
- [6] Gehlaut, S., *et al.* (2002) A “Freely Coasting” Universe. arXiv:astro-ph/0209209v.2.
- [7] Melia, F., *et al.* (2012) *Monthly Notices of the Royal Astronomical Society*, **419**, 2579-2586. arXiv:1109.5189 [astro-ph.CO]. <https://doi.org/10.1111/j.1365-2966.2011.19906.x>
- [8] Melia, F., *et al.* (2013) *Monthly Notices of the Royal Astronomical Society*, **432**, 2669-2675. <https://doi.org/10.1093/mnras/stt596>
- [9] Melia, F. (2015) *Monthly Notices of the Royal Astronomical Society*, **446**, 1191-1194. <https://doi.org/10.1093/mnras/stu2181>
- [10] Melia, F., *et al.* (2016) *Monthly Notices of the Royal Astronomical Society*, **456**, 3422-3431. <https://doi.org/10.1093/mnras/stv2902>
- [11] Tatum, E.T., Seshavatharam, U.V.S. and Lakshminarayana, S. (2015) *International Journal of Astronomy and Astrophysics*, **5**, 116-124. <https://doi.org/10.4236/ijaa.2015.52015>
- [12] Tatum, E.T., Seshavatharam, U.V.S. and Lakshminarayana, S. (2015) *Journal of Applied Physical Science International*, **4**, 18-26.
- [13] Tatum, E.T., Seshavatharam, U.V.S. and Lakshminarayana, S. (2015) *Frontiers of Astronomy, Astrophysics and Cosmology*, **1**, 98-104.
- [14] John, M.V. (2016) *Monthly Notices of the Royal Astronomical Society*, 1-12.
- [15] Milne, E.A. (1935) *Relativity, Gravitation and World-Structure*. The Clarendon Press, Oxford.
- [16] Perlmutter, S., *et al.* (1999) *Astrophysical Journal*, **517**, 565-586. <https://doi.org/10.1086/307221>
- [17] Schmidt, B., *et al.* (1998) *Astrophysical Journal*, **507**, 46-63. <https://doi.org/10.1086/306308>
- [18] Riess, A.G., *et al.* (1998) *Astronomical Journal*, **116**, 1009-1038. <https://doi.org/10.1086/300499>
- [19] Ade, P.A.R., *et al.* (2015) *A&A*, **594**, A13. <http://arxiv.org/abs/1502.01589>
- [20] Suzuki, N., *et al.* (2011) The Hubble Space Telescope Cluster Supernovae Survey: V. Improving the Dark Energy Constraints Above $Z > 1$ and Building an Early-Type-Hosted Supernova Sample.