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On Radiative Decay of Heavy Vector Mesons $V \to P\gamma$ in the Scalar Strong Interaction Hadron Theory

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Abstract

Radiative decay of heavy ground state vector meson $V \to P\gamma$ is treated semi-classically in the scalar strong interaction hadron theory. The treatment successfully employs the new wave function of the vector meson updated here. The ratio of the available J/ψ and $D^{*\pm}$ decay rates agrees with prediction. The values of the predicted rates are also in order of magnitude agreement with measurements. These agreements are the only ones directly computed from a first principles' theory.

Keywords

Radiative Decay, Vector Meson, Scalar Strong Interaction, Semi-Classical

1. Introduction

Radiative decay of heavy ground state vector meson $V \to P\gamma$ has been treated earlier [1] and Section 6.4 of [2], hereafter denoted by I, in the scalar strong interaction hadron theory [3]. The dimensional approximation used was unsatisfactory and the results disagreed with later data [4]. Further, the meson wave functions used in these earlier works have recently been updated [5]. The purpose of this paper is to provide a revised treatment of the decay $V \to P\gamma$ taking into account these two developments as well as including a new gauge field component ignored earlier.

This paper is divided into two following sections. In Sec. 2, the wave function for pseudoscalar mesons at rest I (4.3.2) is replaced by [5 (8a)] and that of the vector meson I (4.3.3) by a corrected wave function in (1.6) below. The wave functions of heavy, slowly moving pseudoscalar mesons treated in I Section 3.5

are replaced by new ones in §2.3 below. These results are then used in the new approach in Sec. 3 to evaluate the $V \to P\gamma$ rates semi-classically.

2. Meson Wave Functions

The scalar strong interaction hadron theory allows inherently for two types of quark-antiquark interactions, a Coulomb type and a harmonic oscillator type [3, §7], I (3.2.8). When the theory was initially being developed in the early 1990's, data suggested that the confining potential is of Coulomb plus linear type [6] [7]. Since a linear confinement at large quark separations is inherent in the theory [3, §7], I (3.2.19) the Coulomb type of potential was, somewhat indiscriminatingly, chosen.

Such a potential could rather successfully account for ground state meson spectra [8], I Section 5.3-4, but failed to account for the excited meson spectra I Section 5.5-7. This was largely remedied by replacing the Coulomb potential by the harmonic oscillator type [5]. Here, a correction to the ground state vector meson wave function in [5] will be made.

In §2.1, the updated ground state meson wave function is given. The wave equations of a nonrelativistic pseudoscalar meson are given in §2.2 and the wave functions for heavy mesons are derived in §2.3.

2.1. Wave Functions of Ground State Mesons at Rest

For ground state singlet mesons, I (3.4.1) or [5 (4)] with $J = 0$ holds. The updated and normalized pseudoscalar meson wave function using the harmonic type of potential reads [5 (8b) with $J = 0$].

$$\psi_{00}(r) = \frac{1}{\sqrt{\Omega}}\alpha_{00}\exp\left(-\frac{d_h}{2}r^2\right),\ \alpha_{00} = \left(\frac{d_h}{\pi}\right)^{3/4} = 0.0577\ \text{GeV}^{3/2},\ J = 0 \tag{1.1}$$

in which $d_h = 0.07\ \text{GeV}^2$ of [5 (11)] has been used.

For ground state triplet mesons, I (3.4.3) or [5 (4)] with $J = 1$ derived from I (3.2.11a) is incorrect in two signs and is replaced by

$$\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r}\frac{\partial}{\partial r} + \frac{2}{r^2} + d_{m0} - d_h^2 r^2 + \frac{E_{10}^2}{4} - M_m^2\right)\psi_1(r) = 0,\ \psi_1(r) = \chi_1(r) \tag{1.2}$$

The nonlinear potential Φ_{c1} there has been dropped in the linearized (1.2) which, like (1.1), has a harmonic oscillator type of solution [5 (8a)]

$$\psi_{10}(r) = \frac{1}{\sqrt{\Omega}}\exp\left(-\frac{d_h}{2}r^2\right)\sum_{\nu=0}^{n} a_\nu r^{s+\nu},\ d_h > 0,\ a_0 \neq 0,\ a_{odd} = 0,\ n = 2n_r \tag{1.3}$$

where n_r is the radial quantum number. An extra subscript 0 has been added to indicate that ψ_{10} is of zeroth order in later perturbational calculations. Following [5 (9)], the series terminates when

$$a_{n+2} = \frac{2d_h\left(s+n-\frac{1}{2}\right) - \frac{1}{4}\left(E_{10}^2 - \left(m_p + m_r\right)^2\right) - d_{m0}}{(s+n+2)(s+n-1)+2}a_n = 0,\ s = 1 \text{ or } 2 \tag{1.4}$$

These gives the vector meson mass E_{10},

$$\frac{1}{4}E_{10}^2 = \frac{1}{4}\left(m_p + m_r\right)^2 - d_{m0} + 5d_h,\ s = 1,\ n = 2 \tag{1.5}$$

Here, the lowest allowed s and n values have been chosen. The choice $n = 0$ would lead to a mass $E_{10} < E_{00}$, the mass of the corresponding pseudoscalar meson, contrary to data. Therefore, the next lowest value $n = 2$ is chosen. This relation is the same as [5 (10)] with $J = 1$ and $n = 0$ for vector meson so that the spectra found in [5] remain unchanged. The only difference is that the wave function [5 (8b) with $J = 1$] is replaced by the updated, normalized wave function for a vector meson obtained from (1.3)

$$\psi_{10}\left(r\right) = \frac{1}{\sqrt{\Omega}}\alpha_{10} r\left(1 - 2d_h r^2\right)\exp\left(-\frac{d_h}{2}r^2\right),\ \alpha_{10} = 2.44\times10^{-3}\ \text{GeV}^{5/2} \tag{1.6}$$

2.2. Wave Equations for Nonrelativistic Pseudoscalar Mesons

The starting point is I §3.5.1 which introduces the small parameter

$$\varepsilon_0 = \left|\underline{K}\right|/E_{00} \ll 1 \tag{1.7}$$

where $\underline{K}$ is the momentum of the nonrelativistic pseudoscalar meson. Expand the meson energy E_K in I (3.1.6) and the wave functions in I (3.1.7a), suppressing the relative time factor in I (3.1.9), in the form

$$\psi_0\left(\underline{x}\right) = \sum_i \psi_{0i}\left(\underline{x}\right),\ \underline{\psi}\left(\underline{x}\right) = \sum_i \underline{\psi}_i\left(\underline{x}\right),\ \psi \to \chi,\ E_K = \sum_i E_{Ji} \tag{1.8}$$

where i denotes the ith order in ε_0. Only freely moving mesons are considered so that the nonlinear potential Φ_c in I (3.2.8) drops out according to (1.1, 3) where $\Omega \to \infty$. Therefore, Φ_m of I (3.2.8a) is independent of the meson wave functions (1.1, 3).

To zeroth order in ε_0, I (3.5.8) goes over to I (3.2.10b) and

$$\chi_{00}\left(r\right) = -\psi_{00}\left(r\right),\ \ \underline{\chi}_0\left(r\right) = \underline{\psi}_0\left(r\right) = 0,\ \ E_K = E_0 = E_{00} \tag{1.9}$$

To first order in ε_0, I (3.5.8), (3.2.8) with the potentials replaced by those in (1.2) leads to

$$\chi_{01} = 0,\ \ \psi_{01} = 0,\ \ E_{01} = 0 \tag{1.10a}$$

$$\begin{aligned}&\left(E_{00}^2/4 - \Delta\right)\underline{\chi}_1 + 2\underline{\partial}\left(\underline{\partial}\underline{\chi}_1\right) + E_{00}\underline{\partial}\times\underline{\chi}_1 + \left(d_{m0} - d_h^2 r^2 - M_m^2\right)\underline{\psi}_1 \\ &= \frac{1}{2}E_{00}\underline{K}\chi_{00} - \underline{K}\times\underline{\partial}\chi_{00}\end{aligned} \tag{1.10b}$$

$$\begin{aligned}&\left(E_{00}^2/4 - \Delta\right)\underline{\psi}_1 + 2\underline{\partial}\left(\underline{\partial}\underline{\psi}_1\right) - E_{00}\underline{\partial}\times\underline{\psi}_1 + \left(d_{m0} - d_h^2 r^2 - M_m^2\right)\underline{\chi}_1 \\ &= \frac{1}{2}E_{00}\underline{K}\psi_{00} + \underline{K}\times\underline{\partial}\psi_{00}\end{aligned} \tag{1.10c}$$

To second order in ε_0, the singlet part of I (3.5.8) becomes

$$\begin{aligned}&\left(E_{00}^2/4 + \Delta\right)\chi_{02} - \left(d_{m0} - d_h^2 r^2 - M_m^2\right)\psi_{02} \\ &= -\underline{\partial}\left(\underline{K}\times\underline{\chi}_1\right) - \left(\frac{1}{2}E_{00}E_{02} + \frac{1}{4}\underline{K}^2\right)\chi_{00} + \frac{1}{2}E_{00}\underline{K}\,\underline{\chi}_1\end{aligned} \tag{1.11a}$$

$$\left(E_{00}^2/4+\Delta\right)\psi_{02}-\left(d_{m0}-d_h^2r^2-M_m^2\right)\chi_{02}$$
$$=\underline{\partial}\left(\underline{K}\times\underline{\psi}_1\right)-\left(\frac{1}{2}E_{00}E_{02}+\frac{1}{4}\underline{K}^2\right)\psi_{00}+\frac{1}{2}E_{00}\underline{K}\underline{\psi}_1 \tag{1.11b}$$

The spherical symmetry present in the $\varepsilon_0=0$ limit is broken by the momentum $\underline{K}$ so that separation of variables in the relative space $\underline{x}$ cannot be carried out. This renders that each of (1.10, 11) consists of two coupled second order partial differential equations containing eight dependent variables and cannot be readily solved analytically.

2.3. Approximative Heavy Meson Wave Functions

For heavy mesons, the last term is small next to the first term on the right sides of (1.10b, 10c) when

$$E_{00}\gg 2\sqrt{d_h}\approx 0.529\text{ GeV} \tag{1.12}$$

Here, the magnitude of $\underline{\partial}$ in (1.10) is d_hr according to (1.1) where r has been replaced by some mean value $r_0\approx 1/\sqrt{d_h}$. This inequality holds roughly for mesons containing b or c quarks but not for kaon and pion. In these cases, an approximate solution can be found. Let

$$\underline{K}=\left(0,0,K\right) \tag{1.13}$$

The approximation consists of making the simplifying ansatz

$$\underline{\chi}_1\left(\underline{x}\right)=-\underline{\psi}_1\left(\underline{x}\right) \tag{1.14}$$

in (1.10b, 10c). Addition and subtraction of thee two equations yields

$$\left(E_{00}^2/4-\Delta+d_h^2r^2-d_{m0}+M_m^2\right)\underline{\psi}_1=\frac{1}{2}E_{00}\underline{K}\psi_{00}-2\underline{\partial}\left(\underline{\partial}\underline{\psi}_1\right) \tag{1.15a}$$

$$-E_{00}\underline{\partial}\times\underline{\psi}_1=\underline{K}\times\underline{\partial}\psi_{00} \tag{1.15b}$$

The ansatz (1.14) thus leads to that these two equations determine only one unknown $\underline{\psi}_1$; there has to be an inconsistency. Solving (1.15b) with (1.13) gives

$$\underline{\psi}_1\left(\underline{x}\right)=\frac{\underline{K}}{E_{00}}\psi_{00}\left(r\right)=\left(0,0,\psi_{1z0}\left(\underline{x}\right)\right) \tag{1.16}$$

which shows that $\underline{\psi}_1$ is of order ε_0. Inserting this expression into (1.15a) using (1.1) gives

$$\frac{1}{2}E_{00}K\psi_{00}\left(r\right)=\frac{1}{2}E_{00}K\psi_{00}\left(r\right)+2\frac{K}{E_{00}}d_h\left(1-\hat{z}\hat{z}d_hr^2\right)\psi_{00}\left(r\right) \tag{1.17}$$

The last term comes from the last term in (1.15a) and causes that (1.17) cannot be satisfied and is inconsistent with (1.16).

However, the last term in (1.17) will be small for heavy mesons having large E_{00} so that (1.17) and the solution (1.16) both hold approximately. Replacing r by the mean value r_0 above and putting $\hat{z}\hat{z}=1/3$ in anticipation of later angular integration, the criterion is that the ratio between the two terms on the right side of (1.17) be small. This gives another requirement for heavy meson approximation

$$\frac{8d_h}{3E_{00}^2} \approx \frac{0.187}{E_{00}^2} \ll 1 \tag{1.18}$$

This ration is 9.6 for pion, 0.762 for kaon, 0.054 for D, and 0.0067 for B meson. Thus, (1.18) is well satisfied by mesons with b or d quark but again not by kaon or pion. This inequality agrees roughly with that given by (1.12). There is however no great loss here; pions as decay products often move relativistically so that the first order (1.10) no longer holds in the first place.

The last terms in (1.15a) will also introduce a correction to (1.16). As an estimate, let the corrected (1.16) be an average of (1.16) and (1.17) multiplied by $2/E_{00}^2$ and generalized to include the x and y components of the last term in (1.15a);

$$\underline{\psi}_1(\underline{x}) = \left(\psi_{1x1}(\underline{x}), \psi_{1y1}(\underline{x}), \psi_{1z0}(\underline{x}) + \psi_{1z1}(\underline{x})\right) \tag{1.19}$$

$$\begin{aligned} &\psi_{1x1}(\underline{x}) = -\frac{2K}{E_{00}^3}\hat{x}\hat{z}d_h^2r^2\psi_{00}(r), \quad \psi_{1y1}(\underline{x}) = -\frac{2K}{E_{00}^3}\hat{y}\hat{z}d_h^2r^2\psi_{00}(r) \\ &\psi_{1z1}(\underline{x}) = \frac{2K}{E_{00}^3}d_h\left(1-\hat{z}\hat{z}d_hr^2\right)\psi_{00}(r) \end{aligned} \tag{1.20}$$

The second order Equations (1.11) are treated analogously. Subtraction and addition of (1.11a) and (1.11b) leads to

$$\chi_{02}(\underline{x}) = -\psi_{02}(\underline{x}) \tag{1.21a}$$

$$\left(\frac{E_{00}^2}{4} + \Delta - d_h^2r^2 + d_{m0} - M_m^2\right)\psi_{02} = -\left(\frac{E_{00}E_{02}}{2} + \frac{K^2}{4}\right)\psi_{00} + \frac{E_{00}\underline{K}\underline{\psi}_1}{2} \tag{1.21b}$$

$$\underline{\partial}\left(\underline{K} \times \underline{\psi}_1\right) = 0 \tag{1.21c}$$

The last equation is satisfied for $\underline{\psi}_1$ given by (1.16). It is slightly violated by (1.19, 20) but the violations contain odd powers of the angles $\hat{r}$ I (3.1.7b) and will vanish upon integration over the angles later. The left operator of (1.21b) is the same as the linearized ones in I (3.4.1) and vanishes for $\psi_{02} \propto \psi_{00}$; ψ_{02} can be absorbed into the zeroth order ψ_{00} and put to 0. Thus,

$$\frac{1}{2}E_{00}\underline{K}\underline{\psi}_1 = -\left(\frac{1}{2}E_{00}E_{02} + \frac{1}{4}K^2\right)\psi_{00}, \quad \psi_{02} = \chi_{02} = 0 \tag{1.22}$$

Inserting (1.16) into the first of (1.22) leads to

$$E_{02} = K^2/2E_{00} \tag{1.23}$$

3. Radiative Decay of Heavy Vector Meson $V \rightarrow P\gamma$

In this section, the wave functions found in Sec. 2 above are applied to the decay of a heavy ground state vector meson V into the corresponding pseudoscalar meson P and a photon γ. The treatment is semi-classical; the electromagnetic field is not quantized. This is in accord with the quantum mechanical nature of the present theory, which cannot be quantized.

Such decays have been treated earlier [1] and I Section 6.4. The estimated $D^{*\pm}$ decay rate turned out to be too small compared to subsequent measurement. This section follows basically [1] with two main differences. Firstly, the assumption that the U(1) gauge field is limited to the two component photon field is removed. Secondly, the Coulomb form of wave functions of the ground state mesons adopted in [1] and I (4.3.2-3) is replaced by harmonic oscillator form (1.1.6). Consistently, the approximative wave functions of I §3.5.3 are replaced by those in §2.3 above.

Eugene Wigner once said: "Once the equation of motion is known, the rest is engineering". In this sense, the present treatment may be such an "engineering" and assumes no "model", as in the literature [9].

The formalism of I Section 6.4 is largely taken over here.

3.1. Wave Functions of Decaying Meson

For a free meson, the meson Equations I (2.3.22) with $\Phi_c = 0$ in I (3.2.8a) hold. If an electromagnetic field A is introduced on the quark level, I (2.3.22) is replaced by I (6.1.10). The magnitude of the difference between these two sets of equations is small, of the order of quark charges. Therefore, the associated decay $V \to P\gamma$ can be formulated as a first order perturbational problem. The wave function of the decaying meson is taken to be a modified form of I (3.1.5, 6, 9) with I (3, 5.6),

$$\psi^{a\dot{b}}(X,x) = \sum_K b_{JK}\left(\delta^{a\dot{b}}\psi_{JK}(\underline{x}) - \underline{\sigma}^{a\dot{b}}\underline{\psi}_{JK}(\underline{x})\right)\exp\left(-iE_{JK}X^0 + i\underline{K}_J\underline{X}\right),\ \psi \to \chi$$
$$b_{JK} = a_{JK} + a_{JK}^{(1)}\left(X^0\right) \tag{2.1}$$

The subscript $J = 0$, 1 refers to pseudoscalar and vector mesons, respectively. a_{JK} is unity here but is in a quantized case in §3.4 below to be elevated to an annihilation operator annihilating an initial meson of spin J and momentum $\underline{K}$. $a_{JK}^{(1)}\left(X^0\right)$ is a small first order amplitude that varies slowly with time and, in the quantized case in §3.4, becomes an operator that "slowly" transforms the initial vector meson to some intermediate state. It is zero at $X^0 = -\infty$. Similarly, a_{JK}^* enters ψ^*; the complex conjugate of ψ, is also unit and is to be elevated to a creation operator creating a final state with the same J and $\underline{K}$. $a_{JK}^{(1)*}\left(X^0\right)$ is the complex conjugate of $a_{JK}^{(1)}\left(X^0\right)$ and, in the quantized case, becomes an operator that "slowly" creates the same final state as that created by a_{JK}^*. These quantization assignments are phenomenological and rudimentary.

In $V \to P\gamma$, $a_{JK}^{(1)}\left(X^0\right)$ is caused by the qA terms in (2.3) below and is hence of first order in quark charge. Picking out only one J value and one $\underline{K}$ value in the summations of (2.1), it leads to the simplified form

$$\chi^{a\dot{b}}(X,x) = \chi_0^{a\dot{b}} + \chi_1^{a\dot{b}},\quad \chi_1^{a\dot{b}} = \left(a_{JK}^{(1)}\left(X^0\right)/a_{JK}\right)\chi_0^{a\dot{b}} \tag{2.2}$$

where the subscripts 0 and 1 denote orders in quark charge. $\chi_0^{a\dot{b}}$ is simply (2.1) with $a_{JK}^{(1)}\left(X^0\right) = 0$.

3.2. U(1) Gauge Field

The photon is observable and the electromagnetic field is thus introduced on the meson level similar to that in [1 (2.1)] and to the U(1) gauge field in I (6.2.3-4). The equivalent of I (6.1.10a) becomes

$$\begin{aligned}&\left(\partial_I^{a\dot{b}}+i\frac{1}{2}q_pA^{a\dot{b}}\left(X\right)\right)\left(\partial_{II}^{f\dot{e}}-i\frac{1}{2}q_rA^{f\dot{e}}\left(X\right)\right)\chi_{(pr)\dot{b}f}\left(X,x\right)\\&-\left(M_m^2-\Phi_m\left(\underline{x}\right)\right)\psi_{(pr)}^{a\dot{e}}\left(X,x\right)=0\end{aligned}\tag{2.3}$$

where I (3.1.3a) and the transition of I (2.3.23) to (3.1.11) have been used. Here, ∂_I and ∂_{II} operate on χ only. Putting the time component A_0 to 0 and expand $\underline{A}(X)$ in plane waves, as in [1 (3.3)],

$$\underline{A}\left(X\right)=\sum_{K_\gamma}\frac{1}{\sqrt{2E_r\Omega}}\sum_T\underline{e}a_T\left(\underline{K}_\gamma\right)\exp\left(-iE_rX^0+i\underline{K}_r\underline{X}\right)+c.c.\quad \underline{e}=\left(e_X,e_Y,e_Z\right)\tag{2.4}$$

where E_γ and $\underline{K}_\gamma$ are the energy and momentum of the photon and Ω a large normalization box. $a_T(\underline{K})$ is the analog of a_{JK} in (2.1) and is set to a unity here but is elevated to an annihilation operator in the quantized case below.

In [1] and I, the unit vector $\underline{e}=\left(e_T,e_Z=0\right)=\left(e_X,e_Y,e_Z=0\right)$ for $\underline{K}_\gamma=\left(0,0,K_\gamma\right)$ representing the photon field associated with the both transverse modes $T=1, 2$. The assumption is removed here and $e_Z\neq 0$ is allowed here even if it is not part of the photon field.

3.3. First Order Relations

Inserting (2.2) into (2.3), multiplying it by $\chi_{\dot{e}a}^*$ and integrating over X and x, the first order part reads

$$\begin{aligned}&\int d^4Xd^4x\chi_{0\dot{e}a}^*\left[\partial_I^{a\dot{b}}\partial_{II}^{f\dot{e}}\chi_{1\dot{b}f}-\left(M_m^2-\Phi_m\right)\psi_1^{a\dot{e}}+i\frac{1}{2}q_pA^{a\dot{b}}\left(X\right)\partial_{II}^{f\dot{e}}\chi_{0\dot{b}f}\right.\\&\left.-i\frac{1}{2}q_r\partial_I^{a\dot{b}}A^{f\dot{e}}\left(X\right)\chi_{0\dot{b}f}\right]=0\end{aligned}\tag{2.5}$$

Applying I (6.1.3) and the second of (2.2) and noting that $a_{JK}^{(1)}\left(X^0\right)$ can with good approximation be moved to the left of ∂_I or ∂_{II} because it varies slowly over X^0, (2.5) becomes

$$\begin{aligned}&\int d^4Xd^4x\left\{\partial_I^{a\dot{b}}\left(a_{JK}^{(1)}\left(X^0\right)/a_{JK}\right)\chi_{0\dot{e}a}^*\partial_{II}^{f\dot{e}}\chi_{0\dot{b}f}+i\frac{1}{2}q_pA^{a\dot{b}}\left(X\right)\chi_{0\dot{e}a}^*\left(\partial_{II}^{f\dot{e}}\chi_{0\dot{b}f}\right)\right.\\&+i\frac{1}{2}q_r\left(\partial_I^{a\dot{b}}\chi_{0\dot{e}a}^*\right)A^{f\dot{e}}\left(X\right)\chi_{0\dot{b}f}-i\frac{1}{2}q_r\partial_I^{a\dot{b}}\chi_{0\dot{e}a}^*A^{f\dot{e}}\left(X\right)\chi_{0\dot{b}f}\\&\left.-\left(a_{JK}^{(1)}\left(X^0\right)/a_{JK}\right)\left[\left(\partial_I^{a\dot{b}}\chi_{0\dot{e}a}^*\right)\left(\partial_{II}^{f\dot{e}}\chi_{0\dot{b}f}\right)+\left(M_m^2-\Phi_m\right)\chi_{0\dot{e}a}^*\psi_0^{a\dot{e}}\right]\right\}=0\end{aligned}\tag{2.6}$$

The $a_{JK}^{(1)}\left(X^0\right)$ factor multiplying the brackets in (2.6) can with good approximation be moved outside the integral sign because it varies slowly with X^0. Substituting I (6.1.3) once more into (2.6), it is seen that the surface term associated with the first term on the right of I (6.1.3) vanishes upon integration. The remaining terms in the bracket also drop out by virtue of the equation of motion I (2.3.22) for steady state mesons. The next to last terms in (2.6), the $-iq_{IIr}\partial_I$ term, is also a surface term and vanishes after integration, noting that $A(X)$ is real and satisfies periodic boundary conditions at large X^μ as in (2.4). Equation (2.6) can now be written as

$$S'_{mAd} = S'_{mAs} \tag{2.7}$$

$$S'_{mAd} = \int d^4 X d^4 x \left[\partial_I^{a\dot{b}} \left(a_{JK}^{(1)} \left(X^0 \right) / a_{JK} \right) \chi_{0\dot{e}a}^* \partial_{II}^{f\dot{e}} \chi_{0\dot{b}f} \right] \tag{2.8}$$

$$S'_{mAs} = i\int d^4 X d^4 x \left\{ \frac{1}{2} q_p \left[A^{a\dot{b}} \left(X \right) \chi_{0\dot{e}a}^* \left(\partial_{II}^{f\dot{e}} \chi_{0\dot{b}f} \right) \right] - \frac{1}{2} q_r \left[A_{\dot{f}e} \left(X \right) \chi_0^{*b\dot{f}} \left(\partial_{Ia\dot{b}} \chi_0^{e\dot{a}} \right) \right] \right\} \tag{2.9}$$

The last relation has been extracted from (2.5) directly.

3.4. Rudimentary Quantization and Decay Amplitude

The semi-classical treatment mentioned in the beginning of this section is analogous to the treatment of time-dependent problems in quantum mechanics. A justification is that the energies involved here are low so that typical field-theoretical effects such as vacuum polarization and self energy are small.

The following rudimentary quantization procedures are accordingly adopted. Let $|0\rangle$ and $\langle 0|$ denote vacuum states, one has conventionally,

$$\langle f | i \rangle = \langle 0 | i \rangle = \langle f | 0 \rangle = 0, \quad \langle 0 | 0 \rangle = \langle f | f \rangle = 1 \tag{2.10}$$

$$|i\rangle = \left| V \left(\underline{K}_1 = 0 \right) \right\rangle, \quad \langle f | = \left\langle P \left(\underline{K} \right), \gamma_T \left(\underline{K}_\gamma \right) \right| \tag{2.11}$$

where V denotes a vector meson at rest which decays into a pseudoscalar meson P with momentum $\underline{K}$ and photon γ_T with momentum $\underline{K}_\gamma$.

Insert (2.2) into (2.8) and sandwich it between $\langle f |$ and $|i\rangle$. a_{10} in χ operated on the initial state $|i\rangle$ picks out the zeroth order initial vector meson at rest,

$$\chi_{0\dot{b}f} |i\rangle \to \underline{\sigma}_{\dot{b}f} \hat{r} \chi_{10} \left(r \right) \exp \left(-iE_{10} X^0 \right) \tag{2.12}$$

where (2.1) and I (3.2.4a, 5b) have been used and $\chi_{10} = \psi_{10}$ is given in (1.6). Since (2.8) does not contain the final state photon $A(X)$, χ^* operating on the final state $\langle f |$ becomes equivalent to the complex conjugate of the initial state (2.12). Using I (3.1.4, 10a) and (3.5.6), letting the a's considered below (2.1) be elevated to operators according to the interpretations there and integrating over X^0 leads to

$$\langle f | S'_{mAd} | i \rangle = -i \frac{1}{4} E_{10} S_{fi} \int d^3 \underline{X} \int d^4 x \chi_{10}^2 \left(r \right) \tag{2.13}$$

$$S_{fi} = \langle f | a_{JK}^* a_{JK}^{(1)} \left(X^0 \to \infty \right) | i \rangle \tag{2.14}$$

S_{fi} corresponds to the conventional S-matrix element and is interpreted as the decay amplitude via the assignments of the a's below (2.1).

Next, place (2.9) between $\langle f |$ and $|i\rangle$ of (2.11) and elevate a_{JK} and a_{JK}^* to annihilation and creation operators, respectively, so that they are on the same level as $a_T(\underline{K}_\gamma)$ in (2.4). Here, (2.12) is applicable. $a^*_{T'}\left(\underline{K}_\gamma\right)$ in *c.c.* part of (2.4) picks out a final state photon of polarization T with momentum $\underline{K}_\gamma$. The final state meson operator χ^*_0 operating on $\langle f|$ picks out a pseudoscalar meson having a momentum $\underline{K}$ with the wave function

$$\langle f|\chi^*_{0\dot{e}a} \to \left(\delta_{\dot{e}a}\chi^*_{0K}\left(\underline{x}\right)+\underline{\sigma}_{\dot{e}a}\underline{\chi}^*_{0K}\left(\underline{x}\right)\right)\exp\left(iE_{0K}X^0-i\underline{KX}\right) \tag{2.15}$$

where $\underline{K}$ stands for $\underline{K}_J$ in (2.1) for $J = 0$. Insert (2.4, 12, 15) into (2.9), apply I (3.1.4, 10a) and (3.5.6) and integrate over X. The result reads

$$\langle f|S'_{mAs}|i\rangle = \frac{i}{\sqrt{2E_r\Omega}}(2\pi)^4\delta\left(E_{0K}+E_r-E_{10}\right)\delta\left(\underline{K}+\underline{K}_\gamma\right)\int \mathrm{d}x^0 I_q \tag{2.16}$$

$$\begin{aligned}I_q = \int \mathrm{d}^3\underline{x}\Big[&\left(q_p+q_r\right)\left(\underline{e}\underline{\chi}^*_{0K}\left(\underline{x}\right)\left(\partial\hat{r}\chi_{10}\left(r\right)\right)+\frac{1}{2}E_{10}\left(\underline{\chi}^*_{0K}\left(\underline{x}\right)\times\underline{e}\right)\hat{r}\chi_{10}\left(r\right)\right)\\ &-\left(q_p-q_r\right)i\frac{1}{2}E_{10}\underline{e}\hat{r}\chi^*_{0K}\left(\underline{x}\right)\chi_{10}\left(r\right)\Big]\end{aligned} \tag{2.17}$$

in which only one of $T = 1$ or 2 in (2.4) has been included. The χ^*_0 wave functions obey I (3.5.8) which has not been solved, as was indicated in I §3.5.3.

For the heavy mesons containing b or c quark, I Table 5.1 shows that the masses of the vector meson and the associated pseudoscalar meson are close to each other so that the latter moves slowly in a radiative decay of the former. For these mesons, the criteria (1.7, 12, 18) are satisfied and approximative solutions have been given by (1.9, 10a, 14, 16, 19, 20) where χ_{00} stands for χ^*_{0K} and $\underline{\chi}_1$ for $\underline{\chi}^*_{0K}$ in (2.17). Inserting these results together with (1.1, 6) into (2.17) and carrying out the angular integrations, terms with odd powers of $\hat{r}$ vanish, and

$$\begin{aligned}I_q &= \left(q_p+q_r\right)\frac{K}{E_{0K}}e_Z I_{dh}\\ I_{dh} &= 4\pi\alpha_{00}\alpha_{10}\int \mathrm{d}r r^2\left(1+\frac{2d_h}{E^2_{0K}}\left(1-\frac{1}{3}d_h r^2\right)\right)\left(1-7d_h r^2+2d_h^2 r^4\right)\exp\left(-d_h r^2\right)\end{aligned} \tag{2.18}$$

Let the photon be directed along the Z axis consistent with (1.13) and put

$$\underline{K}_\gamma = \left(0,0,K_\gamma\right) = -\underline{K} = \left(0,0,-K\right) \tag{2.19}$$

the $i\underline{K}_\gamma\underline{X}$ term in (2.4) and the $-i\underline{KX}$ term in (2.15) cancel out in (2.9) and the last δ function in (2.16) drops out. Equating (2.13) to (2.16) with (2.18, 19) according to (2.7), the decay amplitude becomes

$$S_{fi} = -\frac{4}{E_{10}}\frac{1}{\sqrt{2E_r\Omega}}2\pi\delta\left(E_{0K}+E_r-E_{10}\right)\left(q_p+q_r\right)e_Z I_{dh} \tag{2.20}$$

in which the integrals over the relative time x^0 and the laboratory space $\underline{X}$ have been cancelled out and I (4.2.8) has been consulted.

3.5. Decay Rate

The decay rate is

$$\Gamma\left(V\to P\gamma\right) = \sum\nolimits_{\text{final states}}\left|S_{fi}\right|^2/T_d = \sum\nolimits_T\sum\nolimits_K\left|S_{fi}\right|^2/T_d \tag{2.21}$$

where T_d is a long time during which decay takes place and $\Sigma_T = 2$ for the both photon polarizations mentioned below (2.4). Further,

$$\sum_K = \frac{\Omega}{(2\pi)^3}\int \mathrm{d}^3\underline{K} \to \frac{\Omega}{(2\pi)^3}4\pi\int \mathrm{d}KK^2 \tag{2.22}$$

With (2.19) and $E_r = K_\gamma$, one finds

$$\delta\left(E_{0K}+E_r-E_{10}\right)=\delta\left(\sqrt{E_{00}^2+K^2}+K-E_{10}\right)=\delta\left(K-K_0\right)\left(1-K_0/E_{10}\right) \tag{2.23a}$$

$$K_0=\left(E_{10}^2-E_{00}^2\right)/2E_{10} \tag{2.23b}$$

This δ function in (2.20) will be squared in (2.21). Using (2.23a), this square becomes

$$\left[\delta\left(E_{0K}+E_r-E_{10}\right)\right]^2=\delta\left(K-K_0\right)\left(1-\frac{K_0}{E_{10}}\right)^2\frac{T_d}{2\pi} \tag{2.24}$$

Combining (2.20-22, 24) leads to the decay rate

$$\Gamma\left(V\to P\gamma\right)=\frac{16}{\pi}\frac{K_0^3}{E_{10}^2\left(E_{00}^2+K_0^2\right)}\left(1-\frac{K_0}{E_{10}}\right)^2\left(q_p+q_r\right)^2 e_Z^2 I_{dh}^2 \tag{2.25}$$

Here, e_Z is not fixed here but is a parameter < 1. This is as far as the present semi-classical treatment can carry. Perhaps a quantized version of the present treatment, if possible to be devised, can fix this e_Z.

e_Z is the third, longitudinal, component of the unit verctor $\underline{e}$ in the photon field (2.4) and balances off the momentum $\underline{K}_0$ of the pseudoscalar meson. Its absence in the earlier treatment [1] and I led to the assumption that higher order effects were responsible for such radiative decays. This is now seen not to the case. e_Z, being separate from the photon associated with the transverse components e_X, e_Y in (2.4) is responsible for the decays.

The first order (2.9) can be complemented to include second order terms containing the square of the photon field $\underline{A}(X)$. The coefficient for this square representing the mass of the photon contains only odd powered $\hat{r}$ terms and vanishes after integration over the relative space $\underline{x}$. This verifies that the photon remains massless.

Finally, it is remarked that (2.25) remains unchanged if the employed equation for χ (2.2) were replaced by an equivalent equation for ψ analogous to I (6.1.10b). This is due to that I (6.1.10a, 10b) originate from I (2.2.4a, 4b) which are invariant under the interchanges $\chi \leftrightarrow \psi$ and I $\leftrightarrow$ II. The last one leads to a sign change of the relative coordinate $\underline{x}$ in I (3.1.3a) but does not affect the decay amplitude (2.20).

3.6. Comparison with Data

Table 1 gives the $V \to P\gamma$ decay rate for some heavy mesons most of them listed in I Table 5.1.

Table 1. Decay rate $\Gamma(V \to P\gamma)$ for some heavy mesons. The meson momentum K_0 is given by (2.23b). The expansion parameter ε_0 is defined in (1.7). q_p and q_r are the quark charges. The predicted decay rate $\Gamma(V \to P\gamma)$ is given by (2.25) which holds only for non-relativistic and heavy pseudoscalar mesons satisfying $e_0 \ll 1$ of (1.7) and (1.12, 18). Only the mesons with b or c quark qualify. Equation (2.25) does not apply to K^*. This is indicated by the parentheses around the so-obtained decay rates. e_Z is the longitudinal polarization vector in the unit vector $\underline{e}$ in (2.4) and should be < 1 but is not fixed in the present nonquantized treatment. The underlined entries are the only ones available for direct comparison with predictions.

	K_0 (GeV)	$\varepsilon_0 = K_0/E_{00}$	$\lvert q_p + q_r \rvert$	$\Gamma(V \to P\gamma)/e_Z^2$ keV	Γ (data) [4] keV
$\Upsilon(1S) \to \gamma\eta_b$	0.07	0.0075	2/3	7×10^{-4}	no data
$B_s^{*0} \to \gamma B_s^0$	0.0459	0.009	2/3	2.2×10^{-3}	dominant
$B^{*0} \to \gamma B^0$	0.0454	0.0086	2/3	1.2×10^{-4}	dominant
$B^{*\pm} \to \gamma B^\pm$	0.0451	0.0086	1/3	3×10^{-5}	dominant
$J/\psi \to \gamma\eta_v$	0.1114	0.037	4/3	$\underline{1.02}$	$\underline{1.58 \pm 0.37}$
$D_s^{*\pm} \to \gamma D_s^\pm$	0.139	0.0706	1/3	0.6	<1776
$D^{*0} \to \gamma D^0$	0.137	0.0735	4/3	11.65	<741
$D^{*\pm} \to \gamma D^\pm$	0.136	0.0726	1/3	$\underline{0.69}$	$\underline{1.33 \pm 0.33}$
$K^{*0} \to \gamma K^0$	0.31	0.623	2/3	(1707)	116
$K^{*\pm} \to \gamma K^\pm$	0.309	0.626	1/3	(436)	50.3

For mesons with b quark, there is no data available. The K^* decay rates are put inside parentheses to indicate that $\Gamma$of (2.25) is not applicable. This is due to thatε_0 is too large and does not satisfy the criteria (1.7) and the kaons are too light so that (1.12, 18) are violated. Further, the effect of the singularity associated with lighter mesons mentioned above §5.7.2 has not been investigated and may impact upon the validity of the approximations leading to (2.25). Still the predictions of (2.25) for kaons are included to indicate the trend that, as the mesons get lighter and move faster; the decay rates increase. Here, they are 10 - 15 times too high.

For mesons containing c quark, J/ψ and $D^{*\pm}$ rates from (2.25) can be compared to data. The ratio between these two measured rates is 1.58/1.33 = 1.19 and can be 0.73 up to 1.95 within error limits. The predicted value 1.02/0.69 = 1.48 lies well within these limits. This agreement can hardly be any coincidence inasmuch as the ratio between their $(q_p + q_r)^2$ values is 16.

Putting $e_Z = 1$, the measured rates are greater than the predicted ones by a factor of 1.5 - 1.9. Since $e_Z < 1$ in the present semi-classical treatment, this factor is actually still greater. Nevertheless, the predicted values and data are of the same magnitude and may be regarded to be in basic agreement with each other. Consider that the mass ratios of K^*, D^* and B^* are 0.44:1:2.86 while the ratios of predicted decay rates are 632:1:4.3 × 10^{-5}, the above discrepancy of a factor of 1.5 - 1.9 or greater is negligible. The approximations introduced in (1.19, 20) can however only lead to a correction of ±10% for D^* in (2.25).

These results indicate that the harmonic oscillator type of meson wave functions (1.1, 6) is useful not only for meson spectra but also for decay problems. They provide further support to the scalar strong interaction hadron theory. No other first principles' theory can make a prediction of this kind.

If the present semi-classical treatment can be quantized and e_Z can be fixed, a wholly new problem, a more precise prediction may be found.

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