

SU(3) × SU(2) × U(1) Symmetries of Lattice System and Correspondence to Quantum Field Theory

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Abstract

Three theorems on critical fluctuations and new concepts of isomorph spin, hyperspin, quablocks, and pioblocks are proposed, these new quantities correspond to the same symmetries although they have distinguishable values in different lattice systems. The self-similar transformations take place not only in the ordblocks, but also in the disblocks. In the quablocks sense, the critical fluctuation is analogous to the asymptotic freedom of the quark theory, and the reciprocal transformation between the ordblocks and disblocks is like the transfer between protons and neutrons. There are three transition temperatures, different temperature regions correspond to different symmetries and gauge invariances, leading to that there is no unified quantum theory.

Keywords

Critical Fluctuation, Symmetry, Gauge, Isomorph Spin, Hyperspin

1. Introduction

The basic idea of lattice quantum chromodynamics was introduced by K. Wilson in 1974 [1], based on large-scale Monte-Carlo numerical calculation. The lattice field theory is very similar to the continuous phase transition of the lattice system: Many physical quantities such as thermal capacity and susceptibility will diverge at the critical temperature, which is equivalent to the ultraviolet divergence in quantum field theory. This means the continuum limit of quantum field theory and the critical phenomena are different language descriptions of the same process. In addition to the numerical calculation, we think symmetry analysis and gauge invariance exploration are still elementary theoretical research, which will make the lattice model closer to the quantum field theory. In

turn, the quantum field theory will help us uncover the mysterious nature and details of the continuous phase transition.

In reference [2], we pointed out the fractional side length n^* of the ordered block may destroy the system symmetry, and forces the system adjust the side length to compose of blocks with identical integer side to conserve the symmetry, and the system tries constantly to reach up to the critical point. It's possible that there are two ways for the adjustment: 1) the old blocks break into lattices, and the lattices then construct new blocks; 2) the old blocks transfer directly into another type of spin structures. The second path is reasonable in that the lattices correlation length at the critical temperature is much longer than the lattice constant, and doesn't allow the blocks to disperse to lattices. The critical opalescence reveals that the density fluctuations become of a size comparable to the incident light wavelength, the light is scattered and causes the normally transparent sample to appear cloudy, which is a structure of the size larger than the molecular [3].

The critical ups and downs are not disorganized, there are symmetries and gauge invariance. They govern the fluctuation process and determine the rule of the structure adjustment. What forms will the critical fluctuation be? What symmetries will follow? In this paper we will investigate the two questions. In Section 2, we put forward three theorems on the critical fluctuations, then two new concepts, isomorph spin and hyperspin, are proposed. In Section 3, we discuss SU(3) symmetries including octet and decuplet states, introduce quablocks and pioblocks, find out abnormal symmetries and three transition temperatures by analysis on symmetries. We predict the influence of these transition temperatures on the quantum field theory. Section 4 is conclusion remark.

2. Theory

2.1. Three Theorems on Critical Fluctuations

Let f be the statistical distribution function of the block spins for a system containing only various of block spins at temperature T little higher than the order-disorder transition temperature. Because the mean value of the block spin $\langle S \rangle$ is zero before the transition:

$$\langle S \rangle = \int_{-\infty}^{+\infty} S f dS = 0 \quad (1)$$

While the relevant mean square of the spin $\langle S^2 \rangle$ is

$$\langle S^2 \rangle = \int_{-\infty}^{+\infty} S^2 f dS = 1 \quad (2)$$

The formula will hold only if we take proper adjustment of the function coefficient. Probability theory says that Gaussian distribution has maximum Boltzmann entropy among those functions according with Equations ((1) and (2)). Moreover, Jaynes pointed out that the best approximating function for the unknown density distribution should be that has the maximum Boltzmann entropy, since it's not prejudiced [4] [5]. Using the Gaussian distribution function,

we describe the Ising model, and the high accurate datum of the critical point verify it is the best approximation [2]. It's the Gaussian function that proves the minimum block spin S_{\min} relates to the critical point.

Theorem 1. The fixed-point equation of the side length n of the blocks derived from the minimum fractal dimension D_{\min} of the blocks gives a unique solution n^* .

Proof: From reference [2], the fractal dimension D of a block is defined as

$$D = [\ln N(n)] / \ln n \tag{3}$$

where $N(n)$ is the total number of lattices inside the block. Considering temporarily n as a continuous variable, the derivative of D with respect to n is zero when D takes the minimum,

$$[dD/dn]_{n=n^*} = 0 \tag{4}$$

Equation (4) results in a fixed-point equation for n

$$n = F(n) = N(n) \cdot D / [dN(n)/dn] \tag{5}$$

Equation (5) has a unique solution n^* by the contraction principle [6]. In the configuration space sense, the critical fluctuation is the fluctuation of the lattice correlation lengths. We expand D as a series of n around n^* , ignoring the terms larger than the second power,

$$D = D(n) = D_{\min} + [dD/dn]_{n=n^*} (n - n^*) + [d^2D/dn^2]_{n=n^*} (n - n^*)^2 \tag{6}$$

where $[dD/dn]_{n=n^*} = 0$, and $[d^2D/dn^2]_{n=n^*} > 0$ because of D_{\min} , we get

$$D(n) = D_{\min} + C \cdot (n - n^*)^2 \tag{7}$$

where $C = [d^2D/dn^2]_{n=n^*}$ is a positive constant. The $D(n)$ is an opening-upward parabolic function with D_{\min} , the uniqueness of n^* determines the uniqueness of D_{\min} , which indicates $D(n)$ is a monotonic function if $n > n^*$. On the other hand, it cannot be a self-similar transformation for the fractional side length n^* , which will force the system to adjust the sides to be integers. Hence, there is another theorem:

Theorem 2. The critical fluctuation corresponds to a vibration of integers n about n^* .

The self-similar transformation permits only one kind of block's integer side length for the ordered state, resulting in the following theorem:

Theorem 3. The critical fluctuation around n^* takes place in the transfer from the disordered state to the ordered state between n_+ -blocks and n_- -blocks.

Where the n_+ and the n_- are integer numbers nearest neighbor to n^* , and $n_+ > n^*$, $n_- < n^*$, their blocks are n_+ -blocks and n_- -blocks, respectively.

2.2. Isomorph Spin I_f

In the figures of this paper the n_- -blocks are colored by grey, the n_+ -blocks, white. **Figure 1** and **Figure 2** illustrate the n_+ -blocks and the n_- -blocks turn out spon-

taneously. In the tetrahedron lattice system a n_- -block has four same triangle faces: the first face, for example in **Figure 2**, is colored by grey, the second is adjacent to one face of its nearest n_+ -block, each of the other two faces is respectively oriented to one face of its two nearest neighbor n_- -blocks. The numerical calculation indicates there is no lattice outside block. Both types of blocks are the production of the lattices correlation, they should have fractal dimensions. For the triangle lattice system: $n^* = 14.4955$, $n_- = 14$, $n_+ = 15$, and their fractal dimensions: $D_- = 1.81409160$, $D_+ = 1.81409299$, and the difference is $D_+ - D_- = 0.00000139 \approx 0$. For the tetrahedron lattice: $n^* = 8.7272$, $n_- = 8$, $D_- = 2.45544074$, $n_+ = 9$, $D_+ = 2.45474569$, and $D_- - D_+ = 0.00069505$. In the **Figure 3** and **Figure 4** of reference [2], there are only n_+ -blocks. In **Figure 3** and **Figure 4** of this paper, the sub-blocks of the n_- -blocks lie inside or between the sub-blocks of the n_+ -blocks, sharing partly the carrying space of the n_+ -blocks. This case is like the protons and neutrons inside the nucleus. Below, for simplicity, we call the sub-block as block, and differentiate them unless we have special needs. For the two systems, the side lengths must take odd integers

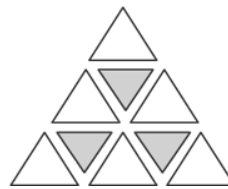


Figure 1. The triangle lattices.

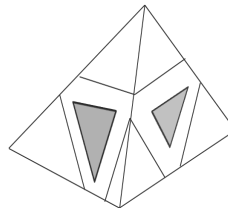


Figure 2. The tetrahedron lattices.

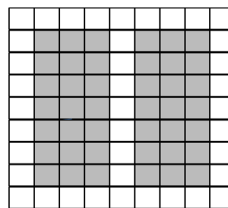


Figure 3. The plane square lattices.

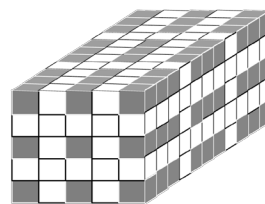


Figure 4. The simply cubic lattices.

because of their symmetries. For the plane square lattice system, $n^* = 7.8400$, $n_- = 7$, $n_+ = 9$, $D_- = 1.7810$, $D_+ = 1.7804$, and $D_- - D_+ = 0.006$. For the simply cubic lattice system, we see that two n_+ -blocks stand in line, each of them has side length $n_+ = 5$, and contains four sub-blocks. Twenty seven other sub-blocks of grey color with $n_- = 3$ belong to the n_- -blocks, each of which has four sub-blocks: $n^* = 4.7491$, $D_- = 2.5237$, $D_+ = 2.4785$, and $D_- - D_+ = 0.0452$. These results show that the fractal dimensions difference is so small that the n_+ -block and the n_- -block can be interpreted as two states of a sort of spin for the same block, the new spin is called isomorph spin, denoted by I_f , and its third component I_{f3} is

$$I_{f3} = -I_f, -I_f + 1, \dots, I_f - 1, I_f \tag{8}$$

Obviously, the isomorph spin is only meaningful to the blocks. Like the isospin space, the isomorph spin space is abstract. A disordered block, simply written as disblock, is a non-zero vector in the isomorph spin space although it is supposed to an empty set in the spin space. The ordered blocks, simply as ordblocks, and disblocks keep the original symmetries, respectively. For the n_+ -blocks and the n_- -blocks, only one type of them can become the ordblocks, another are the disblocks, otherwise the original symmetries will be destroyed. The isomorph spin is analogous to the isospin, they have the same symmetries: SU(3) and SU(2) groups and Lie algebras.

For an ordblock, $I_f = 1/2$, $I_{f3} = 1/2$, its wave function $|ord\rangle$ of the isomorph spin is

$$|ord\rangle \equiv \chi_{1/2,1/2} = |1/2, 1/2\rangle \tag{9}$$

For a disblock, $I_f = 1/2$, $I_{f3} = -1/2$, its wave function $|dis\rangle$ is

$$|dis\rangle \equiv \chi_{1/2,-1/2} = |1/2, -1/2\rangle \tag{10}$$

Equations ((9) and (10)) show the smallest non-trivial multiple of SU(2). For example in **Figure 3**, the wave function of an ordered structure composed of one n_+ -block, labeled by “1”, and one n_- -block, labeled by “2”, can be built by the Equations ((9) and (10)) as the following two forms:

$$|I_f = 0, I_{f3} = 0\rangle = [\chi_{1/2,1/2}(1)\chi_{1/2,-1/2}(2) - \chi_{1/2,-1/2}(1)\chi_{1/2,1/2}(2)]/\sqrt{2} \tag{11}$$

This is the singlet state of the isomorph spin, another is one of the triplet state:

$$|I_f = 1, I_{f3} = 0\rangle = [\chi_{1/2,1/2}(1)\chi_{1/2,-1/2}(2) + \chi_{1/2,-1/2}(1)\chi_{1/2,1/2}(2)]/\sqrt{2} \tag{12}$$

2.3. Hyperspin Y_f

There are only two energy states for the coupling of two isolated lattice spins: excitation state out of their spins antiparallel, and basic state from the spin parallel. This case is available to the statistical calculation. Another situation should be much accounted of when we analyse symmetry: A lattice in a system has Z (coordinate number) nearest neighbors, the number of the coupling energy states for this lattice is more than two, *i.e.* the complexity of the coupling energies

makes the lattice spin have much more excitation states. More states are equivalent to that a lattice spin has multiple spin states from the smallest S_{\min} to the largest spin S_{\max} , not just the singlet one. Generally, $S_{\min} + S_{\max} \neq 0$, and the spin multiple is not necessarily located symmetrically around the origin of the spin axis. Hence, the center of spin may differ from zero. A new concept, hyperspin Y_f , is introduced:

$$Y_f = S_{\min} + S_{\max} \tag{13}$$

Apparently, the spin symmetry center is $Y_f/2$, its unit is S_b determined by the Equations ((6.1) and (6.2)) of reference [2], so does I_{f3} . By the similarity to the isospin, the block spin S is expressed as

$$S = Y_f/2 + I_{f3} \tag{14}$$

For these new variables, different lattice systems give different numerical values.

3. Discussion

3.1. Octet and Decuplet

The isomorph spin and the hyperspin manifest themselves $SU(2)$ symmetries, respectively, and form $SU(3)$ symmetry together. **Figure 5** and **Figure 6** are the octet and the decuplet of the blocks spin states. In **Figure 5**, the label A represents ordblock states, the B , disblock states. A_+ , A_- and B_+ , B_- are the

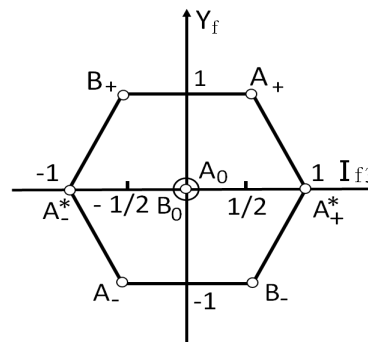


Figure 5. The octet of states.

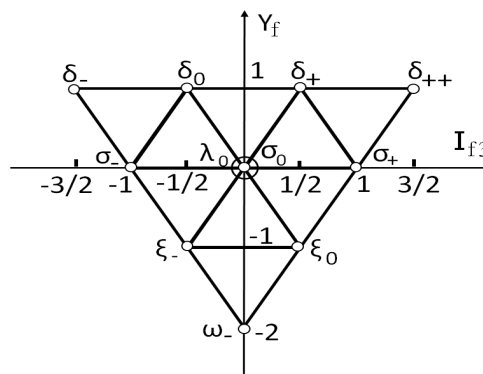


Figure 6. The decuplet of states.

basic states for them. A_0 and B_0 are their intermediate states when they transfer from A_+ and B_+ to A_- and B_- , and A_+^* and A_-^* are the excitation states for the ordblocks.

There are more possible excitation states for them in **Figure 6**. Not every lattice system has all of these states, the number of the states for a system depends on its individual structure.

3.2. Quablocks and Pioblocks

Considering the resemblance of the isomorph spin to the isospin [7] [8] [9], we introduce quablocks q_u, q_d, q_s , and $\bar{q}_u, \bar{q}_d, \bar{q}_s$, being respectively congruent relationship to the quarks u, d, s , and $\bar{u}, \bar{d}, \bar{s}$. The quablocks states are expressed by $|Y_f, I_{f3}, S\rangle$, consequently, $|q_u\rangle = |1/3, 1/2, 2/3\rangle$, $|q_d\rangle = |1/3, -1/2, -1/3\rangle$, $|q_s\rangle = |-2/3, 0, -1/3\rangle$, $|\bar{q}_u\rangle = |-1/3, -1/2, -2/3\rangle$, $|\bar{q}_d\rangle = |-1/3, 1/2, 1/3\rangle$, $|\bar{q}_s\rangle = |2/3, 0, 1/3\rangle$. The quablocks exist only in the isomorph spin space since there is no set scale inside the blocks in the configuration space, where we are unable to find the occurrence of the quablocks. In the isomorph spin space these quablocks can produce pioblocks P^-, P^0 , and P^+ , which states are given by

$$|P^-\rangle = |q_d \bar{q}_u\rangle, |P^0\rangle = |q_u \bar{q}_u - q_d \bar{q}_d\rangle / \sqrt{2}, |P^+\rangle = |q_u \bar{q}_d\rangle \quad (15)$$

A possible transfer between the ordblocks and disblocks is represented as

$$P^- + A_+ \Leftrightarrow P^0 + B_+ \quad (16)$$

where $|A_+\rangle = |q_u q_u q_d\rangle$, $|B_+\rangle = |q_u q_d q_d\rangle$. The blocks and pioblocks are viewed as the bound states of the quablocks, and the three pioblocks as an isomorph spin triplet ($I_f = 1, I_{f3} = -1, 0, 1$), or spin triplet ($S = -S_b, 0, -S_b$).

3.3. Correspondence to Quantum Field Theory

The regular distribution of the disblocks cannot be explained if the interaction is emphasized only for the ordblocks. There must be a non-spin coupling between the disblocks, along with a non-spin interaction among the ordblocks and disblocks. The non-spin interaction implies the isomorph spin symmetries and gauge invariance. It's well known the so-called spin coupling constant is an exchange integral [10], generally, it is related to an electric potential for the ferromagnetic phase transition, the non-spin interaction is actually the nearest electromagnetic interaction. However, in the time-lattice model [11], the potential is only associated with the strong interaction independent of the spins. The choice of the ordblock state for the first block compels all the other blocks of identical size to take in an order relative to this state, the attempt to disturb this order creates the elastic waves, showing U(1) symmetry. In like manner, there are elastic waves and U(1) symmetry in another kind to retain disordered. The critical point and the critical temperature are two things, although the former appears only at the later. The critical point is a specific value (n^*) that a quantity (n) approaches but never reaches. Such behavior is analogous to the asymptotic freedom

of the quark theory [12], signifying there exists non-Abelian gauge. We may interpret this phenomenon in the sense of quablocks: At first, a non-spin attraction among the quablocks make the n_+ -blocks (or n_- -blocks) ordered to shorten the distance between the system state and the critical point, resulting in a strong non-spin repulsion among the quablocks. Then the repulsion coerces the quablocks to make the n_+ -blocks (or n_- -blocks) disordered such that the quablocks non-spin attraction becomes stronger again, making the n_- -blocks (or n_+ -blocks) ordered and the system approach closely to the critical point once more. The closer to the critical point the system is, the stronger the non-spin repulsion will be, and the weaker the attraction will become, and vice versa.

The self-similar transformations for the ordblocks and disblocks take place on the limit hierarchies. After infinity iteration the whole system becomes an ordblock containing some disordered regions, which are the disblocks on the former hierarchy. On the limit hierarchy the system always conserves the SU(3) symmetry, which breaks only on the infinite hierarchy. In the point of view of symmetries, we find three transfer temperatures. The first is T_{cf} , linking to the order-disorder transition, the local lattice spins symmetry breaks to form the n_+ -blocks and the n_- -blocks on the limit hierarchies to set up the SU(3) symmetry, evolving the isomorph spin symmetry and the hyperspin symmetry. There are elementary excitation with U(1) symmetry [13], while the lattice correlation length is limited. The block side length changes into longer and longer with the temperature decreasing after the first transition. There are n -blocks with side length n , acting as steady ordblocks, and n' -blocks with side length n' nearest to n , $n' < n$, acting as steady disblocks, without the order-disorder transfer between them. Hence, the SU(3) symmetry still holds until the second transition temperature T_{cs} . The lattice correlation length is of infinity at T_{cs} , and the whole system represents for an ordblock, and the disblocks disappear, then the local SU(3) symmetries of the blocks are broken. When $T < T_{cs}$, the lattices hyperspin symmetry holds locally until the third transition temperature T_{ct} , *i.e.* Bose-Einstein transition one, $0 < T_{ct} < T_{cs}$, the local lattices hyperspin symmetry is broken.

A crucial key is that all phase transitions are mutation processes, which cannot be described by any dynamic equation. Different temperature regions correspond to different symmetries and different gauge invariances. This states that there is no unified field theory to discuss about the particles properties, each characteristic region should have its special equation. This scene provides theoretical basis for Tomonaga's "Principle of renunciation": One should give up the hope that the theory is perfect and that everything can be calculated [14].

If the lattices have only charges, but no spins, we may discuss the charge order phase transition [15].

3.4. Abnormal Symmetries

It's said the higher the temperature rises, the more chaotic the system is, and the

more number of symmetries will be. However, the additional symmetries such as the U(1) out of the elementary excitation and the SU(3) of the isomorph spin and hyperspin only appear in the region $T_{cs} < T \leq T_{cf}$ rather than in the higher region $T_{cf} < T < +\infty$. The abnormal symmetries are associated with the scaling. For particles, it may occur in the same region, where the T_{cf} is concerned in the time order-disorder transition, the T_{cs} , in the electro-weak transition [12]. This implies those particles with the isospin symmetries may be born in the region.

4. Conclusion

There exist symmetries and gauge invariances in the lattice system, which are as the same as in the quantum field theory. The analysis on the symmetries helps us understand deeply the critical phenomena, which, in turn, makes us realize there is no unified theory for the quantum field.

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