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CMB—A Geometric, Lorentz Invariant Model in Non-Expanding Lobachevskian Universe with a Black Body Spectral Distribution Function

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Abstract

In the present paper, based on Lobachevskian (hyperbolic) static geometry, we present (as an alternative to the existing Big Bang model of *CMB*) a geometric model of *CMB* in a Lobachevskian static universe as a homogeneous space of horospheres. It is shown that from the point of view of physics, a horosphere is an electromagnetic wavefront in Lobachevskian space. The presented model of *CMB* is an Lorentz invariant object, possesses observable properties of isotropy and homogeneity for all observers scattered across the Lobachevskian universe, and has a black body spectrum. The Lorentz invariance of *CMB* implies a mathematical equation for cosmological redshift for all *z*. The *global* picture of *CMB*, described solely in terms of the Lorentz group— $SL(2C)$, is an infinite union of double sided quotient spaces (double fibration of the Lorentz group) taken over all parabolic stabilizers $P \subset SL(2C)$. The *local* picture of *CMB* (as seen by us from Earth) is a Grassmannian space of an infinite union all horospheres containing origin $o \in L^3$, equivalent to a projective plane RP^2. The space of electromagnetic wavefronts has a natural identification with the boundary at infinity (an absolute) of Lobachevskian universe. In this way, it is possible to regard the *CMB* as a reference at infinity (an absolute reference) and consequently to define an absolute motion and absolute rest with respect to *CMB*, viewed as an infinitely remote reference.

Keywords

CMB, Cosmological Redshift, Non-Expanding Universe,
Hyperbolic Geometry, Horospherical Electromagnetic Waves, Hubble's Error

1. Introduction

In the present paper, we give an alternative model of *CMB* in a non-expanding Lobachevskian (hyperbolic) $3D$ real space (universe) and we define absolute motion and absolute rest in accordance with the developed geometric model of *CMB*.[1]

We do not accept the "official" interpretation of the origin of the *CMB* as a consequence of the so-called Big Bang origin of the universe, since we do not accept the Big Bang "event" in the first place. In [1], we already gave a complete mathematical model of cosmological redshift in a static Lobachevskian universe, with all its measurable properties. We also explicitly showed why and at which point Edwin Hubble misinterpreted his observations, resulting in an illusion of apparent space inflation.

Nevertheless, we have to address existing experimental observations and give them a rational description in terms of existing mathematics and physics, not referring to any supernatural occurrences of the Big Bang, and that is the goal of the present paper.

This paper may be seen as a continuation of a series of our articles on electromagnetic phenomena in Lobachevskian spaces [2] [3] [4] [5].

At the same time, this paper is the first and the only one in the scientific literature which gives an alternative to the Big Bang, namely a mathematical, Lorentz invariant, model of *CMB* with all its experimentally detected properties including homogeneity, isotropy, and black body spectrum, which the Big Bang hypothesis of *CMB* failed to derive.

CMB and cosmological redshift are presented as "indisputable" proof of a Big Bang occurrence. We hope that alternative mathematically based models for both phenomena will contribute to a revision and dismissal of the Big Bang hypothesis in years to come and to the development of cosmology based on rational foundations.

We give definitions for each introduced notion. The figures we supply along the text to visualize the mathematics are *faithful representations* of Lobachevskian geometry in a Poincare $2D$ ball model.

There are several models of Lobachevskian geometry [6] [7] [8]. In the text we use, without special notification, models of Lobachevskian geometry which are the most convenient for particular task.

Lobachevskian geometry has wide range of application in physics. The best known one is the physical interpretation of Lobachevskian velocities space, with Gaussian curvature $K = -c^{-2}$, c being the speed of the light in a vacuum, known as Special Relativity (*SR*).

2. Some Homogeneous Spaces Related to SL(2C)—The Lorentz Group

Our primary object of discussion is Lobachevskian geometry and its application

[1]Nikolai Ivanovich Lobachevski, 1792-1856, a Russian mathematician of Polish ancestry. He put hyperbolic geometry into its final form as a self-contained geometrical system. William Clifford called him the "Copernicus of Geometry". For more info on Lobachevski, his life and work, go to: https://en.wikipedia.org/wiki/Nikolay_Lobachevsky.

to the universe around us. Lobachevskian space L^3 is a locally compact, globally non-compact, simply connected, metric space of constant negative Gaussian curvature $K < 0$. For convenience, it is assumed that $K = -1$ (in the case of Lobachevskian velocities space that is equivalent to a choice of physical units in which $c = 1$).

It is known [9] [10] that a $3D$ Lobachevskian geometry has its group of motions (group of isometries) being the group $SL(2C)$, *i.e.* the Lorentz group. Therefore it is quite natural that we start with the group $SL(2C)$ since it is a well understood and well established mathematical structure.

In this chapter we follow closely Gelfand, Grayev, Vilenkin [7], and for details, we refer the reader to that work.

With a Lorentz group one can relate several homogeneous spaces on which the Lorentz group acts transitively. In general, those homogeneous spaces are constructed from $SL(2C)$ by picking up some podgroup $H \subset SL(2C)$ called a stabilizer and taking the quotient $SL(2C)/H$. Depending on what we want to stabilize we get different homogeneous spaces $X = SL(2C)/H$.

1) Thus, taking as origin in Lobachevskian space, the unity 2×2 matrix e we see that its stabilizer is: $e = g^{*}eg = g^{*}g$ which means that $g \in SU(2)$ is in the form $\begin{Vmatrix} a & b \\ -\bar{b} & \bar{a} \end{Vmatrix}, |a|^2 + |b|^2 = 1$. The resulting homogeneous space $X =$ $SL(2C)/SL(2) = L^3$ is a Lobachevskian real $3D$ (hyperbolic) space L^3 which we call the space at *vicinity*. Points in $L^3 = SL(2C)/SU(2)$ are represented by 2×2 Hermitian positive definite matrices with determinant +1. A point $x \in L^3$ having homogeneous coordinates x_0, x_1, x_2, x_3, corresponds to a positive definite Hermitian matrix h with determinant +1, $h = \begin{Vmatrix} x_0 - x_3 & x_2 - ix_1 \\ x_2 + ix_2 & x_0 + x_3 \end{Vmatrix}$ $\in SL(2C)/SU(2)$, $\det h = 1 = x_0^2 - x_1^2 - x_2^2 - x_3^2$. Motions (isometries) in matrix representation are: $h' = g^{*}hg$ [2].

2) Taking a stabilizer as the subgroup $P \subset SL(2C)$, of matrices $\begin{Vmatrix} \mathrm{e}^{-i\varphi} & 0 \\ z & \mathrm{e}^{i\varphi} \end{Vmatrix}$ we end with $X = SL(2C)/P$, which is the boundary at infinity $L^3(\infty)$ of Lobachevskian space L^3. We call it the space at *infinity*.

3) Taking the stabilizer in the form $H = \begin{Vmatrix} a & b \\ \bar{b} & \bar{a} \end{Vmatrix}, \begin{Vmatrix} b & a \\ -\bar{a} & -\bar{b} \end{Vmatrix}, \left|a^2\right| - \left|b^2\right| = 1$ we get the homogeneous space $SL(2C)/H$ which is a one sheet hyperboloid. We call it the space at *transfinity* just to specify that it lies beyond infinity.

The reason we call 1, 2, and 3 in this way is: any two points in 1 can be joined by geodesics with finite length. Points in 2 are at infinite distance from any point

[2]Since matrices g and $-g$ in $SL(2C)$ correspond to the same motion, they are identified. Thus it would be correct to call the $PSL(2C) = SL(2C)/(-e, e)$ —projectivization of the $SL(2C)$ as a Lorentz group in the above representation—but we stay with the $SL(2C)$) notation, remembering mentioned identification of matrices which differ only by sign.

in 1. An infinite distance is required to reach them from any internal point in 1. Points in 3 are beyond reach, either in a finite distance or in an infinite distance—they are at transfinity.

Other models of the spaces listed in 1, 2, and 3 above are:

1) Lobachevskian space L^3 as an upper sheet of a two sheet $3D$ hyperboloid, $x^2 = [x,x] = 1 = x_0^2 - x_1^2 - x_2^2 - x_3^2$

2) Space at infinity $\partial L^3(\infty)$, upper cone, $x^2 = 0$

3) Space at transfinity, a one sheet $3D$ hyperboloid, (imaginary Lobachevskian space), $x^2 = -1$ where $x(x_0, x_1, x_2, x_3)$ are homogeneous coordinates.

We note that the $3D$ real Lobachevskian space will be perceived by an observer as:

1) The interior of a ball B^3 equipped with standard hyperbolic metric (Poincare ball), when viewed from the inside (when viewed from the vicinity)[3].

2) A paraboloid when viewed from the boundary at infinity (when viewed from infinity).

3) A hyperboloid when viewed from beyond the infinity (when viewed from transfinity).

In the Poincare representation of Lobachevskian geometry in Euclidean space E^3:

1) Lobachevskian space will be an interior of $3D$ ball B^3 equipped with a hyperbolic metric.

2) The space at infinity (boundary at infinity) will be the sphere $S^2 = \partial L^3(\infty)$ viewed as the boundary of the ball B^3.

3) The space at transfinity will be the complement of a closed ball $\bar{B}^3$ to the entire space E^3.

The spaces we discuss here for the purpose of the present paper are Lobachevskian real $3D$ space L^3, (item #1) and its boundary at infinity $\partial L^3(\infty)$, (item #2).

3. Horospheres in Real 3*D* Lobachevskian Space. Geometric Model Of CMB as a Projective Plane RP²

Intuitively a horosphere in Lobachevskian space L^3 can be understood due to the following construction.

In a Lobachevskian real $3D$ space L^3 we pick up an arbitrary point o and we draw a sphere of hyperbolic radius r centered at o, $S^2(o,r)$. Next we increase the radius $r \to \infty$, requiring that in the process the sphere S^2 will pass through the some fixed point $o \in L^4$. In the limit (such a limit exists) the sphere S^2 will become a horosphere—the sphere in Lobachevskian space having its center at infinity. Internal geometry on horospheres is Euclidean [10]. In that sense horospheres are an analogue of Euclidean planes E^2.

[3]The ball $x_1^2 + x_2^2 + x_3^2 = 1$ is a common part of the cone $x^2 = 0$ and a plane $x_0 = 1$.

[4]Origin o in homogeneous space L^3 is set arbitrarily. Any point can be regarded as an "origin". For practical purposes in physical applications the origin usually is the place where an observer is located.

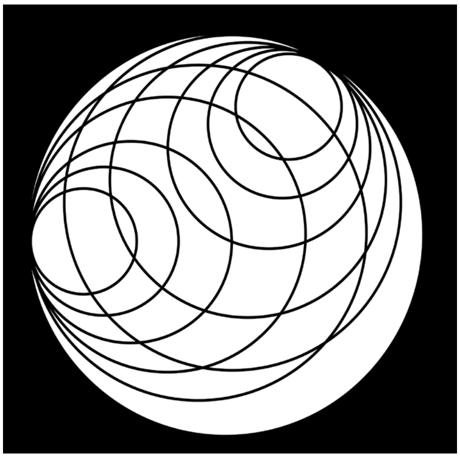

Figure 1. The global picture of CMB in Lobachevskian universe. For clarity, only two equivalence classes of parallel EM horospherical wavefronts are shown. Interference of EMhorospherical wavefronts will create a "spotty" pattern of small non-uniformities already observed by PLANCK [11] [12].

In the Poincare $2D$ model of Lobachevskian geometry (Lobachevskian plane) the horospheres in **Figure 1** are represented by circles tangent to the boundary at infinity (Hawaiian earring space). The horospheres are orthogonal to the congruence of parallel geodesics which emanate from the common point of tangency at infinity.

Now we give a formal description.

Given a Lobachevskian real $3D$ space L^3 and its boundary at infinity $\partial L^3(\infty)$, we define a bilinear mapping of $L^3 \times \partial L^3(\infty) \to R$ as follows.

Let $x(x_0, x_1, x_2, x_3)$ be homogeneous coordinates of $x \in L^3$, $x^2 > 0$, $x_0 > 0$, and $\xi(\xi_0, \xi_1, \xi_2, \xi_3)$ be homogeneous coordinates of $\xi \in \partial L^3(\infty)$, $\xi^2 = 0$, $\xi_0 > 0$, then the equation of the horosphere $h_\xi(x)$, centered at ξ and passing through *x*, in Lobachevskian space L^3 is:

$$x_0\xi_0 - x_1\xi_1 - x_2\xi_2 - x_3\xi_3 = [x, \xi] = \text{const} > 0. \tag{1}$$

Normalizing homogeneous coordinates in an appropriate way (if necessary) we can get the equation of the horosphere in canonical form,

$$[x, \xi] = 1. \tag{2}$$

Equation (1) is Lorentz invariant. That means it is preserved if we move the point $x \to gx$ and point $\xi \to g\xi$ simultaneously with some $g \in SL(2C)$.

$$[x, \xi] = [gx, g\xi] \tag{3}$$

The Lorentz invariance of the equation of horosphere has the utmost significance for physics. In fact the Equation (3) is the equation of *cosmological redshift* and related optical phenomena in Lobachevskian spaces.

3.1. From Geometry to Physics. What is the Physical Meaning of the Notion of Horosphere?

To see what the horosphere represents for a physicist we have to assign a physical meaning to mathematical entries in Equation (1).

By a standard notation we interpret homogeneous coordinates ξ, $[\xi,\xi]=\xi^2=0$, $\xi_0>0$, as the coordinates (components) of the *wave vector of an electromagnetic wave* (*EM* wave) in a vacuum $k(k_0,k_1,k_2,k_3)$, $[k,k]=k^2=0$, $k_0>0$. Therefore the equation of horosphere $h_k(x)$, centered at $k\in\partial L^3(\infty)$, and passing through $x\in L^3$ for physicist will be:

$$[x,k]=\text{const}>0 \tag{4}$$

Equation (4) says:

- The horosphere is a $2D$ surface of constant (positive) phase of an electromagnetic horospherical wave.
- The phase of a horospherical electromagnetic wave in a vacuum is Lorentz invariant.

The $2D$ surfaces of constant phase (of an electromagnetic wave) in physics are called *wavefronts.*

It follows that:

Definition 1. *The horosphere in real* $3D$ *Lobachevskian space is a wavefront of an electromagnetic* (*EM*) *wave in a* $3D$ *Lobachevskian vacuum.*

For the horosphere passing through the origin, $o\in L^3$, $o(o_0,o_1,o_2,o_3)$, $[o,o]=1$, $o_0>0$ (*i.e.* where observer is located) we will have, $h_k(o)=[k,o]$

Origin *o*, called also a *base point*, is set arbitrarily.

Definition 2. *The horosphere* $h_k(o)$ *is called the reference horosphere*, *the reference EM wavefront*.

What information can we extract directly from the group invariance of a horospherical electromagnetic wavefront (3) in Lobachevskian universe?

We claim that:

3.2. Group Invariance Relation (3), $[x,k]=[gx,gk]$, Is Geometrically Encoded Formula for Cosmological Redshift for All *z*

The proof of the above statement is done by direct calculations using the Poincare $3D$ ball model, equipped with the so-called Weierstrass (homogeneous) coordinates. We observe that from an algebraic point of view, Equation (1) is a bilinear form resulting in a real number and as such, due to a group invariance (3), it does not depend on the coordinates in which it is computed. First we compute Equation (1) in "general position", that is where source is located *y* ($y_0=\frac{1}{\sqrt{1-d^2}}$, $y_i=\frac{x_i}{\sqrt{1-d^2}}, i=1,2,3$), and then we compute (1) again in the origin (center) $o=y'(1,0,0,0)$ *i.e.* where the observer is located. Here $d=d(y,y')$ denotes the Euclidean distance from the origin $o=y'$ (center) to the arbitrary point *y*, in the unit Poincare ball model, $R=1$, with Gaussian negative curvature $K=-1$.[5]

By the group invariance under the rigid motions (isometries) we get:

[5]Note that same scaling $K=-1$ when Lobachevskian geometry is represented by velocities space results in choice of units where $c=1$.

$$[y,k]=\frac{k_0-\boldsymbol{xk}}{\sqrt{1-d^2}}=\frac{k_0-|\boldsymbol{x}||\boldsymbol{k}|\cos\alpha}{\sqrt{1-d^2}}=k_0\frac{1-d\cos\alpha}{\sqrt{1-d^2}}=[gy,gk]=k_0'$$

Taking into account that observations are done along the line of sight, which means $\alpha=0$, $\cos\alpha=1$, we get:

$$\frac{k_0'}{k_0}=\frac{\lambda}{\lambda'}=\frac{1}{1+z}=\sqrt{\frac{1-d}{1+d}},$$

$$z=\sqrt{\frac{1+d}{1-d}}-1 \tag{5}$$

It is easy to check (just use definition of $\tanh(\cdot)$ and solve for $z+1$) that Equation (5) is just a *different form* of Equation (6).

$$d=\tanh\left(\ln(1+z)\right) \tag{6}$$

Equation in the form (6) was derived, see [1] from the fundamental formula of Lobachevskian geometry which gives the rate of divergence of geodesics.

The cosmological redshift given by (5) follows from group invariance. That is a very powerful and elegant result. On the other hand, the same cosmological redshift derived from the divergence of geodesics [2] in Lobachevskian universe has a more geometric "flavor". It gives explicitly the distance d versus z and a telescope equipped with diffraction gratings can be calibrated accordingly to give as an output distance $d\in[0,1)$ to the object (a star, a galaxy) in a Poincare model of Lobachevskian universe with $K=-1$. So the formula (6) is a kind of a "*stick*" to measure distances in the Lobachevskian universe.

Both forms (5) and (6) show the beauty and simplicity of geometric physics.

3.3. What Did Edwin Hubble Actually Observe Looking nto His Telescope?

This question is easy to answer. First we note that Equation (5) gives $z(0)=0$, means at distance zero there will be no redshift. Next we go away from zero, by expanding the redshift equation via a Taylor series, $z(d)=z(0)+z'(0)d+\cdots$. This yields:

$$z(d)=d \tag{7}$$

Edwin Hubble, when he looked into his telescope, saw the linear term of a Taylor expansion of cosmological redshift (5) in a Lobachevskian universe, which we call Lobachevskian Hubble redshift. In a ball model with $R=1$, the redshift z is simply equal to the (Euclidean) distance d from the center of the ball where Hubble's telescope was located [2]. Approximations like this are called linearizations or Euclideanizations of phenomena modeled on curved geometries. In the Lobachevskian velocities space with $c=1$, this is called linear Lobachevski-Doppler shift $z=\upsilon$. Edwin Hubble was a fine astronomer but, as it appears to us, he was not familiar with Lobachevskian geometry. His reasoning was like that: since he found that experimentally $z=d$ and he knew the linear Doppler effect $z=\upsilon$, he concluded that $d=\upsilon$, meaning velocity rises with distance, what is now the "famous" Hubble distance velocity "law", and the error that started "space inflation". From a logical point of view his reasoning on the source of redshift is an elementary logic error of a sort: Cat is a mammal = true, dog is a mammal = true ⇒ cat is a dog = false. This is a classic example when two separately correct statements result in a false conclusion. It is quite remarkable that the linear term in the Taylor series of redshift formula (5) is called in all literature on the subject as "observable proof of galaxies' receding".

3.4. Group Theoretical Description of CMB. The Global Picture

In this section we will describe *global CMB* solely in terms of a $SL(2C)$ group. We follow a simple geometric construction. Given a Lobachevskian space $L^3 = SL(2C)/SU(2)$ we decompose it into a set of parallel horospherical waves, in an analogue way as the $3D$ Euclidean space is decomposed into a set of plane waves $\mathrm{e}^{[ikx]}$, and then we take an infinite union of those decompositions. In the case of a model of Lobachevskian space in the unit ball—this decomposition is pictured in **Figure 1** in a Poincare ball model of the Lobachevskian space—horospheres are represented by $2D$ spheres tangent to the boundaries at infinity.

Definition 3. *A (sub)group* $P \subset SL(2C)$ *is called a parabolic if it fixes the center* $k \in \partial L^3(\infty)$ *of some horosphere* $h_k(\cdot)$ *in Lobachevskian space* L^3.

Definition 4. *The horospheres (horospherical EM wavefronts) in Lobachevskian space are called mutually parallel if they have a common center. We call parallel horospheres equivalent.*

Given a point $k \in \partial L^3$ a group $P \subset SL(2C)$ is selected which stabilizes *k*, $Pk = k$. Now we decompose (*slice*) the Lobachevskian space L^3 with respect to parabolic group *P*, meaning we take the quotient
$P \setminus L^3 = P \setminus SL(2C)/SU(2) = [h_k(\cdot)] \subset Hor(L^3)$, see **Figure 1**, and we take the union of all equivalence classes of parallel horospheres generated by the set of all parabolic groups all over the boundary at infinity $\partial L^3(\infty)$.

That way we get the set of *all* horospheres in L^3, which is the *global* picture of *CMB*.

$$CMB_{\text{Global}} \simeq \cup_P P \setminus SL(2C)/SU(2) \tag{8}$$

The symbol $\cup_P$ in formula (8) denotes the infinite union of double sided quotient spaces $P \setminus SL(2C)/SU(2)$ over the set of all stabilizers $P \subset SL(2C)$ applied to all points $k \in \partial L^3(\infty)$, see **Figure 2**.

Definition 5. *Globally viewed Cosmic Microwave Background—CMB—is the infinite union of homogeneous, double sided quotient spaces (double fibration) of the Lorentz group,* $\cup_P P \setminus SL(2C)/SU(2)$, *where* P *is the parabolic group.*

The $SL(2C)$ acts transitively on its double sided $P \setminus SL(2C)/SU(2)$ quotient space [8] and so it does on the union—meaning on the *CMB*. This shows that *CMB* is a Lorentz invariant object

We can describe the global *CMB*, see **Figure 1**, in an equivalent way to the one given by formula (8).

The set of parallel horospheres $\left[h_k(\cdot)\right]$ is an equivalence class for any given horosphere $h_k(\cdot)$ containing all horospheres parallel to it. The set of parallel horospheres is orthogonal to the set of parallel geodesics having common point at boundary at infinity which is the center of class $\left[h_k(\cdot)\right]$.

Definition 6. *Globally viewed Cosmic Microwave Background* (*CMB*) *is a set of all equivalence classes* $\left[h_k(\cdot)\right]$ *of horospherical electromagnetic wavefronts in Lobachevskian* 3*D universe.* L^3

$$CMB_{\text{Global}} \simeq \cup_k \left[h_k(\cdot)\right] \tag{9}$$

3.5. Local Interpretation of *CMB* in Lobachevskian Universe. Picture of *CMB* as Seen by Us

It is needless to say that we have no ability to take a "*global*" look at a real Lobachevskian universe. The *global information* can be deduced only from models. All our physically acquired information is by necessity *local*. Thus we have no other option than to collect, process, and to draw conclusions from local information only.

Such information acquisition was done e.g. by COBE, WMAP and PLANCK experiments [11] [12].

Here we describe what a local experimenter will see (experimenter at origin o) adopting the presented geometric model of *local CMB* in Lobachevskian universe L^3.

A local observer at some arbitrary point o (origin) will see only those representatives $h_k(o) \in \left[h_k(o)\right]$ from each class of horospherical wavefronts $\left[h_k(\cdot)\right]$ which passes through o; see **Figures 2-5**. Therefore for an observer at o, the observable local *CMB* will be given by the following formula (10).

$$CMB_{\text{Local}} \simeq \cup_k h_k(o) \tag{10}$$

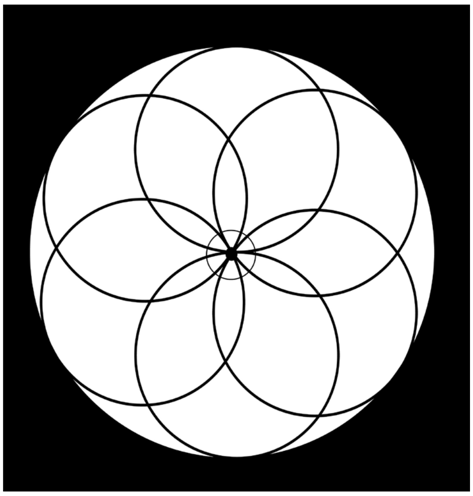

Figure 2. The local view of CMB as recorded by an observer at the origin of the Lobachevskian universe taken as the "center" of space. An observer will see homogeneous and isotropic CMB over the visual sphere around his location, represented as the black point. CMB will appear as a projective plane RP^2.

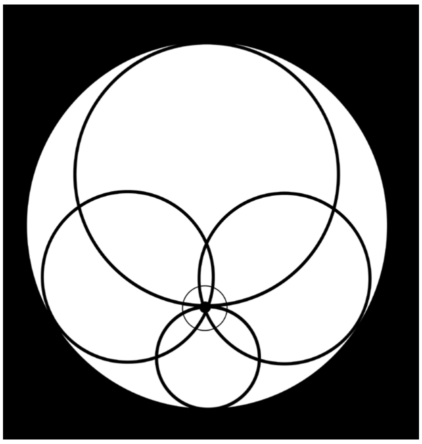

Figure 3. The conformal automorphism of Lobachevskian universe and the corresponding CMB pattern shown in **Figure 2**. All observers at any arbitrary point will see the same picture of homogeneous and isotropic CMB.

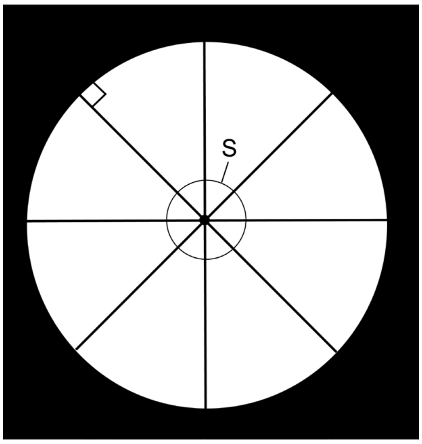

Figure 4. The "same" as **Figure 2** but it is given in terms of geodesics. Clearly, the isotropy and homogeneity of CMB over the sphere of visibility at the center is shown.

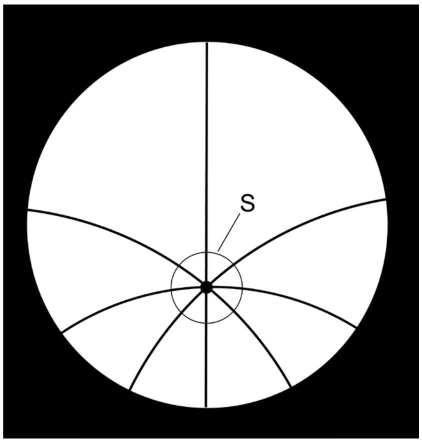

Figure 5. Corresponding to **Figure 3**, shows the conformal automorphism of Lobachevskian universe in terms of geodesics (light rays). An observer at any point in the Lobachevskian universe will see isotropic and homogeneous CMB over the sphere of visibility; however, there will be a redistribution of the ends of the geodesics at the boundary at infinity which the local observer won't notice anyway.

Horospheres containing the origin o are also regarded as subspaces in Lobachevskian space L^3 and this fact leads us to one more geometric interpretation of CMB.

Geometry is built upon relations between its objects. Projective geometry, for example, is built upon relations of *incidence*. From that point of view relations are first, and objects are subject to interpretation.

This view on geometry was developed by the German mathematician and physicist Hermann G. Grassmann (1809-1877) and today spaces built from more general objects than points and lines are called Grassmannian spaces.[6]

Thus a projective geometry is a Grassmannian geometry of all lines E^1 passing through the origin o (one dimensional linear subspaces) in Euclidean space E^3, denoted as Gr_3^1. The Grassmannian geometry Gr_3^2 describes the space of all Euclidean 2*D* planes in 3*D* Euclidean space passing through the origin, that is the space of two dimensional linear subspaces $E^2 \subset E^3$.

This construction has an immediate extension to non-Euclidean geometry, in our case to Lobachevskian geometry. What we only need to do (in the latter case) is to replace Euclidean 3*D* space by Lobachevskian 3*D* space, and the set Euclidean subspaces $E^2 \subset E^3$ by the (sub) set of all *horospheres* $Hor\left(L^3\right)$ containing origin *o*, $Hor\left(L^3, o\right) = \bigcup_k h_k\left(o\right) \subset Hor\left(L^3\right)$.

What we get is the non-Euclidean Grassmannian space $Gr_3^2 = Hor\left(L^3, o\right)$. Since we have already identified $Hor\left(L^3\right)$ with the global *CMB*, thus in context of Grassmaniann spaces we have the following definition of the local *CMB*

Definition 7. *The local view of CMB in a Lobachevskian* 3*D universe is the Grassmaniann space* Gr_3^2 *of horospheres i.e. horospherical electromagnetic wavefronts.*

This identification opens new ways to investigate *CMB* by working just with Grassmannian geometry. It also shows the close relationship, actually an identity, between geometry and physics in this case. Grasmannian spaces have interesting decompositions in so called *Schubert cells*. But this is beyond the scope of the present paper.

We can go one step further in the interpretation of local observations of *CMB*. Grassmannian spaces Gr_n^p and Gr_n^{n-p} are in fact isomorphic. In the particular case of Gr_3^2 and Gr_3^1 isomorphism follows from fact that horosphere $h_k\left(o\right)$ through origin o is orthogonal to the geodesic through the origin *o*, see **Figures 2-5**.[7]

We have already mentioned that *horospheres carry Euclidean geometry*. We also note that in domains of linear size of 10^5 light years or less, Lobachevskian space will appear as Euclidean space [2]. Thus, a local observer may regard the *CMB* as the space Gr_3^1 which is the real *projective* space RP_3^1, which is a projective plane RP^2.

Conclusion 8. *For an observer at an arbitrary point in a Lobachevskian*

[6]On life and work of Hermann Grassmann see Wikipedia: https://en.wikipwedia.org/wiki/Hermann_Grassmann.

[7]In the case of E^3, 2D planes through the origin are defined by a vector from the origin and orthogonal to that plane.

Universe the local geometry of CMB will appear as the geometry of the real projective plane RP^2.

Real projective space RP^2 is a *compact* space of dimension 2. It is known that the real projective plane RP^2 is isomorphic with the sphere S^2 with antipodal points x and $-x$ identified. Since the sphere S^2 covers the RP^2 twice, the real projective plane may be represented by a *closed hemisphere* with the opposite points on a great circle identified. This gives yet another interpretation of what we see as a local *CMB*.

4. From Lobachevski to Planck. Spectral Content of CMB

We already know that *CMB* in the framework of Lobachevskian geometry of the universe is identified with the space of all horospherical electromagnetic wavefronts; see **Figure 1**. The natural question which arises at this point is about its spectral properties. In other words, which spectral distribution function follows from the presented geometric model. In this paragraph we will show that from fundamental theorem of Lobachevskian geometry, the spectral distribution of *CMB* is the Planck black body distribution function.

Before going to this problem we have to elaborate a little bit on a notion of *distance* and *size* in Euclidean and Lobachevskian worlds. Those seemingly unrelated and "evident" notions, at least in Euclidean geometry, have immediate impact on physics measurements in the Lobachevskian world.

The essence is in the fact that in Euclidean geometry there is no *internally defined* size scale, so all the measurements of a size (length) are relative to the some arbitrary standard of length which is brought to Euclidean geometry from outside. On the other hand, Lobachevskian geometry has such internal standard of scale, and with respect to that standard—called characteristic length (characteristic constant)—all sizes have an absolute meaning. If that characteristic length will be denoted by $\varkappa$, then distance (size) in Lobachevskian space is always denoted as $\frac{d}{\varkappa}$ being a *dimensionless* number. Here is hidden a deeper idea that mathematics is and physics should be built on relations and only on dimensionless magnitudes (ratios) which reflect the true laws of physics. That is our point of view. Some people share it, some not.

Now we derive the Planck distribution formula from the Lobachevskian geometry.

It has been shown [2] [4] [6] that a horospherical electromagnetic wave in Lobachevskian universe will experience a change in frequency *i.e.* redshift z due to fundamental formula of Lobachevskian geometry which relates the divergence rate of geodesics (light rays) to the distance passed.

$$e^{\frac{d}{\varkappa}} = z + 1 \tag{11}$$

Here $z = \frac{\lambda_o}{\lambda_s}$, is the standard redshift notation, λ_o is the observed wavelength, and λ_s is the source wavelength.

From the formula (11) we get Lobachevskian *geometric spectral density distribution function* of horospherical wavefronts as:

$$\frac{1}{z} = n(d, \varkappa) = \frac{1}{e^{\frac{d}{\varkappa}} - 1} \tag{12}$$

The Equation (12) is a geometric model of a Planck black body distribution function. It represents the law of geometry. To get the physical interpretation of a geometric model we need only to assign a physical meaning to entries in Equation (12). Since we are interested in the energy distribution function we interpret the distance d (separation of parallel horospheres) in terms of energy. Thus:

1) We send $d \to E = \hbar k_0 = \hbar\omega$

2) Since the exponent in (12) is dimensionless, the characteristic length must also be in the same units, and natural choice here is $\varkappa \to kT$, where k is Boltzman constant and T is the absolute temperature.

After substitution we come to the famous Planck black body spectral distribution formula as a direct consequence of Lobachevskian geometry.

$$n(\omega, T) = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} \tag{13}$$

The best way to write Lobachevski-Planck formula (as we call it) (13) would be to work with units in which $c = k = \hbar = 1$. Then in the exponent in formula (13), it would just be the ratio $\frac{T_r}{T_T}$ of the radiation temperature T_r to thermodynamical temperature (reservoir temperature) T_T which makes physics simple and homogeneous.

Ending this section we conclude:

Conclusion 9. *The space of horospherical electromagnetic wavefronts in Lobachevskian universe has Lobachevski-Planck black body spectral distribution function* (13).

There is an interesting relation between the (thermodynamical) temperatures T at some point x in the Lobachevskian universe and the average temperature at the boundary at infinity. Those results are due to Schwartz, Neuman, Bocher and are presented in detail in the book by Needham [13]. We refer the reader to that source and here we just recall the major result given in [13] for the Lobachevski plane.

An observer located at an arbitrary point $x \in L^2$, regarded as Poincare disc model, making a full 2π turn can relate the average temperature $\langle T \rangle_x$ at boundary at infinity (circle S^1) to the temperature at his position $T(x)$ via the simple formula:

$$T(x) = 2\pi \langle T \rangle_x \tag{14}$$

We just conjecture that in the case of $3D$ Lobachevskian universe, viewed as Poincare ball model with boundary at infinity S^2, we can replace 2π angle by 4π

angle and we will get relation between (thermodynamical) temperature at *x*, and the average temperature on the sphere S^2 viewed as the boundary at infinity (horizon) ∂L^3 in Lobachevskian universe.

$$T(x) = 4\pi \langle T \rangle_x \tag{15}$$

4.1. Lobachevskian Universe as a Hyperbolic EM Cavity Filled with CMB

We already have seen that Lobachevskian geometry when interpreted in terms of a physical reality, resulting in a correct equation of cosmological redshift, gives us a correct model of CMB with is observable properties of homogeneity, isotropy and a Lobachevski-Planck black body spectrum, the facts of which are experimentally confirmed.

The black body radiation distribution function indicates anotherinterpretation of *CMB*.

It has been said that Lobachevskian space, for an internal observer (any observer embedded in L^3), will be perceived as the interior of a $3D$ ball equipped with a standard hyperbolic metric. That is the Poincare model. The interior of $3D$ Poincare ball filled with the set of horospherical electromagnetic waves having black body Lobachevski-Planck distribution spectral function has a natural interpretation as a hyperbolic electromagnetic cavity.

In the case of Euclidean space, the electromagnetic field in spherical cavities was studied for quite some time [14], and resulted in many fine applications.

The case of a hyperbolic $3D$ cavity is a simple non-Euclidean analog where the metric inside the cavity is now a hyperbolic metric and the Laplace operator is replaced by the Beltrami Laplace operator.

$$\frac{\partial^2 u}{\partial t^2} = z^2 \Delta u - z \frac{\partial u}{\partial z} + u \tag{16}$$

Its solutions are called horospherical waves [10].

In Equation (16), the upper half space model, Δ is a standard Laplace operator and the last term u in *RHS* of Equation (16) ensures that the spectrum lies on the half line $(-\infty, 0)$,

The wave equation in Lobachevskian space is well studied and we refer reader to the relevant literature [8].

Proposition 10. *The Lobachevskian universe, for an internal observer, may be regarded as a* $3D$ *hyperbolic cavity filled with homogeneous and isotropic electromagnetic radiation, having a Lobachevski-Planck black body spectral distribution function, which is the CMB.*

4.2. Existence of Absolute Motion and Absolute Rest in a Lobachevskian Universe

In this concluding paragraph we are going to show that the problem of the existence of absolute motion and absolute rest in Lobachevskian universe is defined correctly and has an affirmative solution.

In our point of view, the origin of the historical misunderstanding of absolute motion/absolute rest is due to the fact that the whole (or almost whole) of physics development was done in compact spaces. The compactness of an underlying space, even if not mentioned up-front, was somehow behind the scenes when physics phenomena were analyzed.

We need to specify first what we understand under the name of "reference". Intuitively it is desirable that a reference would be a "stable" object.

First we get some guidance from astronomy. Astronomers in their observations pick up a very distant reference, so-called "fixed star", and make their calculations of celestial motions with respect to the fixed star. Being at a *finite* distance from us, the "fixed star" is not fixed in a mathematical sense but it is "far enough" from us. Far enough means that due to the limited resolving power of our instruments we *cannot* detect any position change of that "fixed star" with respect to other "fixed stars".

The above example leads us to an idea that if we would be able to move the reference to infinity, then it would make sense to speak about a relative-to-reference-at-*infinity* motion and respectively about absolute rest. Thus, we give the definition of absolute motion/absolute rest.

Definition 11. *An absolute motion* (*an absolute rest*) *is a state of a motion* (*rest*) *recorded with the respect to the reference at infinity in Lobachevskian space.*

Now we have to specify what we mean by reference at infinity. From an operational point of view, the direct selection of an object at infinite distance is simply not feasible since all our instruments operate only locally. In this context, local distances mean arbitrarily big *finite* distances which are any way larger than the range of our instruments. Thus we have to proceed in some indirect way.

We already mentioned that the family of parallel horospherical *EM* wavefronts, see Figure 1 decomposes Lobachevskian space into an infinite number of constant phase, $2D$ horospherical surfaces ("slices") in an analogous way as $3D$ Euclidean space is decomposed into an infinite set of constant phase surfaces which are *plane waves,* $\mathrm{e}^{i[k,x]}$.

The above *one to one* correspondence between points $k \in \partial L(\infty)$ and equivalence classes of parallel horospherical *EM* wavefronts $\left[h_k(\cdot)\right]$ leads us to the following conclusion of isomorphism.

Conclusion 12. *The boundary at infinity* $\partial L^3(\infty)$ *of Lobachevskian space* L^3 *is isomorphic with the set of all equivalence classes of parallel horospheres* $\left[h_k(\cdot)\right]$ (*with the set of equivalence classes of parallel EM horospherical wavefronts.*)

$$\partial L^3(\infty) \simeq \bigcup_k h_k(\cdot) \tag{17}$$

We note that any equivalence classes of parallel horospherical *EM* fronts is *uniquely defined* by any of its representatives $h_k(\cdot)$, and in particular by the representative wavefront passing through origin o, $h_k(o)$ being where an observer sits. Thus the set of all representatives of equivalence classes is isomorphic to the boundary at infinity, since every horosphere through o has its center at $\partial L(\infty)$ and for any point at $\partial L^3(\infty)$ there will be the unique horosphere passing through o. See Figures 2-5.

$$\partial L^3 \simeq CMB_{\text{local}} \simeq \bigcup_k k(o) = RP^2 \tag{18}$$

Since we identified the boundary at infinity with the set of locally observed electromagnetic wavefronts (horospheres), which is the local *CMB*, it is natural to regard local *CMB* as an *absolute reference.* Therefore we come to the following definition:

Definition 13. *The absolute motion/absolute rest in a Lobachevskian universe* L^3 *is the state of motion/rest relative to the CMB viewed locally as a real projective plane* RP^2.

It is well known that the Big Bang based space inflation hypothesis requires a *flat Euclidean geometry of space*. Both Euclidean space and Lobachevskian space are non-compact/locally compact spaces. The difficulty with Euclidean space is due to the fact that*there is no mathematical object at infinity* in Euclidean geometry which has a *clear physical interpretation,* and the concept of absolute motion/absolute rest in the Euclidean spaces is simply*meaningless*. In contrary to that, in Lobachevskian geometry such geometrical object, the space of horospherical wave fronts, meaning the *CMB*, exists and it is identified as the reference at infinity.

5. Summary

The essence of a geometric model of *CMB* as a *space of horospherical electromagnetic wavefronts* in static Lobachevskian universe may be summarized as follows:

- It is a natural consequence of the Lobachevskian geometry of a large scale negatively curved vacuum.
- All observers regardless of their position inside a Lobachevskian universe will record the isotropic distribution of *CMB* over their spheres of visibility, see Figures 2-5. For the visibility sphere around the Earth, this is already confirmed experimentally.
- The presented *CMB* displays a Lobachevski-Planck black body spectral distribution function. Big Bang cosmology fails to give a mathematical derivation of the *CMB* spectral distribution function—only experimental measurements exist.
- From the global view, the *CMB* is a homogeneous space $\bigcup_P P \backslash SL(2C)/SU(2)$, of a real dimension 3, on which the Lorentz group $SL(2C)$ acts transitively. In other words, the model of the *CMB* presented here is a Lorentz invariant object.
- The $SL(2C)$ group invariance of the *CMB* implies a simple equation for *cosmological redshift* (5) with all its experimentally observed features; see also [1] [5]. It follows that both phenomena, the *CMB* and the *cosmological*

redshift, are inherently interconnected as two features having a common root—Lobachevskian geometry.

- From a local view (what we observe), we see the local *CMB* as a Grasmannian space G_3^2 of all horospherical electromagnetic wavefronts $\bigcup_k h(k,o)$ passing through origin o in L^3, **Figures 2-5**, equivalently as a real projective plane RP^2.
- The isomorphism of the *CMB* with the boundary at infinity $\partial L^3(\infty)$ of Lobachevskian universe L^3 leads to natural choice of *CMB* as an absolute reference. Thus it makes sense to talk about absolute motion and/or absolute rest with respect to *CMB* in our model.
- The scientific significance of the identification of *CMB* with the boundary at infinity $\partial L^3(\infty)$ in Lobachevskian universe L^3, is that the *CMB* is the *first physical realization* of the mathematical notion of infinity never before done in science.
- The set of horospherical electromagnetic waves in Lobachevskian universe, viewed in Poincare model as an interior of a $3D$ ball equipped with the hyperbolic metric, may be regarded as *hyperbolic* $3D$ *cavity*, having black body spectral distribution (Lobachevski-Planck).
- Superposition of horospherical electromagnetic wave fronts, and perhaps the Sunayev-Zeldovich (SZ) effect, over the sphere of visibility will create for a local observer a "spotty" pattern already recorded by WMAP [12]. In the discussed model, this spotty WMAP, PLANCK pattern—when properly decoded—may result in finding the radial position of the Earth in the Lobachevskian universe.

Obviously, the physical universe is far more complex and contains a variety of astrophysical phenomena such as nucleosynthesis, light elements distribution, galaxy formation, just name a few. These problems (and many other topics) are extensively explained (with no relation to the Big Bang) in works of Fred Hoyle, Geoffrey Burbidge, and Jayant Narlikar. Their results and the results of other researchers are summarized in their book [15]. On the page 188 of that book, the state of Big Bang cosmology is shown, as seen by the authors of [15]; it is worth a look.

Finally we would like to comment on some notions and extrapolations more or less routinely used in cosmology. For example, the backward extrapolation of Hubble's erroneous reasoning which gave birth to the idea, characterized mockingly by Fred Hoyle, as a Big Bang.

Suppose we investigate a Hausdorf space X with respect to some property P. It turns out that the property P holds in the closed neighborhood $\overline{U_x}$ of each of the points $x \in X$. Can we conclude that the entire space X poses the property P?

In the case of a compact spaces X such extrapolation is valid and answer is *yes*. In the case of locally compact, non-compact spaces e.g. Lobachevskian space, such extrapolation (in general) is not valid and answer is *no*. Let property P in question be a compactness. Lobachevskian space (as well as Euclidean space) is locally compact (compact in the closed neighborhood of each of its points) but globally is not compact. This example shows that in non-compact spaces unwarranted extrapolation of the local data, acquired in arbitrary finitely big domains, onto an entire space cannot be granted of being true.

In our opinion, the Lobachevskian Universe is unlimited, in the past and in the future temporal extent as well, and some notions like "time" are simply not applicable to the Universe as a whole. The notion of time while applicable to each and every object in the universe—an atom, a man, a star, a galaxy—*does not apply to the universe as a whole* in the same way e.g. as the notion of a set—while applicable to each and every aggregation—does not apply to the object called the *set of all sets.*

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