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An Experiment to Improve the Model of Lumped Element Circuit

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Abstract

The model of lumped element circuit ignores the finite time of signals to propagate around a circuit. However, using modern oscilloscope, the time of nanoseconds in a circuit can be measured. Then the speed of alternating electricity can be obtained in a RL circuit. A typical RL circuit is formed by a power source, wire, resistance and inductance. The basic formula is: $U(t) = I(t)R + L\,\mathrm{d}I(t)/\mathrm{d}t$. It can be derived from the Ohm's law and Kirchhoff laws. Based on our experimental results, this paper has discussed the new explanation of this equation in a RL circuit. As a result, the speed of alternating electricity is greater than light in a special RL circuit. The model of lumped element circuit can be improved when considering the finite time of signals.

Keywords

Lumped Element Circuit Model, RL Circuit, Finite Time, Speed of Alternating Electricity

1. Introduction

The differential form of Maxwell equations consists of four equations [1]:

$$\nabla \cdot \boldsymbol{D} = \rho \tag{1a}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{1b}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial B}{\partial t} \tag{1c}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{1d}$$

The constitutive relationships are

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \tag{2a}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{2b}$$

Equation (1a) is the differential form of Coulomb's law. Equation (1b) is the differential form of the modified Ampere's law, where the first term on the right side is the conduction current density. The second term on the right side is the displacement current density, which is the theoretical contribution of Maxwell. Equation (1c) is the differential form of Faraday's law. Equation (1d) indicates that there is no magnetic charge. ε in the formula (2a) is the dielectric constant; μ in (2b) is the permeability coefficient.

By solving the Maxwell equations, the wave equation of the electromagnetic field can be derived. The propagation speed of the electromagnetic wave is the reciprocal of the square root of the dielectric coefficient and the permeability coefficient, which is exactly equal to the speed of light. Thus, Maxwell boldly concluded that light is also an electromagnetic wave. The Maxwell equations are the foundation for the theory of electromagnetic wave, and it opens up a new era of radio communication.

In many electromagnetism textbooks, there is a chapter of circuit theory. Many authors argue that the circuit theory is a simplification of the Maxwell equations at low frequencies [2]-[7]. However, this is a misunderstanding. In a typical RL circuit, the elements are the power source, wire, resistance and inductance. Its basic formula can be derived from the Ohm's law and Kirchhoff laws.

In the classical circuit theory, the model of lumped element circuit ignores the finite time of signals to propagate around a circuit [8]. However, using modern oscilloscope, the time of nanoseconds in a circuit can be measured. Then the speed of alternating electricity can be obtained in a RL circuit. Based on our experimental results, this paper has discussed the new explanation of this equation in a RL circuit. As a result, the model of lumped element circuit can be improved when considering the finite time of signals.

2. A Typical RL Circuit

In 1827, the Ohm's Law was discovered by a German physicist G. S. Ohm. In 1847, the law of current (KCL) and the law of voltage (KVL) are presented by a German physicist G.R. Kirchhoff. The hardware of a circuit is power supply, wire, resistance, inductance, capacitance and other components. The classical circuit theory formula is Ohm's law and Kirchhoff's two laws. The function of the circuit can be summarized as two aspects: First, the power supply of electricity, transmission, distribution and conversion, such as power systems and transmission lines. Secondly, the devices to generate and transmit low-frequency signals are applied, such as telephone, cable television.

The main physical quantities of a circuit are electromotive force, current and voltage. The electromotive force is the physical quantity that represents the nature of the power supply, which can be expressed in $U(t)$, in units of volts (V).

The electromotive force U of the power supply is numerically equal to the power force (non-electrostatic force) to move the unit positive charge from the low electric potential side to the high potential side.

The conduction current $I(t)$ represent the directional motion of the charge. A large number of free electrons in a metal conductor move under the action of an electric field. The charge per unit time through the conductor cross-section is called the current intensity, referred to as current $I(t)$. It can be expressed as: $I(t) = \mathrm{d}q/\mathrm{d}t$, in unit of Amperes (A).

Voltage represents the potential difference across a resistor. It represents the unit positive charge from a point A to point B, the unit is also the volt (V). Figure 1 is a simple RL circuit.

In Figure 1, V_s is the power supply, L_1 is the circuit inductance, R_1 is a resistor. In this circuit the metal wire is connected to the power supply, and the Ohm's law plays an important role due to the presence of non-electrostatic field and the large number of free electrons in the metal wire. Ohm's law is an empirical law. For a electric circuit, if the current is $I(t)$, the resistance R, the voltage across the resistor is:

$$V_R(t) = I(t)R \tag{3}$$

The differential form of Ohm's law can be derived from (3), which is written as:

$$\boldsymbol{J} = \sigma \boldsymbol{E} \tag{4}$$

Here $\boldsymbol{J}$ is the current density vector. Its direction is same as the direction of the electric field $\boldsymbol{E}$; σ is the conductivity, which is characterization of the conductive properties of metal materials.

According to the classical microcosmic theory of current in the metal wire, the free electrons are drifted along the direction of the electric field $\boldsymbol{E}$, and the potential of the free electrons is converted into kinetic energy. The electrons are constantly colliding with the metal lattice during the movement, losing part of the kinetic energy, and then re-accelerating the movement under the action of the electric field, and repeatedly collide with the metal lattice. The average distance traveled by electrons between two collisions is the mean free path. The

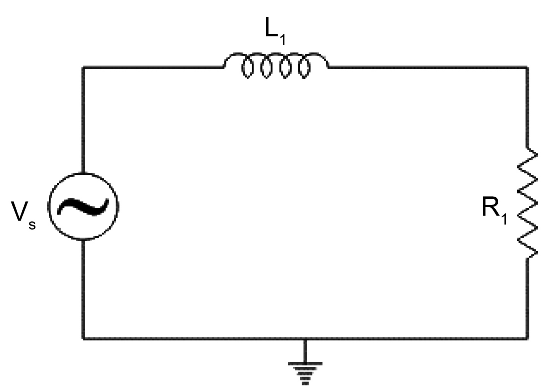

Figure 1. A simple RL circuit.

average drift velocity of electron motion is denoted by u, the free electron density in the conductor is n, the mass of electron is m, and the electron charge is e. τ is the average time interval between collisions. From the classical microcosmic theory, it can be deduced that the relationship between the internal current density and the electric field strength is:

$$\boldsymbol{J} = \left(ne^2\tau/m\right)\boldsymbol{E} \tag{5}$$

Comparing Equations (3) and (4), the conductivity, $\sigma = ne^2\tau/m$ can be obtained. The velocity of the current, *u*, that is the average drift velocity of the electron motion, is very small, less than 1 mm/s.

Kirchhoff's two laws are the basic laws of lumped parameter circuits. (KCL) means: for any node in the lumped parameter circuit, the algebra current flowing into or out of the node at any time are zero. (KVL) means: Algebra sum equal to zero for all branch voltages of one cycle around any circuit at any time in the lumped parameter circuit.

The self-inductance was discovered by American scientist Henry in 1832. He connects the magnetic flux Φ of a circuit with the current: $\Phi = LI$, where L is the self-inductance.

In many electromagnetism textbooks, circuit theory is considered a simplification of the Maxwell equations at low frequencies. It is a misunderstanding. A typical example is a textbook: "Electromagnetic Wave Theory" written by Professor Jin Ou Kong [2]. In this book, there is a topic: 2.1. A circuit theory.

The first sentence of this section is: Circuit laws are limiting cases of the Maxwell equation. Then he wrote: Kirchhoff current law (KCL), which states that the current flowing into a node must equal those that flow out of the node, is a result of the continuity law. When no charge accumulation at a point, partial derivative $\mathrm{d}\rho/\mathrm{d}t = 0$, $\nabla \cdot \boldsymbol{J} = 0$, the integral form is zero. Thus the current flowing into and out of the point must add to zero.

The Kirchhoff voltage law (KVL), which states that the voltage drops along a closed loop must sum up to zero, is a static limit of Faraday's law. Where there is no time varying field lining a closed loop, when we have Curl $\boldsymbol{E} = 0$, or its integral form equals zero. Thus the voltage drops along a closed loop add up to zero.

However, Kong added a note. Note: From the point of view of the Maxwell equations, let the term partial derivative $\mathrm{d}\boldsymbol{B}/\mathrm{d}t = \mu_o\,\mathrm{d}\boldsymbol{H}/\mathrm{d}t = 0$, in free space is mathematically equivalent to letting $\mu_o = 0$. Thus the speed of light can be thought of being infinite in the circuit theory.

Obviously, Kong noted the difference between the loop integral of Faraday's law and the Kirchhoff's voltage law (KVL). He assumed the right side of Faraday's law to be zero, and he called it as it the static limit of Faraday's law. Indeed, it is an inappropriate approximation.

Taking the RL circuit of **Figure 1** as an example, the Kirchhoff voltage law with self-inductance can be written in a form:

$$U(t) = I(t)R + L\,\mathrm{d}I(t)/\mathrm{d}t \tag{6}$$

In Equation (6), $U(t)$ represents the electromotive force of the power supply. The first item on the right side is Ohm's law and the second item on the right side can be derived from Faraday's law.

In fact, in the case of alternating current, Kirchhoff's law of voltage is consistent with the law of conservation of energy. It is a basic equation developed in parallel with the Maxwell equations. Equation (6) is just an example of the Kirchhoff's voltage law (KVL) applied to RL circuit.

3. The Speed of Alternating Electric Field in a RL Circuit

In the last three years, we have experimentally studied the speed of alternating current [9] [10] [11]. In our experiments, we have used two irregular RL circuits. We use L1 and L2, respectively, on behalf of the short wire and long wire self-inductance. $R_1 = R_2 = 1\ \text{M}\Omega$ is the oscilloscope's input impedance of two channels. In the experiment, the operating frequency is 2 MHz, and the entire circuit scale does not exceed 4 meters. The short wire loop and the long wire loop are arranged in different directions. In such a circuit design, the circuit length is less than 5% of the equivalent wavelength *(c/f)*, thus it cannot be calculated by using the "transmission line theory".

In order to calculate the time delay of a RL circuit, we use the Kirchhoff voltage law with inductance, *i.e.* Equation (6).

In our experiments, L_1 and L_2 represent the distributed inductances of short and long wires, respectively, which are not separate inductance elements. That is, the distributed inductance of the conductor is related to the length of the wire. This design is different from the single inductive element shown in **Figure 1**.

The distribution inductance of the straight conductor is calculated by the following approximate formula [6]:

$$L = 2l\left[\ln(2l/r) - 0.75\right]10^{-7} \tag{7}$$

In the Equation (7), l is the wire length, and r is the radius of the wire. The length is taken as a unit of m, and the result unit is Henry. The distribution inductance of a copper conductor with a diameter of 1.0 mm and a length of 0.4 m is 530 nH; the distribution inductance of a copper conductor with a length of 6.4 m is 11.2 μH. Since the distribution inductance of the 1.0 m conductor is approximately 1.0 μH, its impedance is about 12 Ω for a 2 MHz signal; the distribution inductance of the 6.8 m wire is 12 μH and the impedance is about 150 Ω for a 2 MHz signal. The input impedance of the oscilloscope is 1 MΩ, thus whether it is 12 Ω or 150 Ω, they are very small when comparing with relative to 1 MΩ input impedance.

Our experimental data show that when the signal frequency exceeds 3 MHz, the oscilloscope shows a significant displacement of the phase. This phenomenon is likely to be caused by the inductance of the conductor and the capacitance inside the oscilloscope. There are typically 10 - 20 pF capacitors inside the oscilloscope, and the distributed capacitance nearby have some effect on the time delay of the alternating signal. However, in our experiments, we are concerned about the time difference between the two alternating signals, so the effect of the oscilloscope capacitance is cancelled out. In the process of the experiment, we first exchange two oscilloscope channels, indicating that no change in the alternating signal. It indicates that the oscilloscope's two channels are equal.

In the following, we calculate the time delay of the voltage on a resistor relative to the electromotive voltage in a RL circuit. For a RL circuit, the alternating current can be expressed exponentially

$$I(t) = I_o \exp(j\omega t) \tag{8}$$

Here, j represents the imaginary number. Thus, the Kirchhoff voltage law with inductance (6) can be written as:

$$U(t) = I(t)R + j\omega L I(t) \tag{9}$$

Under the condition of $\omega L \ll R$, Equation (4) can be rewritten as

$$U(t) = I(t)R\exp(j\omega t + j\Phi) \tag{10}$$

In Equation (10), Φ represents the phase angle:

$$\Phi = \omega L/R = \omega \Delta t \tag{11}$$

Therefore, in the RL circuit, the voltage on the resistor R comparing to the electromotive force $U(t)$ has a time delay:

$$\Delta t = L/R \tag{12}$$

This time delay is independent of the operating frequency. If the inductance L is 10 μH and the resistor R is 1 M ohms, then Δt is about 10 ps.

According to Equation (7), the inductance of the wire is positively correlated with the total length of the wire. Thus the time delay of Equation (12) is related to the speed of the alternating current. Here we introduce the following working definition: the total length of the loop of the RL circuit is d, which approximates the total length of the wire. We define the alternating electric field velocity in the wire as:

$$V_{AC} = d/\Delta t = dR/L \tag{13}$$

For a wire, the inductance of the unit length of the wire $L_d = L/d = 2\left[\ln(2d/r) - 0.75\right]10^{-7}$, then the speed of alternating electric field is

$$V_{AC} = R/L_d \tag{14}$$

here R is the total resistance of the circuit loop, L_d is the inductance of the unit length.

For a 6-meter copper wire, the inductance per unit length is approximately 1.8 μH. In our experiments, the resistance R is 1 MΩ. According to Equation (14), we get the theoretical value: $V_{AC} \approx 1800c$, here c represents the speed of light in vacuum.

From our measurements, for frequencies below 3.0 MHz, the speed of the longitudinal electric field exceeds the speed of light by more than 20 times, and this result is independent of frequency. This result is consistent with the theoretical value of Equation (14).

The measured time delay is about 500 ps = 0.5 ns. Since this amount has reached the lower limit of the oscilloscope's accuracy, the actual time delay should be 0.5 ± 0.5 ns. For two RL loops, the length difference is 6.0 m and the operating frequency is 2.0 MHz. We define the speed of the alternating electric field is the difference in length divided by the time delay: $v = dx/dt$, the time difference dt is given by the display of the oscilloscope; dx is the length difference of loops. The experimental results show that the speed of signals and electrical energy in the metal wire can be superluminal.

The key to this experiment is: the distributed capacitance in the circuit is less than 1 pF, and it can be ignored. The time delay of the longitudinal alternating electric field in the RL circuit is caused by the inductance of the single wire itself, and the self-inductance of the wire has a certain correlation with the length. Under such circuit conditions, the phase difference of the two loops is equal to the time difference multiplied by the working frequency. For a given frequency, the phase difference is equivalent to the time difference. The experimental results of the following figure show that when the frequency is less than 3 MHz, the time difference between the two loops is less than 1 nanosecond. We have also tested the length difference 9.0 m, the results shown below.

In Figure 2, the red curve represents data with a wire length difference of 9 m, and the blue curve represents data with a wire length difference of 6 m. In Figure 2, two horizontal lines represent the speed of light as reference line. That is, if it is assumed that the alternating electric field in the metal wire moves at the speed of light, it needs 30 ns to passes through a length of 9 m. it needs 20 ns to passes through a length of 6 m. The data above the horizontal reference line is

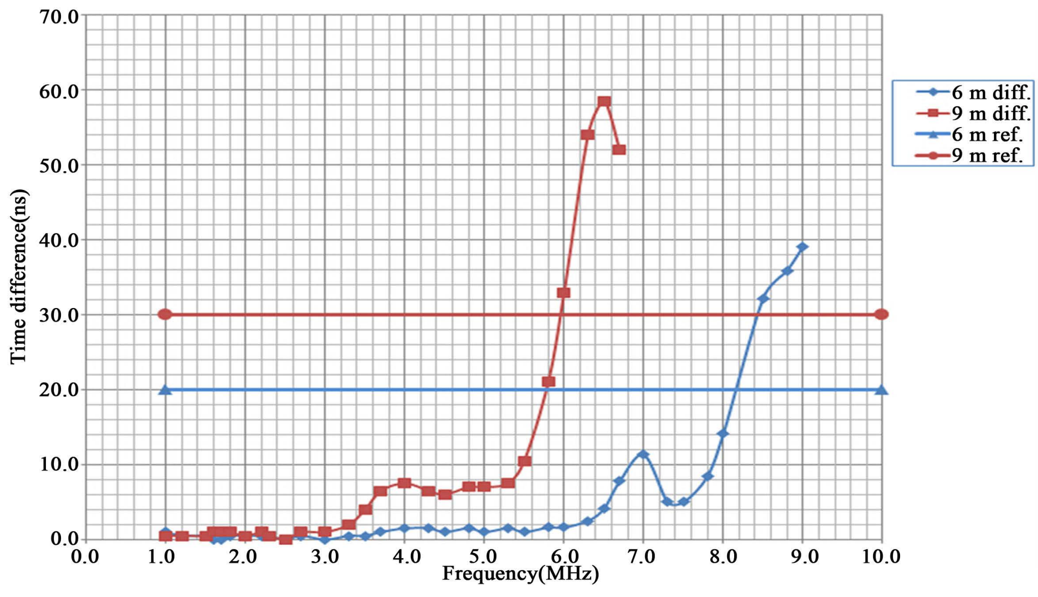

Figure 2. Time difference vs. frequency curve.

the subluminal, and the data below the reference line is the superluminal.

We have replaced the length of the common ground line of the two RL loops, from 0.4 m to 3.0 m, and the time difference displayed by the oscilloscope are not changed. We have also conducted a conditional test in which the two input channels of the oscilloscope are connected in parallel with a 1 kΩ resistor, and we have got $V_{AC} \approx 2c$. This experimental result is consistent with the theoretical value of V_{AC} obtained in Equation (14).

From our many experiments, it is shown that the signals with low-frequency in wires can be greatly superluminal, which is the result of the RL circuit according to Equation (6).

4. The Integrity of Circuit and the Scalar Time

When the operating frequency is 2 MHz, its equivalent wavelength (c/f) is 150 meters, and our experimental circuit length is less than 10 meters. Within the laboratory scale, circuit experts use models of lumped parameter circuit. The model does not calculate the length of the wire. In fact, it implies a hypothesis that the speed of the alternating electrical signal in wires is infinite. Therefore, our experimental results show that under certain conditions, the speed of the alternating electrical signal in the wire is more than 20 times faster than light. It is not only reasonable, but also improves the model of the lumped parameter circuit model.

The alternating electrical signal propagated within conductor is not an electromagnetic wave. It is shown by experiments that the speed of low-frequency electromagnetic waves in the metal conductor is very small, such as about 10 m/s when frequency is 400 Hz [12].

The alternating electric field in the circuit has the characteristics of frequency and phase, but it is not "traveling wave". Since the alternating electric signal in the circuit is not an electromagnetic wave, it is the "longitudinal synchronous oscillation" of the electric field in each part of the wire. In the design of the oscilloscope, it has a periodic scanning function. It shows a curve for voltage vs. time in the oscilloscope display.

From Equation (6), the phase delay of the voltage on the large resistor comparing with the electromotive force can be calculated, which is equivalent to the time delay. This time delay is caused by the distributed inductance of the wire. Therefore, the distribution inductance of the wire is "cause", and the phase delay is "result".

We emphasize that Equation (6): $U(t) = I(t)R + L\,\mathrm{d}I(t)/\mathrm{d}t$, is a scalar equation. Since the power supply $U(t)$ is an independent component of the circuit, it is caused by external forces acting on the charge inside the power supply. In principle, Equation (6) cannot be written in a differential form. In other words, Equation (6) reflects the integral characteristics of the circuit.

In Equation (6), the current $I(t)$ and the time variable t are also scalar. It shows that at any moment, the current $I(t)$ is the same at different locations in the circuit. It reflects the absolute nature of the same time.

Since Equation (6) embodies the integrity of a circuit, Kirchhoff's two laws imply an assumption that the alternating longitudinal electric field is nonlocal. That is, the alternating electromotive force acts the instantaneous transmission of the electric field to each part of the wire and the resistor in the circuit. The magnitude and the phase of the current at each point in the circuit are the same at some point. When time goes, the magnitude and phase of the current in a circuit are changed as a whole. Therefore, in the metal wire, the transmitting signal is a macro non-local effect. "Non-local" superluminal phenomena exist not only in the field of quantum mechanics, and also exist in the macro world.

From the Maxwell equations, the speed of the electromagnetic wave in the free space is constant. However, the electromagnetic waves are transmitted and received by the circuit. The application of electromagnetic waves cannot be separated from the circuit.

However, the starting point of our experiment is the Kirchhoff's voltage equation (KVL) applying to a RL circuit, which contains the Ohm's law. The Ohm's law is an empirical law, which is independent of the Maxwell equations. It does not obey the Lorentz transformation. The conditions in this experiment are: the use of a single wire, reducing the distribution of capacitance, constituting a long and short RL loops. In the experiment, the time delay of the alternating electric field is caused by the inductance of the wires itself. The self-inductance of the wire has a certain positive correlation with the length.

More specifically, in the case of an AC circuit, the alternating electromotive force generated by the power supply directly transfers the energy and signals to the various parts of the circuit. This way of transmitting energy is completely different from electromagnetic waves.

We have measured the time difference of 1 nanosecond in our experiments. Thanks to advances in oscilloscope technology since the beginning of this century. Our contribution is only the choice of the experimental method and the specific conditions of the circuit in the experiment. The error of the time measurement is determined by the accuracy of the oscilloscope. The accuracy of the oscilloscope we used is 0.5 nanoseconds. Thus the measured time difference is 0.5 ± 0.5 ns, for a wire with a length difference of 6.0 m. The measured speed of the alternating electric field in the metal wire is 20 times greater than the speed of light.

Our experimental results are consistent with the classical circuit theory, and there is no contradiction with the Maxwell equations. We emphasize that Equation (6): $U(t) = I(t)R + L\,\mathrm{d}I(t)/\mathrm{d}t$, used in the RL circuit is a scalar equation. In principle, it cannot be written in a differential form. The left side of this equation is the electromotive force of the power supply, which does not belong to the Maxwell equations. The right side of this equation is the Ohm's law and does not belong to the Maxwell equations as well. Only $L\mathrm{d}I/\mathrm{d}t$ is related to Faraday's law.

The time delay of the voltage on the resistor R with respect to the electromotive force $U(t)$: $\Delta t = L/R$, can be deduced by the Kirchhoff's voltage law, Equation (6). Since the initial time value of the electromotive force $U(t)$ is arbitrary in our experiment, we design a long and a short RL circuits. A short RL loop is close to the power supply and the voltage across resistor R_1 is used as a reference for comparison with the voltage on resistor R_2 in the long RL loop. We define the speed of the alternating electric field is the difference in length divided by the time delay: $v = \mathrm{d}x/\mathrm{d}t$. This definition is due to the integrity of the circuit, that is, non-local effect. Although it is different from the definition of localized propagation of electromagnetic waves, these two definitions of speed can be compared experimentally. For example, if the power supply has a distance of 3 meters from the load resistor, then the experimental AC signal delay is 0.5 ns, while the light needs 10 ns. Therefore, the speed of AC signal is 20 times faster than the speed of light.

The model of lumped element circuit ignores the finite time of signals to propagate around a circuit. However, by using modern oscilloscope, the time of nanoseconds in a circuit can be measured. Then the speed of alternating electricity can be obtained in a RL circuit. Based on our experimental results, the model of lumped element circuit can be improved when considering the finite time of signals.

This paper has discussed the new explanation of this equation in a RL circuit. The experimental results support our new explanation. Equation (6) reflects the overall characteristics of the circuit. Among them, the current $I(t)$ and the time variable t are scalar. It shows that at any moment, the current $I(t)$ is the same at different locations in the circuit. It reflects the absolute nature of the scalar time.

We believe that special relativity may not apply to circuits. In the universe, the speed of the longitudinal electric field is not limited by the speed of light in vacuum. Our experiments show that n low-frequency, alternating signals in the metal wire may be superluminal, which is a macroscopic non-local effect [13] [14].

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