

Mach's Principle to Hubble's Law and Light Relativity*

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How to cite this paper: Zhang, T.X. (2018) Mach's Principle to Hubble's Law and Light Relativity. *Journal of Modern Physics*, 9, 433-442.

<https://doi.org/10.4236/jmp.2018.93030>

Received: November 25, 2017

Accepted: February 10, 2018

Published: February 13, 2018

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Abstract

A new redshift-distance relation is derived from Mach's principle with light relativity that describes disturbance of a light on spacetime and influence of the disturbed spacetime on the light inertia or frequency. A moving object or photon, because of its continuously keeping on displacement, disturbs the rest of the entire universe or distorts/curves the spacetime. The distorted or curved spacetime then generates an effective gravitational force to act back on or drag the moving object or photon, so that reduces the object inertia or photon frequency. Considering the disturbance of spacetime by a photon is extremely weak, we have modeled the effective gravitational force to be Newtonian and thus derived the new redshift-distance relation that can not only perfectly explain the redshift-distance measurement of distant type Ia supernovae but also inherently obtain Hubble's law as an approximate at small redshift. Therefore, the result obtained from this study does neither support the acceleration of the universe nor the expansion of the universe but prefers to Einstein's simplest cosmology of the static universe or Zhang's static or dynamic cosmology of the black hole universe.

Keywords

Cosmology, Redshift, Supernovae, Hubble Law, Gravitation

1. Introduction

In the end of 1920s, Hubble [1] discovered that light from all galaxies was red-dened and the redshift of light was linearly proportional to the distance of galaxies. He further interpreted the redshift as the Doppler shift $Z \sim v/c$ and developed Hubble's law as $v = H_0 D$ (or in the redshift form as $Z = H_0 D/c$). Here Z is the redshift of light, v is the recessional velocity of galaxies, c is the speed of

*New redshift-distance relation.

light in the vacuum, H_0 is the Hubble constant, and D is the light traveling distance. According to this interpretation of galactic redshifts, Hubble's law became the fundamental observation for the expansion of the universe and the basic pillar for the big bang origin of the universe. Although Hubble himself was disillusioned with the recession interpretation of the redshift, scientists nowadays have widely accepted that the universe is expanding and had an origin from a non-physical big bang even though that severely violates fundamental laws of physics.

Almost immediately after Hubble's discovery, Zwicky [2] proposed that the reddening effect was not due to motions of the galaxy, but to an unknown phenomenon that caused photons to lose energy as they travel through space. He considered that most likely to be a drag (or tired light) effect in which photons transfer momentum to surrounding masses through gravitational interactions. However, the redshift-distance relation that was obtained by Zwicky from the tired light effect was a linear relation, which is not sufficient to explain the redshift and distance measurements of distant galaxies and type Ia supernovae. There are some other mechanisms that can also produce a distance-proportional redshift, beside the space expansion or Doppler effect from the recessional motions of galaxies [3] [4] [5] [6]. All of these mechanisms can fully address Hubble's law for nearby galaxies, but lack of precisely interpreting further the measurements of distant type Ia supernovae.

The time dilation measured for type Ia supernovae is a direct evidence for the expansion of the universe and thus does not support the tired light effect as the redshift physics [7] [8]. But the extremely redshifted quasars and gamma-ray bursts (GRBs) do not show any time dilations and hence do not support the redshift from the expansion of the universe [9] [10]. Lopez-Corredoira [11] recently pointed out that a possible criticism to the time dilation of supernovae could be the time under the light-curve depending on the intrinsic brightness of supernovae, which would vary significantly the redshift. Recently, Crawford [12] found that the standard templates contained an anomaly in that the width of light curve is proportional to the emitted wavelength. This anomaly exactly removes any time dilation from the original supernova data and thus supports the static universe. The width-luminosity relation was derived using low-redshift type Ia supernovae where the time dilation effect is, if any, negligible [13] [14] [15] [16] [17]. It may not be valid for distant supernovae. Inserting a redshift factor $(1+Z)$, due to the width of light curve to be proportional to the wavelength, into the width-luminosity relation, one can remove the time dilation of type Ia supernovae.

In this paper, we will quantitatively investigate the tired light effect according to Mach's principle and light relativity and derive a new redshift-distance relation, which inherently approximates to Hubble's law for nearby galaxies (*i.e.* $Z \ll 1$) and meantime fully explains the redshift and distance measurements of distant type Ia supernovae. The result obtained from this study,

therefore, does not support the expansion and acceleration of the universe but prefers for Einstein's simplest cosmology of the static universe or Zhang's recently well-developed static and dynamic black hole universe.

2. Mach's Principle and Light Relativity

According to Mach's Principle, first phrased by Albert Einstein in 1918, the inertia of an object results from the gravitational interaction on the object by the rest of entire universe [18]. How much amount of inertia that an object holds depends directly on how much amount of matter there is and how the matter distributes in the universe. For an isotropic and homogeneous universe as the Cosmological Principle describes so, matter uniformly distributes throughout the space and the inertia of an object should thus depend on the actual density (ρ) of matter in the universe.

In principle, any disturbance (e.g. a variation of the density) over there in the universe can alter the inertia of the object at here. A moving object including a photon due to its keeping on displacement would inevitably cause a certain amount of disturbance or distortion, no matter how small it is, to the universe or spacetime. The disturbed universe or distorted spacetime will then act/affect back on the moving object to vary its inertia or total non-gravitational energy (its frequency for a photon). This effect on the inertia can be significant if the speed of the object is not small in comparison to the light and/or if the distance traveled by the object or photon is great. In an empty universe (or at $\rho \rightarrow 0$) with a single object, the object and its motion will be inertia-less.

Six decades ago, Sciamia [19] developed a theoretical model to incorporate Mach's Principle and obtained $GM_{eff}/(c^2R_{eff})=1$, where M_{eff} and R_{eff} are the effective mass and radius in the universe [18] [20], where

$G=6.67\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2$ is the gravitational constant. Four years later, Davidson [21] showed that Einstein's general relativity, a theory that describes the effect of matter on spacetime, is fully consistent with Sciamia's interpretation of Mach's Principle. He further modified the relation of effective mass and radius in the universe with a factor of 2 as

$$\frac{2GM_{eff}}{c^2R_{eff}}=1, \quad (1)$$

from which we can see that the ratio of the effective mass to the effective radius is a big constant $c^2/(2G)\sim 6.7\times 10^{26}\text{ kg/m}$. Matter outside the effective radius does not effectively interact with the object that is placed at the center. Equation (1) is also the relation between mass and radius for the Schwarzschild black hole.

From the measured density of the universe and Equation (1), we can determine the values of the effective mass and radius, respectively, as

$$R_{eff}=c\sqrt{\frac{3}{8\pi G\rho}}=\frac{c}{H_0}\sqrt{\frac{\rho_c}{\rho}}=\frac{c}{\sqrt{\Omega_M}H_0}=\frac{D_H}{\sqrt{\Omega_M}}, \quad (2)$$

$$M_{\text{eff}} = \frac{c^2}{2G} \sqrt{\frac{3}{8\pi G \rho}} = \frac{c^3}{2GH_0} \sqrt{\frac{\rho_c}{\rho}} = \frac{c^3}{2GH_0 \sqrt{\Omega_M}} = \frac{c^2 D_H}{2G \sqrt{\Omega_M}}, \quad (3)$$

where the Hubble constant is currently measured to be

$$H_0 \sim 70 \text{ km/s/Mpc} \sim 2.3 \times 10^{-18} \text{ s}^{-1} \quad [1] [15]-[22];$$

$\rho_c = 3H_0^2 / (8\pi G) \sim 9.2 \times 10^{-27} \text{ kg/m}^3$ is the critical density of the universe; and

$D_H = c/H_0 \sim 1.3 \times 10^{26} \text{ m}$ is the Hubble distance of the universe. According to

measurements [23] [24], the matter density of the universe is $\Omega_M = \rho / \rho_c \sim 0.3$,

i.e. $\rho = \Omega_M \rho_c \sim 2.76 \times 10^{-27} \text{ kg/m}^3$. Then, the effective radius and mass of the

present universe are $R_{\text{eff}} \sim 2.4 \times 10^{26} \text{ m}$ and $M_{\text{eff}} \sim 1.6 \times 10^{53} \text{ kg}$, which are also

the observable universe radius and mass. For $\Omega_M = 1$, *i.e.* without the dark

energy component, we have the effective radius to be the same as the Hubble

distance, $R_{\text{eff}} = D_H$.

Mass, which is usually defined as the amount of matter contained within an

object, can be also interpreted as a measure of inertia of the object. Einstein [25]

in his special relativity, a theory that describes the effect of motion on spacetime,

related the mass of an object to its speed, $m = m_0 / \sqrt{1 - v^2/c^2}$, where m_0 is the

rest mass and v is the speed of the object. An object increases its inertia as it is

accelerated from a work done by a net external force. It should be noted that the

gravitational interaction on the object by the distant matter of the universe is

transmitted through the light speed. Usually, Mach's principle can be shortly

stated as that the mass there affects the inertia here. The mass here depends on

the speed relative to there. Using the mass-speed expression, one can easily

derive the energy E and momentum P of a moving particle as $E = \sqrt{m_0^2 c^4 + P^2 c^2}$

and $P_c = Ev/c$. For light with frequency ν or wavelength λ , the energy and

momentum of a mass-less photon are $E = h\nu$ and $P = h/\lambda$, respectively. Here

h is the Planck constant.

The inertia of an object, usually defined as the tendency of the object to keep

moving in a straight line at a constant velocity, can be expressed in terms of the

total non-gravitational energy contained in the object, which is usually called as

Einstein's energy-mass relation,

$$m = \frac{E}{c^2}, \quad (4)$$

where E is the total non-gravitational energy and m is the total inertia or mass of

the object. For light, thus, we have the inertia of a photon to be proportional to

its frequency,

$$m_\nu = \frac{h\nu}{c^2}. \quad (5)$$

As a photon of light emits, it travels at c with its initial inertia obtained from

the source of emission or the emitter. The matter behind the photon does not

effectively interact with the photon due to unable catching up. The matter in the

front of the photon however is disturbed by the propagation of the photon. The

disturbed matter acts back on the photon and changes the photon inertia (*i.e.* frequency). In other words, light distorts spacetime and the distorted spacetime drags or resists the light. This implies that light should decrease its inertia or frequency (and thus redshift) in the direction of propagation. The more distance it travels, the more redshift it has. Light relativity describes the effect of photon on spacetime and the variation of photon inertia by the affected spacetime.

3. A New Redshift-Distance Relation

As a photon is traveling through, the spacetime is locally disturbed or distorted. The distorted spacetime generates an effective gravitational field, which acts back on or drags the photon (*i.e.* tires the light). Here, we model the effective gravitational field generated by the distorted spacetime in a Newtonian gravitational field of an equivalent sphere as

$$\mathbf{g} = -\frac{GM}{R^2} \hat{r} = -\frac{4\pi G\rho}{3} R \hat{r}, \quad (6)$$

where R is the radius of the equivalent sphere, which is determined as the following expression to the effective radius, and M is the mass within the sphere of this radius,

$$R = \frac{\alpha}{(1+Z)^\beta} R_{eff}, \quad (7)$$

with α and β being two constants that depends on the matter density of the universe and the frequency of light emitted. \hat{r} is the unit vector along the radial direction. The redshift factor $(1+Z)$ is included in Equation (7) because we have considered that as the light is tired or reddened, the disturbance of spacetime by the photon becomes weaker and thus the effective gravitational field generated by the disturbed spacetime becomes weaker.

Due to the gravitational force dragging and work done, the photon decreases its energy as

$$hd\nu = m_\nu \mathbf{g} \cdot d\mathbf{l}, \quad (8)$$

where $d\mathbf{l}$ is the photon displacement element vector. Substituting Equations (6) and (7) into Equation (8), we have

$$hd\nu = -\frac{h\nu}{c^2} \frac{4\pi G\rho}{3} \frac{\alpha R_{eff}}{(1+Z)^\beta} dl, \quad (9)$$

Separating Equation (9) variables and then integrating it, we have

$$\ln \frac{\nu_e}{\nu_o} = -\frac{4\pi G\rho}{3c^2} \frac{\alpha R_{eff}}{(1+Z)^\beta} D, \quad (10)$$

with ν_e and ν_o are the emission and observation frequencies of the light. Then from Equation (10), we can derive a new redshift-distance ($Z-D$) relation as

$$Z = \exp \left[\frac{H_0}{c} \frac{\sqrt{\Omega_M} \alpha}{2(1+Z)^\beta} D \right] - 1. \quad (11)$$

Here, we have used the following relations $\rho = \Omega_M \rho_c$, $\rho_c = 3H_0^2 / (8\pi G)$, $R_{\text{eff}} = D_H \Omega_M$, and $D_H = c/H_0$. It is seen that when $Z \ll 1$ (i.e. for nearby galaxies), we have Hubble's law $Z = H_0 D/c$ at $\alpha = 2/\sqrt{\Omega_M}$, which gives $\alpha = 2$ for $\Omega_M = 1$ and $\alpha = 4$ for $\Omega_M = 0.25$. In the case of $Z \ll 1$, the constant β is insensitive and can be valued around the unity.

To see more quantitatively Hubble's law from Equation (11), we plot in **Figure 1** the redshift as a function of the distance. Here, we have chosen $\alpha = 2/\sqrt{\Omega_M}$ and $\beta = 1.2$. With these values of α and β , we can estimate the magnitude of the effective gravitational field generated by the spacetime that is distorted by a traveling photon is $g = 4\pi G \alpha D_H \rho / \left[3\sqrt{\Omega_M} (1+Z)^\beta \right] \propto \rho$ proportional to the density of the universe and about $g = 6.8 \times 10^{-10} \text{ m/s}^2$ at $Z = 0$. This gives the drag force on a photon with frequency of $6 \times 10^{14} \text{ Hz}$ to be $F_g \sim 3 \times 10^{-45} \text{ N}$. This universal acceleration may also be applicable to any moving object. Then, Newton's third law of motion could be revised as: "Without a force acting on, an object at rest remains at rest and an object in motion with speed v decelerates at a rate of about 1 nm/s^2 and stops after passed about v billion seconds or traveled about $v^2/2$ billion meters in average."

The redshift-distance relation, Equation (11), with the same values of α and β chosen above for the plot of Hubble's law, also perfectly explains the redshift and distance measurements of distant type Ia supernovae [26] [27]. **Figure 2** plots the redshift-distance relation (red line) according to Equation (11) along with the type Ia supernova measurements (blue dots), which are credited from the Union 2.1 compilation of 580 SNeIA data from Supernova Cosmology Project [28] [29]. In this plot the Hubble constant is chosen to be $H_0 = 70 \text{ km/s/Mpc}$. Here, the vertical axis is the distance modulus, defined by $\mu = 5 \log_{10} D - 5$ with D in parsecs and is plotted as a function of redshift. It is seen that the newly derived redshift-distance relation (Equation (11)) is perfectly consistent with the redshift and distance measurements of distant type Ia supernovae.

Therefore, in a static universe, not only does the redshift-distance relation, newly obtained from Mach's Principle and light relativity, derive Hubble's law but it also explains the measurements of distant type Ia supernovae. This paper provides an alternative but simple and complete solution to the mysteries of cosmological redshift and dark energy. The result completely rules out dark energy for the acceleration of the universe as well as utterly excludes the expansion of the universe, so that this work strongly implies the big bang to be unnecessary for the origin of the universe and robustly supports Einstein's simplest cosmology of the static universe. The author strongly believes that the physics must be simple and the cosmology must be physical. At present, however, scientists have made the cosmology too complicated with too many (or an increasing number of) unphysical assumptions or hypothetical entities that can never be physically examined and validated. To new observations of the universe, they

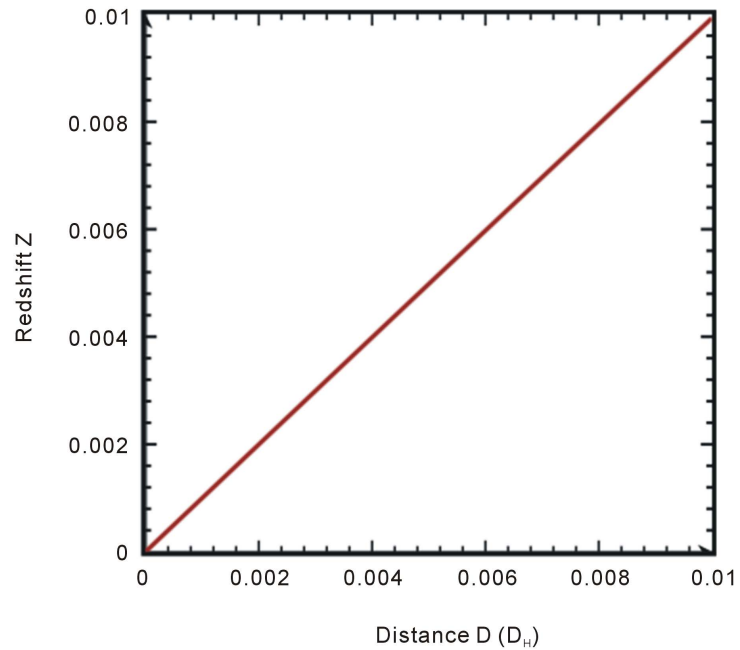


Figure 1. The new redshift-distance relation is plotted for Hubble's law. In the case of a small distance in comparison to the Hubble distance, $D \ll D_H$ (*i.e.* $Z \ll 1$), which is defined by the speed of light in the free space dividing by the Hubble constant, $D_H = c/H_0$, we have Hubble's law, $Z = D/D_H$. It is seen that the new relation perfectly models Hubble's law.

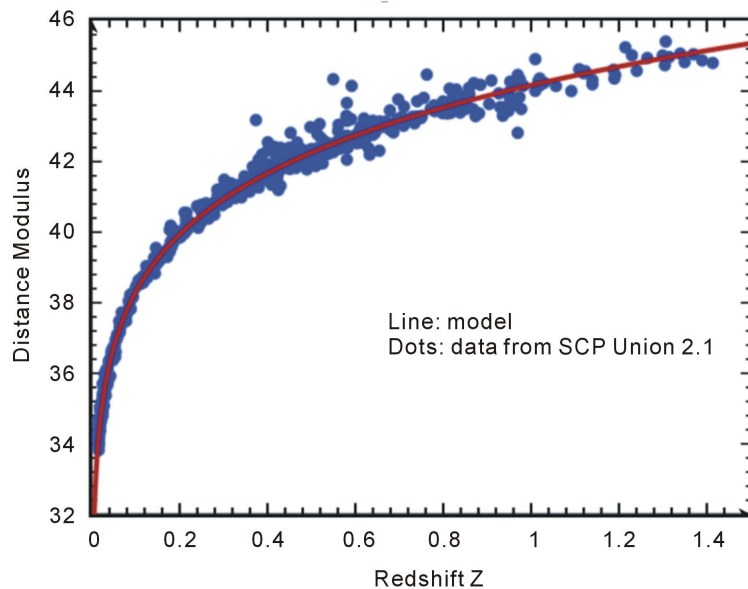


Figure 2. The new redshift-distance relation is plotted for explaining the measurements of type Ia supernovae. The blue dots are the measurements of type Ia supernovae, which are credited by the Union 2.1 compilation of 580 SNeIA data from Supernova Cosmology Project [28] [29]. The red line is the analytical result obtained from this study, *i.e.* the new redshift-distance relation Equation (11), and plots the distance modulus as a function of the redshift. It is seen that the new redshift-distance relation perfectly models the measured data of type Ia supernovae.

have been usually not to dig out the physics behind the observations, instead of blindly and unphysically assuming dark this and that. Dark cannot be the only reason for one to be not able to see. It is the time to turn our thinking and study of the universe in the way of physics that describes the universe simply and effectively, rather than empirically and hypothetically.

Recently, by slightly modifying the standard big bang theory via postulating the spacetime black hole equivalence, this author [30] [31] developed a new cosmological model called black hole universe, which is consistent with Mach's principle, governed by Einstein's general relativity, and able to explain all existing observations of the universe without encountering difficulties such as the flatness, horizon, inflation, dark matter, and dark energy. With this new cosmological model, my colleagues and I have addressed various aspects of the universe including the origin, structure, evolution, expansion, acceleration, cosmic microwave background radiation, gamma ray burst, quasar emissions, and X-ray flares from galactic centers [32] [33] [34] [35] [36]. The black hole universe is usually static with spacetime equilibrium, but becomes dynamic and expands only when it accretes ambient matter or merges with other black holes. The black hole model of the universe does not exclude any tired light mechanism for the galactic redshifts. The tired light effect plus the Doppler effect from the expansion of the universe due to periodically accreting or merging with others may be able to account for the measurement of redshift quantization [37].

4. Discussions and Conclusions

The discovery of a linear redshift-distance relation (e.g. Hubble's law) for nearby galaxies in the end of 1920s has instigated scientists to widely accept expansion of the universe, originated from a non-physical big bang around 13.8 billion years ago. The finding of the redshift-distance relation to be weaker than linear for distant type Ia supernovae nearly two decades ago has further precipitated scientists to largely agree the acceleration of the universe, driven by the mysterious dark energy. The direct observational evidence for the expansion of the universe is the measurement of the time dilation for type Ia supernovae. However, an anomaly was recently found in the standard templates for the width of light curve to be proportional to the emitted wavelength. This anomaly exactly removed the supernova time dilation, and thus is meant to the recessional motions of galaxies or the space expansion to be not the cause of the redshifts. In addition, quasar and gamma ray bursts that do not show any similar time dilations also imply other causes for the redshifts.

We have derived a new redshift-distance relation from Mach's principle with light relativity. A moving object or photon, because of its continuously keeping on displacement, disturbs the rest of the entire universe or distorts/curves the spacetime. The distorted or curved spacetime then generates an effective gravitational force to act back on or drag the moving object or photon, so that reduces the object inertia or photon frequency. Considering the disturbance of spacetime

by a photon is extremely weak, we have modelled the effective gravitational force to be Newtonian and derived the new redshift-distance relation that have not only perfectly explained the redshift-distance measurement of distant type Ia supernovae but also inherently obtained Hubble's law as an approximate at small redshift. The result obtained from this study does not support the expansion and acceleration of the universe, but prefers to Einstein's simplest cosmology of the static universe or my recently weill-developed static or dynamic cosmology of the black hole universe.

Acknowledgements

This work was partially supported by NSF/REU (Grant #: PHY-1263253 and PHY-1559870) at Alabama A&M University.

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