Journal of Modern Physics, 2018, 9, 1043-1051
http://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Relativistic Paradox Characteristic for Belt Transmission with the Bell-Effect

Vladimir A. Leus

Sobolev Institute of Mathematics, Novosibirsk, Russia
Email: vladalex@liv.ac.uk

How to cite this paper: Leus, V.A. (2018) Relativistic Paradox Characteristic for Belt Transmission with the Bell-Effect. *Journal of Modern Physics*, **9**, 1043-1051.
https://doi.org/10.4236/jmp.2018.95066

Received: March 28, 2018
Accepted: April 25, 2018
Published: April 28, 2018

Abstract

Several contradictions inherent for relativistic dynamics get evident in the case of mechanical systems of cyclic type. In the present paper a version of situation taking place in the moving belt transmission is examined. It is shown that non-Euclidean intrinsic geometry, appearing during acceleration, does not abolish the real paradox in this mechanism. An unavoidable discrepancy between Special and General relativities is established. So, the very existence of wormholes becomes a moot point.

Keywords

Bell Effect, Transmission Paradox, Collapsar, Imbedding Theorem, Redundant Dimensions

1. Introduction

The so called "twin paradox" in special relativity theory (SRT) is a scientific thought experiment, which enjoys widespread popularity due to its subject of discussions about interstellar travel in a human lifetime. In brief summary, the "twin paradox" posits that one of two identical twins stays on Earth while the other travels deep into space, and when the traveling twin is reunited with its sibling he/she is younger. There are many scientific papers dedicated to detailed discussion of niceties of the journey. A relativistic explanation for the discrepancy in ageing results from the unavoidable dynamic asymmetry: the astronaut twin experiences an acceleration-deceleration event while the terrestrial twin remains at all time in the same inertial frame. According to French: "*There is no paradox, and the asymmetrical ageing is real*" ([1], p. 156).

The SRT, which marked its 110-th anniversary in 2015, represents a very elaborated theory. All the internal problems are believed solved. Satisfaction may have been complete if an exception having a century behind does not attract until now interest of researchers. Here is an excerpt from rather recently published article: "*No question perhaps in modern physics has been discussed as much as the* (*in*) *famous twin paradox in Einsteinian relativity. Since it was first mentioned by Einstein and other forefathers*, *it has been the subject of hundreds of papers and continues to this day to supply a continuous flurry of articles*" [2]. The author is referring twenty odd articles concerning the "twin paradox" and exclaims: *"Is it worthwhile* (*re*) *visiting the twin paradox?*" [2]. Although most physicists now enthusiastically accept the SRT, it is not forbidden to re-examine and perhaps revise widely accepted ideas. It is unfortunate but largely true that the scientific community have converted Einstein's achievement into a form of "Holy Grail". However, the great scientist A. Einstein was no infallible superman, but a human therefore not immune for the mistakes we all make. His legacy deserves the same scrutiny as that of anyone else.

The above mentioned 20 odd articles, concerning the "twin paradox", are merely the tip of the iceberg, so it would be relevant to include some more [3] [4] [5] [6]. These have a common trait: anthropomorphic version using acceleration and, therefore, non-inertial reference frames. "*Note that it is only in the last reference that the gedanken experiment* (mental experiment — German) *was formulated in terms of twins*, *a scenario which will meet with an unending success*" [2]. The author implies the book "Space, Time, Matter" by H. Weyl [7] appearing in 1919, fourteen years after the pioneering paper by Einstein [8] was published. As it is clear from the titles of articles, published during more than a half-century period, manned journey remains beyond comparison. Discussions deal with living persons but not about devices measuring time — clocks.

When a commentary focuses on theoretical niceties, it should be discerned what paradoxes, real or imagined, are under consideration. In order to underline absence of any discrepancy modern authors use either inverted commas as in [6], or prefix *pseudo* as in [2]. Despite being popularly named "twin paradox", actually it does not contain any irreconcilable divergence or is controverted to relativistic laws. Distinct from "paradoxes", the real *Triplet paradox* is considered in [9]. It calls into questioning the notion "relativity of simultaneity" — a characteristic feature of the SRT. In this paradox the accelerated motion also plays an essential role that is believed to be thoroughly compatible with inertial reference frames (*IRF*). There is a non-inertial reference frame RF' that is moving with speed being a function of time $V(t)$. Let a time interval $\Delta T = T - t$ elapse between moments t and T as shown by the un-primed clock in *IRF*. The question is: what time interval $\Delta T'$ elapsed between the corresponding moments t' and T' shown by primed clock in RF'?

The un-primed *IRF* with space-time coordinates (x, y, z, t) and the primed IRF' with space-time coordinates (x', y', z', t') are reciprocally connected through the Lorentz transformation [1]. The simplest form of relation that "bridges the gap" between the coordinates take place when each primed axis is parallel to corresponding un-primed axis, and vector-velocity $\boldsymbol{V}=(V,0,0)$ is parallel to the *x*-axis (1). The velocity-coordinate $V>0$, if the primed system is moving in positive direction of the *x*-axis. The primed space-time coordinates are expressed through un-primed by the following equations:

$$x'=\frac{x-Vt}{\sqrt{1-V^2/c^2}}=\gamma(x-Vt),\ y'=y,\ z'=z,\ t'=\frac{t-\frac{V}{c^2}x}{\sqrt{1-V^2/c^2}}=\gamma\left(t-\frac{V}{c^2}x\right). \quad (1)$$

Here c is absolute value of light speed — the common constant for every reference frame, and the symbol γ denotes relativistic factor $\left(1-V^2/c^2\right)^{-1/2}$. The inverse transformations have the form

$$x=\frac{x'+Vt'}{\sqrt{1-V^2/c^2}}=\gamma(x'+Vt'),\ y'=y,\ z'=z,\ t=\frac{t'+\frac{V}{c^2}x'}{\sqrt{1-V^2/c^2}}=\gamma\left(t'+\frac{V}{c^2}x'\right). \quad (2)$$

A clock, positioned at a point (x',y',z') of the primed IRF', shows time t'. Let us find the time interval Δt corresponding to the time elapsed between moments t_1' and t_2' shown by the primed clock.

Using the formulae (2), we have

$$t_1=\gamma_1\left(t_1'+\frac{V}{c^2}x'\right),\ t_2=\gamma_2\left(t_2'+\frac{V}{c^2}x'\right).$$

Whence it follows, $\Delta t=t_2-t_1=\gamma\left(t_2'-t_1'\right)=\gamma\Delta t'$ and

$$\Delta t'=\frac{\Delta t}{\gamma}=\Delta t\sqrt{1-V^2/c^2}, \quad (3)$$

where velocity V is a constant value.

Now we can calculate how much time elapsed on the own clock of the non-inertial RF' between moments S and T of the un-primed time. Let us partition the time interval $[T-S]$ for a set of n equal segments $\Delta t=\frac{T-S}{n}$. During the stretch of time $\Delta t_i=t_{i+1}-t_i$, according to the formula (3), in the non-inertial RF' the stretch of primed time $\Delta t_i'\approx\Delta t\sqrt{1-\frac{V_i^2}{c^2}}$ approximately elapsed. Here V_i is a value of the velocity function $V(t)$ taken for some $t\in\Delta t_i$ in accordance with definition of the Riemannian integral. When the number n tends to infinity, the sum $\sum_1^n\Delta t_i'$ gives us in limit the wanted formula

$$\Delta t'=\int_S^T\sqrt{1-\frac{V^2(t)}{c^2}}\mathrm{d}t. \quad (4)$$

Certainly, this formula is valid for an integrable function $V(t)$.

In a new thought experiment [9] three different observers — triplet — are presented. As distinct from the notorious "twin paradox" the paper suggests a case where two persons are moving in opposite directions in a manner symmetrical to the basic reference frame IRF staying at rest. This consideration also uses two non-inertial frames of reference RF' and RF'' moving with velocities $\boldsymbol{V}$ and $-\boldsymbol{V}$ correspondingly (**Figure 1**). The clocks of both traveling coevals began to go at the same moment S — reading on the un-primed clock of the triplet-homebody. The final meeting of all triplets occurs at a reading T on the clock of the triplet-homebody. According to the formula (4) both readings T' and T'' on the clocks of the first and second travelers correspondingly turn to be equal. Indeed, the velocity coordinate is present in quadric power, and the space-time is isotropic and homogeneous. So, the integrable function is one and the same for both integrals, therefore the results of integration will coincide: $T' = T''$. It is just this circumstance that belies the notion of "relativity of simultaneity".

The contraction of a moving body along the direction of motion is the second feature of the SRT. Just this extraordinary phenomenon constitutes the subject of a logical analysis suggested in the present paper. As a result of the implementation of the relativistic axioms and laws, a true controverting paradox is incontestably revealed.

2. Lorentz-Fitzgerald's Contraction of Moving Bodies

A rigid rod, lying along the x'-axis of its own IRF', is moving with velocity $\boldsymbol{V} = (V, 0, 0)$ relative to a IRF, having coordinates (x, y, z) and taking as being at rest. At any moment t' the ends of the rod have the abscissae x_1' and x_2', where $x_1' < x_2'$, therefore its own length is $l' = x_2' - x_1'$. Measurement of this length is reduced to two events — (x_1', t_1') and (x_2', t_2'). Implementation of the formulae (2) gives

$$x_1 = \gamma(x_1' + Vt_1'), t_1 = \gamma\left(t_1' + \frac{V}{c^2}x_1'\right); x_2 = \gamma(x_2' + Vt_2'), t_2 = \gamma\left(t_2' + \frac{V}{c^2}x_2'\right).$$

A simple subtraction of un-primed abscissae

$$x_2 - x_1 = \Delta x = \gamma(x_2' - x_1' + Vt_2' - Vt_1') = \gamma\left[\Delta x' + V(t_2' - t_1')\right] = \gamma(l' + V\Delta t')$$

does not provide the real length of the rod in the IRF because the positions of both ends were fixed in different moments of time t:

$$t_2 - t_1 = \Delta t = \gamma\left(t_2' - t_1' + \frac{V}{c^2}x_2' - \frac{V}{c^2}x_1'\right) = \gamma\left(\Delta t' + \frac{V}{c^2}\Delta x'\right).$$

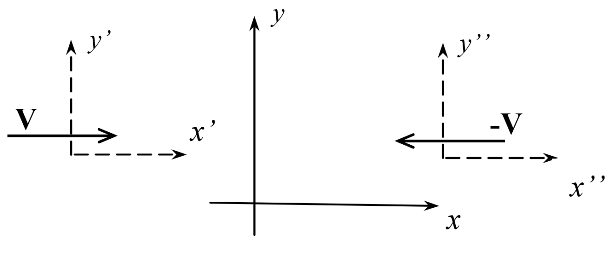

Figure 1. Symmetrically moving IRFs.

In contrast to the own IRF', it is inadmissible here, as soon as during the time interval Δt, elapsed from the first measurement until second one, the rod has been shifting for the distance $\Delta l = V\Delta t$. After this patent correction is introduced we will attain the genuine length of the rod in the un-primed *IRF*:

$$l = \Delta x - \Delta l = \gamma\left(l' + V\Delta t'\right) - \gamma\left(V\Delta t' + \frac{V^2}{c^2}l'\right)$$

$$= \gamma l'\left(1 - \frac{V^2}{c^2}\right) = \left(1 - \frac{V^2}{c^2}\right)^{-\frac{1}{2}}\left(1 - \frac{V^2}{c^2}\right)l' = l'/\gamma .$$

As it can be readily seen, the longitudinal size of a moving body is contracted by relativistic factor γ relative to its longitudinal size measured in the own IRF'. This phenomenon is widely accepted to be named in honour of the Irish physicist George Francis Fitzgerald, even though he attributed his revolutionary guess-work (1892) to a motionless (relative to an Earthed observer) apparatus — the interferometer used in the Michelson-Morley experiments. The addition of the name of Lorentz restores a due terminological balance.

3. Special Relativity Consideration of a New Case with Two Symmetrically Moving Reference Frames

There are two pulleys with parallel axles set on the x'-axis of a laboratory's *IRF*, assumed to be at rest (**Figure 2**). The centre of the left pulley has abscissa $x_1 = -L/2$, the centre of the right one has abscissa $x_2 = L/2$, so that the distance between axels is *L*. The radii of both pulleys are equal to the same value *r*. A flexible drive belt is put on pulleys and sectored for four parts painted into different colours. The rectilinear segment between tangency points A and B is white, the lower rectilinear segment between tangency points C and D is black, and both have the same length value *L*. The right semi-circular segment between tangency points B and C is blue, the left semi-circular segment between tangency points D and A is red, and both have the same length value πr.

Let both pulleys rotate clockwise with a constant angle speed $\mathbf{\Omega}$ (**Figure 3**). The upper part of the drive belt is moving between the output point and the input point in positive *x*-direction with velocity $\boldsymbol{v} = (v, 0, 0)$. The lower part of the drive belt is moving between its "*output*" and "*input*" in negative *x*-direction with velocity $-\boldsymbol{v} = (-v, 0, 0)$. Naturally, the colours of the belt segment between the output and input points are alternating cyclically according to the period of pulleys rotation in the consequence

$$(A, D) \to (D, C) \to (C, B) \to (B, A) \to (A, D) \to \cdots \quad (5)$$

It should be noticed that in the laboratory both pulleys together with adjoining parts of the belt retain Euclidean geometry. On the upper and lower horizontal parts of the belt it is possible to choose IRF' and correspondingly IRF'' in such a way that they are moving symmetrically relative to the laboratory's *IRF*, and their coordinate systems are reciprocally connected through the transformations (1) and (2). Due to the Lorentz-Fitzgerald's contraction at a

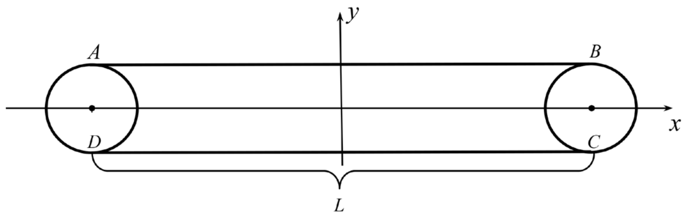

Figure 2. Belt transmission at rest in the laboratory.

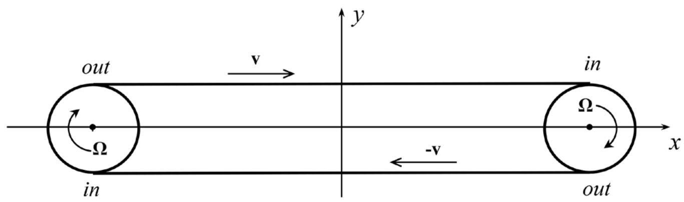

Figure 3. The belt transmission in motion.

sufficiently high speed of rotation the coloured fragments are compressed so much that their number would be increased for an observer being at rest in the IRF. Between the points *out* and *in* several copies of the sequence (5) could be located, and its number depends on the relativistic factor $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$.

4. Belt Transmission Paradox

Here the general relativity comes into operation. If we take into account some other relativistic effect, the above adduced contradiction vanished [10]. From the very inception a still mechanism has to be set in motion. While the transmission is accelerating its internal geometry is undergoing constant change: Euclidean space-time gives up its place to Riemannian one with a negative curvature. The length of the belt's rectilinear fragment increases incessantly.

The point is that the character of current accelerated motion depends on the position of an observer. For example, in the reference frame of railway station a locomotive is uniformly accelerated (a constant acceleration), but in its proper reference frame acceleration rises [11]. The distance between two locomotives, moving in the same direction with the same constant acceleration, does not changes relative to the station, but progressively increases in their proper reference frames. If the locomotives are tied together with rope, it would be broken when sooner or later the elasticity limit of the material is overstepped. That is the essence of so called Bell "paradox".

At a moment, when the maximum rotational speed of the transmission is reached (but elasticity limit is not), any rectilinear fragment of the belt turns to be γ times longer than it was in the stationary position: $L_v = \gamma L$. Just in the same ratio the proper length of the belt is diminished for a stationary observer because of the length contraction, characteristic for the special relativity. Thus, we have a full reciprocal compensation of two effects — Bell's on one hand and

Lorentz-Fitzgerald's on the other hand. Owing to such annihilation the multiplying of belt fragments does not occur, but this does not still mean absence of a real paradox. Let us consider a rectilinear belt part (for instance, lower one with the IRF'') from point of view of the upper part with the IRF'. According to the relativistic rule for velocities addition, the velocity of the IRF'') relative to the stationary IRF' is a vector with absolute value

$$V = \frac{v+v}{1+(v\times v)/c^2} = \frac{2v}{1+v^2/c^2} > v. \tag{6}$$

On the base of (6) the corresponding relativistic factor $\Gamma = \left(1-\frac{V^2}{c^2}\right)^{-1/2}$. The following calculation

$$1-\frac{V^2}{c^2} = 1-\frac{4v^2/c^2}{\left(1+v^2/c^2\right)^2} = \frac{1+2v^2/c^2+v^4/c^4-4v^2/c^2}{\left(1+v^2/c^2\right)^2} = \frac{\left(1-v^2/c^2\right)^2}{\left(1+v^2/c^2\right)^2}$$

gives us an expression for new gamma-factor:

$$\Gamma = \left(1-\frac{V^2}{c^2}\right)^{-1/2} = \left(\frac{\left(1-v^2/c^2\right)^2}{\left(1+v^2/c^2\right)^2}\right)^{-1/2} = \frac{1+v^2/c^2}{1-v^2/c^2}. \tag{7}$$

Using the expression (7), we find the ratio of the new gamma-factor to the previous one:

$$\frac{\Gamma}{\gamma} = G = \frac{1+v^2/c^2}{\left(1-v^2/c^2\right)^{1/2}} = \left(1+v^2/c^2\right)\gamma > \gamma. \tag{8}$$

It is easy to understand that none of reciprocal longitudinal compensation could occur in the case under consideration. As it is evident from (8), the Lorentz-Fitzgerald's contraction (Γ — factor of external geometry) always exceeds the Bell's dilatation (γ — factor of intrinsic geometry). If the speed v increases, approaching light speed, the ratio G grows unlimitedly. Therefore, between points *out* and *in* on the lower rectilinear part still novel and novel coloured fragments appear for the upper observer. In as much as the sequence of fragments one after another is strictly determined, their multiplying is inevitable. Hence, the upper observer must behold himself in duplicate, in triplicate… and virtually in arbitrary number of copies. That is a true paradox, a paradox without inverted commas. Of course, the same phenomenon remains valid for the upper part viewed from the position of the lower observer.

Let, for example, speed $v = \sqrt{0.75}c$. Then $\gamma = 2$ and according to the formula (7) we have:

$$\Gamma = \frac{1+0.75}{1-0.75} = \frac{1.75}{0.25} = 7,$$

where from $G = 3.5$. If the pulley's radius does not exceed the distance between centres divided by π, *i.e.* $r_p \le L/\pi$, then all the coloured fragments in order could be located on the lower rectilinear part of the moving belt (IRF'') visible from the upper part (IRF'). So, the upper observer, while riding the way between points *out* and *in*, would inevitably meet a copy of himself. Together with growth of the velocity v the velocity V also increases according to the formula (6). In consequence of that, duplet, triplet and further multiplets (when v tends to *c*) of the upper observer would appear. As it could be seen, the laws of special relativity and of general relativity enter in an irreconcilable contradiction between themselves. Moreover, this collision is always present irrespective of whether Bell effect is taken into consideration or not.

5. Conclusions

As distinct from many bogus paradoxes criticised in [12] [13], it felt increasingly obvious that the above established true paradox proves the existence of an irreconcilable contradiction between special and general theories of relativity. This may be a consequence of some discrepancies present in the special relativity or/and in the general relativity. It is necessary to handle with care both them. As to a wormhole, this imagery phenomenon is closely tied with a collapsed star, named “black hole”. But the notion of the “black hole” itself is a very poor choice. A singularity of infinite density having no tangible external border cannot be considered as a physical object. The Russian scholarship, founded by academician Anatoly Logunov, proved that “black hole” is an artefact due to violation by the general relativity of fundamental laws — conservation energy, momentum, and moment of momentum. In the consistently developed “Relativistic theory of gravitation” [14] a collapsed star — collapsar — has a definite external border and its density is limited by the Plank one.

Moreover, the mathematical topology requires existence of some redundant dimensions for the coming into being of a wormhole owing to the bending of our physical space-time. For example, according to the so called imbedding theorem, the two-dimensional Klein bottle requires four-dimensional space to be present without self-intersections. Where these necessary for origin of a wormhole additive dimensions could be taken from?

Acknowledgements

The author sincerely thanks Mr Michael Kyle for the valuable assistance rendered during the preparing of this article. He also thanks senior engineer Oleg V. Zajzev at the State Institute “Gradient” (Rostov-Don), for the useful discussion on the topic.

References

[1] French, A.P. (1981) Special Relativity. Thomas Nelson & Sons, Great Britain.

[2] Lévy-Leblond, J.-M. (2015) *European Journal of Physics*, **36**, 1-7. https://doi.org/10.1088/0143-0807/36/6/065023

[3] Lass, H. (1963) *American Journal of Physics*, **31**, 274-276. https://doi.org/10.1119/1.1969430

[4] Perrin, R. (1979) *American Journal of Physics*, **47**, 317-319. https://doi.org/10.1119/1.11835

[5] Good, R.H. (1982) *American Journal of Physics*, **50**, 232-238. https://doi.org/10.1119/1.12862

[6] Müller, T., King, A. and Adis, D. (2008) *American Journal of Physics*, **76**, 360-373. https://doi.org/10.1119/1.2830528

[7] Weyl, H. (1919) Raum, Zeit, Materie. Springer, Berlin. https://doi.org/10.1007/978-3-662-43111-5

[8] Einstein, A. (1905) *Annalen der Physik*, **XVII**, 891-921. https://doi.org/10.1002/andp.19053221004

[9] Leus, V.A. (2015) *American Journal of Modern Physics* (*Special Issue*: *Physics of Time*: *Theory and Experiment)*, **4**, 26-33. https://doi.org/10.11648/j.ajmp.s.2015040201.15

[10] Sokolovsky, U.I. (1964) Teorija Otnositelnosti v elementarnom izlozhenii. [Theory of Relativity in Elementary Narrative.] Publishing House "Science", Moscow. (In Russian)

[11] Ugarov, V.A. (1977) Specialnaja Teorija Otnositelnosti. [Special Theory of Relativity.] Publishing House "Science", Moscow. (In Russian)

[12] Leus, V.A. (2016) *Physical Science International Journal*, **12**, 1-5. https://doi.org/10.9734/PSIJ/216/28188

[13] Leus, V.A. (2018) Critical Comments on the Paper "On the Logical Inconsistency of the Special Theory of Relativity". *Journal of Applied Mathematics and Physics*. https://doi.org/10.4236/jamp.2018.64079

[14] Logunov, A.A. (2001) Relativistskaja Teorija Gravitazii. [Relativistic Theory of Gravitation.] Publishing House "Science", Moscow. (In Russian)