

# Gravitational Field of Moving Masses with Symmetrical Transformation for Relative Motion

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## Abstract

A symmetrical transformation is constructed to analyze the gravitational interactions between two fast moving masses based on the retarded potential without resorting to general relativity. The anomalous precession of the perihelion of orbital stars or planets can be explained with the same results as given by general relativity. By introducing an effective mass for photons, the gravity-induced frequency shift and light deflection in the trajectory by the gravity are derived, which can be reduced to the results based on general relativity under special conditions. The gravity-induced time delay of radar signals and gravitational radiations from binary pulsars are analyzed. The symmetrical transformation between two moving coordinates under zero gravity will also be discussed.

## Keywords

Gravity, Retarded Potential, Symmetrical Transformation, Light Deflection, Anomalous Precession

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## 1. Introduction

The theory of general relativity has acquired wide acceptance since its establishment with successful explanation for several major gravitational phenomena [1] [2] [3] including: 1) Deflection of starlight by a massive object, with a deflection angle of 1.75" for the Sun, which is twice the value of 0.87" as given by Newton's theory [4] [5] [6]; 2) The anomalous precession of the perihelion of Mercury with a precession angle of 43.11" per century; 3) Gravity-induced frequency shift as light traveling away from or toward a massive object; 4) Gravitational radiation from the binary pulsars causing a reduction in the orbital radius and period

[7] [8] [9].

It has been a main stream understanding that the correct value of the bending angle for starlight deflected by the gravity cannot be obtained based on the classical Newton's theory, and that the anomalous precession as well as the gravity-induced frequency shift can only be explained based on the space-time curvature of general relativity. So far, it has been widely accepted that, Newton's law of gravity yields a deflection angle of  $0.87''$  for light beam from a remote star passing near the Sun, and the deflection angle as calculated by general relativity is  $1.75''$ , twice the value based on the classical theory.

Due to the profound nature of space-time curvature in general relativity, the derivations of the related formulations for the above phenomena require complicated mathematics. In this paper, we present a different approach to the analyses of gravitational field for fast moving objects, which can also explain the above phenomena without resorting to general relativity.

By considering a symmetrical transformation between two fast moving masses based on the retarded potential, which generates extra gravitational fields in addition to the classical Newton's law of gravity, the anomalous precession of the perihelion of orbital stars or planets can be explained, with the same result as given by general relativity.

As a special case for the relative motion without considering the symmetrical transformation, by assuming an effective mass for the photons under the gravitational field, the gravity-induced frequency shift as well as the bending angle for starlight deflected by a massive object can be derived, which can be reduced to the results as given by general relativity at the low density limit. For high-density objects with a much larger mass to radius ratio, the deflection angle as given by general relativity based on Schwarzschild's metric will be invalid.

A century-long misunderstanding about the classical Newton's theory with a wrong bending angle for the light deflection will be clarified. The wrong deflection angle is due to the mistakes in solving the related orbital equation. When the orbital equation is solved under the correct boundary conditions, the result obtained by Newton's law of gravity will be more accurate than that as so far obtained by general relativity.

The gravity-induced time delay (or retardation) of radar signals will be examined. The gravitational radiations from binary pulsars will be analyzed, and a similar expression as general relativity for the quadruple radiation can be obtained. The symmetrical transformation between two moving coordinates under zero gravity will also be discussed, which provides a simple approach to derive the Lorentzian factor in special relativity.

## 2. Theoretical Analyses

We start from the retarded potential of Newton's law of gravity between a mass  $m$  orbiting around another mass  $M$ . The classical Newton's law of gravity  $F = GmM/r^2$  is a stationary equation valid for slow moving objects, where  $G$  is the

gravitational constant,  $r$  is the distance between the two masses. In order to consider the gravity between two fast moving masses, we refer to the retarded potential for Coulomb's interactions proposed by Lienard and Wiechert in the early 1900s [10],

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} q/r' = \frac{1}{4\pi\epsilon_0} q/(\mathbf{r} - \mathbf{r} \cdot \mathbf{v}/c), \quad (1)$$

where  $q$  is the electrical charge,  $\epsilon_0$  is the permittivity of vacuum, and  $c$  is the speed of light. Assuming the radius vector  $\mathbf{r}$  and the speed  $\mathbf{v}$  are in the same direction, when the source  $M$  is at rest, and the observation point is moving at a speed  $v_1$ , the gravitational potential  $\phi_1(r, t)$  is similar to the retarded potential of Equation (1),

$$\phi_1(r, t) = GM/[r(1 - v_1/c)], \quad (2)$$

due to the finite propagation speed  $c$  of the gravitational field, which can be taken as the same as the speed of light based on the comparisons between theoretical results and experimental observations. The gravitational retarded potential of Equation (2) has the same form as the electrical retarded potential of Equation (1), since Newton's law of gravity has the same form as Coulomb's interaction for the charged particles, with  $q$  replaced by  $M$ , and  $1/(4\pi\epsilon_0)$  replaced by  $G$ , the corresponding gravitational permeability  $u_g$  is given by  $4\pi G/c^2$ , and  $c^2 = 1/(\epsilon_0 u_0)$ .

When the source  $M$  is moving at a speed  $v_2$ , with the observation point at rest, the gravitational retarded potential is,

$$\phi_2(r, t) = GM(1 + v_2/c)/r. \quad (3)$$

When  $v_1$  and  $v_2$  are equal to  $v$ , and much smaller than  $c$ , then  $1/(1 - v/c) \sim 1 + v/c$ , the above two potentials  $\phi_1(r, t)$  and  $\phi_2(r, t)$  become equivalent.

For high speed moving objects, under the condition that the source and the observation point are indistinguishable in relative motion, and only a relative speed  $v$  can be designated, the above two potentials should be equal, which can be achieved by introducing a symmetric factor  $\gamma$ ,

$$\phi_1(r, t) = \gamma(1 + v/c)GM/r = \phi_2(r, t) = GM/[\gamma r(1 - v/c)], \quad (4)$$

with  $\gamma = 1/\sqrt{1 - v^2/c^2}$ . The symmetric factor is the same as Lorentzian factor in special relativity. The retarded potential is

$$\phi(r, t) = GM/[\gamma r(1 - v/c)]. \quad (5)$$

The related gravitational field is given by,  $\mathbf{E} = -\nabla\phi(r, t)|_{(t'=\text{constant})} + \nabla t' \times [\partial\phi(r, t)/\partial t']$ , which can be obtained through Lorentzian transformation (or referred to as symmetrical transformation) between the stationary coordinates and the moving coordinates [10]. In the stationary coordinates,  $\mathbf{E}_{01} = -GM\mathbf{e}_r/r_0^2 = -GMx_0/r_0^3$ , with  $\mathbf{e}_r$  the unit vector. When the source is moving along the x-axis at a speed  $v$ ,  $E_1 = E_{10}$ ,  $E_2 = \gamma E_{20}$ ,

$E_3 = \gamma E_{30}$ , the coordinates are in the stationary system of  $\mathbf{r}_0$ . Transforming back to the moving coordinates of  $\mathbf{r}$ , and observing at the same time,  $x_0 = \gamma x$ ,  $y = y_0$ ,  $z = z_0$ , then  $\mathbf{E} = \mathbf{E}_0 / \left[1 - (v^2/c^2)\right]^{1/2}$ , with  $\mathbf{E}_0 = -GM\mathbf{e}_r/r^2$ . When the angle between  $\mathbf{v}$  and  $\mathbf{r}$  is  $\theta$ , the gravitational field considering the symmetric transformation is  $\mathbf{E} = \mathbf{E}_0 (1 - v^2/c^2) / \left[1 - (v^2/c^2) \sin^2 \theta\right]^{3/2}$ .

When the acceleration is considered, the gradient of the potential yields an additional term in the gravitational field,  $\mathbf{E}_2 = -\mathbf{e}_r GM (\mathbf{a} \cdot \mathbf{e}_r) / (rc^2)$ . For a nearly circular orbit, the acceleration is  $\mathbf{a} = \mathbf{e}_r GM/r^2$ , when the speed is perpendicular to, and the acceleration is parallel to the direction of  $\mathbf{r}$ , with  $v \ll c$ , then  $\sin \theta = 1$ ,  $1 / \left[1 - (v^2/c^2)\right]^{1/2} \sim 1 + (v^2/c^2)/2$ , the total field is,

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1 + \mathbf{E}_2 = \mathbf{E}_0 + \frac{1}{2} (v^2/c^2) \mathbf{E}_0 - \mathbf{e}_r GM a / (rc^2) = \mathbf{E}_0 + \zeta_0 (v^2/c^2) \mathbf{E}_0 \quad (6)$$

where  $\zeta_0 = 3/2$ . The above derivation is similar to the retarded potential for Coulomb's interactions. Due to the analogy between Newton's law of gravity and Coulomb's interaction, as well as the corresponding retarded potentials, it can be predicted that, some of the major features associated with the electrical fields will be replicated in the gravitational fields including the existence of gravitational waves and radiations, if the additional field  $\zeta_0 (v^2/c^2) \mathbf{E}_0$  in Equation (6) could be proven to be effective for the explanation of some fundamental gravitational phenomena.

### 3. Calculation and Discussion

We discuss the impact of the additional term  $\zeta_0 (v^2/c^2) \mathbf{E}_0$  in the gravity for several major gravitational phenomena including the anomalous precession of the perihelion, light deflection by gravity, photon frequency shift, gravity-induced time delay of radar signals, and the gravitational radiation.

#### 3.1. The Anomalous Precession

First, we consider the anomalous precession of the perihelion of orbital stars or planets. Based on Newton's law of gravity  $\mathbf{F}_0 = -\mathbf{e}_r GmM/r^2$ , for a planet or star  $m$  rotating around a central mass  $M$ , Binet's orbital equation is [11]

$$\left(d^2u/d\varphi^2 + u\right) = u_0, \quad (7)$$

where  $u = 1/r$ ,  $u_0 = GM/H^2$ ,  $H = r^2 (d\varphi/dt)$ , with  $H$  the angular momentum per unit mass, the origin of the polar coordinates is located at the central mass  $M$  such as the Sun, and the mass  $m$  is at position  $P(r, \varphi)$ . The general solution to the above unperturbed equation of Equation (7) is  $u = (1 + e \cos \varphi) / p_0$ , where  $e$  is the eccentricity, for an elliptical orbit  $e < 1$ , and

$$p_0 = 1/u_0 = H^2 / (GM) = a(1 - e^2), \text{ with } a \text{ the semi-major axis.}$$

For the gravitational field taking into account of the symmetrical transformation, with  $\mathbf{E} = \mathbf{E}_0 + 3/2 (v^2/c^2) \mathbf{E}_0$ , the corresponding potential is,

$$\phi = \phi_0 + \phi_s = -GM/r - \frac{3}{2} (v^2/c^2) GM/r. \quad (8)$$

Based on the energy conservation, the total energy  $U$  is constant,  
 $U = \frac{1}{2}mv^2 + m\phi$ . To the first order approximation and neglecting the  $GM/(rc^2)$  term, then  $v^2 = 2(U/m + GM/r)$ ,  $\phi = -GMu - 3(U/m + GM/r)GM/(rc^2)$ . The orbital equation becomes  
 $(d^2u/d\varphi^2 + u) = GM/H^2 + 3(U/m)(GM/c^2)/H^2 + 6(GM/c^2)(GM)u/H^2$ , or  
 $d^2u/d\varphi^2 + (1-\eta_1)u = u_{01}$  (9)

with  $u_{01} = u_0 [1 + 3U/(mc^2)]$ ,  $\eta_1 = 6u_0(GM/c^2)$ . The solution to the above orbital equation of Equation (9) contains  $\cos(\delta\varphi)\cos\varphi$  or  $\sin(\delta\varphi)\sin\varphi$ , with an unperturbed term  $\cos\varphi$  of the original unperturbed equation of Equation (7), multiplied by a slow-varying modulation function  $\cos(\delta\varphi)$ .

Assuming the general solution to the above orbital equation of Equation (9) is  $u = \{1 + e \cos[(1-\delta)\varphi]\}/p$ , with  $\delta$  much less than unity, and substituting the general solution in the above equation with the  $\delta^2$  term neglected, we can obtain  $\delta = \eta_1/2$ . The precession angle per period is

$$\alpha = 2\pi\delta = 6\pi u_0(GM/c^2) = 6\pi GM/[a(1-e^2)c^2] \tag{10}$$

which is the same as given by general relativity [1] [3], with a calculated value of 42.99"/century for Mercury, and in consistence with the observed value of 43.13"/century.

Using a similar modulation function  $\cos(\delta\varphi)\cos\varphi$  in the general solution, another form of the orbital equation  $d^2u/d\varphi^2 + u = u_0 + \eta_2 u^2$ , with  $\eta_2 = 3u_0(GM/c^2)$ , yields the same precession angle of  $\alpha = 6\pi GM/[a(1-e^2)c^2]$ . In general, the additional gravitational fields induced by the high-speed movement with symmetrical transformation will contribute to non-linear terms in the orbital equation,  $(d^2u/d\varphi^2 + u) = u_0 + \sum_j \eta_j u_j^j$ ,  $j = 1, 2, 3, \dots$ , leading to various abnormal behaviors different from the classical Newton's gravity.

For a binary system with two masses  $m_1$  and  $m_2$  rotating relative to each other, the orbital equation is similar to Equation (9), with the central mass  $M$  replaced by the total mass  $M_0 = m_1 + m_2$ , the precession angle is  $\alpha = 6\pi GM_0/[a(1-e^2)c^2]$ . Taking the binary pulsar PSR1913+16 as an example [7] [8] [9], when  $m_1 = 1.44m_0$ ,  $m_2 = 1.387m_0$ ,  $e = 0.6171$ ,  $a = 2.803R_0$ , with  $R_0$  the solar radius, and  $m_0$  the solar mass, the calculated precession  $\alpha$  is 4.2242 degree/year, which is consistent with the total mass  $M_0$  and the observed value of 4.2266 degree/year.

### 3.2. Light Deflection by Gravity and Critical Radius for Black Holes

We analyze light traveling in a gravitational field as a special case for the orbital motion, without considering the additional retarded perturbation potential  $\phi_s = -\frac{3}{2}(v^2/c^2)GM/r$ , or equivalently, without considering the symmetrical transformation for the light, since light is traveling much faster than the massive

object.

For photons of energy  $h\nu_0$ , we introduce a photon effective mass  $m_0$  according to  $h\nu_0 = m_0c^2$ , with  $h$  Planck's constant and  $\nu_0$  the frequency. The orbital equation for a light ray from a remote background star passing near the surface of a massive object such as the Sun with a mass  $M$ , is similar to Binet's orbital equation for a regular mass [11], with a different coefficient,

$$\left(\frac{d^2u}{d\varphi^2}\right) + u = u_1 \tag{11}$$

where  $u_1 = GM/(2H^2)$ . Similar to other regular masses orbiting around a massive mass  $M$ , the effective mass  $m_0$  of photons is not involved in the orbital equation, since  $m_0$  is much smaller than the massive mass  $M$ , and will be cancelled at both sides of the equation. The only impact of the photon mass to the equation is the coefficient of the mass-energy relation.

Under certain conditions, the general solution to the above equation is  $u = A\cos\varphi + B\sin\varphi + u_1$ , with  $u_1 = GM/(2H^2) = u_0/2 = 1/p_1$ ,  $H^2 = p_1(GM/2)$ . At the closest spot  $S$  from the light beam to the Sun, with  $R$  the solar radius or the shortest distance from the light beam to the center of the Sun, the orbital equation yields,  $u = A\cos\varphi + B\sin\varphi + u_1 = B + u_1 = 1/R$ ,  $B = 1/R - u_1$ . The asymptotic conditions at the negative and positive infinite distances yield  $A\cos\alpha_1 + B\sin\alpha_1 - u_1 = 0$ , and  $A\cos\alpha_1 - B\sin\alpha_1 + u_1 = 0$ . Combining the above two conditions, we get  $B\sin\alpha_1 - u_1 = 0$ , and  $\sin\alpha_1 = u_1/B = u_1/(1/R - u_1)$ .

The eccentricity of the hyperbola orbit is

$$e_1 = \left[1 + (U/m)H^2/(GM)^2\right]^{1/2} = \left[1 + \frac{1}{2}(1 + e_1)/k_0\right]^{1/2}, \text{ where } k_0 = GM/(Rc^2), U$$

is the total energy, and  $e_1 = [1/(2k_0) + 1]$ . The deflection angle  $\alpha_1$  at one side is  $\sin\alpha_1 = 1/e_1 = 2k_0/(1 + 2k_0)$ . The total deflection angle at both sides is [12]

$$\alpha = 2\alpha_1 = 2\arcsin\left[2k_0/(1 + 2k_0)\right]. \tag{12}$$

For light rays following a hyperbola orbit, the above solution is analytically exact without simplification, with related assumptions for the photon energy in the form of  $m_0c^2$ , and the gravitational force of  $Gm_0M/r^2$ . Under the small angle approximation, along with a small value of  $k_0$  for the Sun of  $2.1227 \times 10^{-6}$ , then  $\sin\alpha_1 \sim \alpha_1$ ,  $1 + 2k_0 \sim 1$ ,  $\alpha_1 \sim 1/e_1 = 2k_0$ , the total deflection angle reduces to  $\alpha \sim 4k_0/(1 + 2k_0) \sim 4k_0$ . The approximated deflection angle of  $4k_0$  is the same as given by general relativity based on Schwartzchild's metric [1] [3], and equal to 1.751" for the Sun [5]. For other stars or galaxies of much larger masses and smaller radii, with  $k_0$  close to or even larger than unity, the deflection angle of  $4k_0$  as given by Schwartzchild's metric based on general relativity will be invalid.

Numerically, the present solution yields a deflection angle more accurate than the result as so far obtained by general relativity for a wider range of angles, with a deviation  $\Delta\alpha$  roughly on the order of  $8k_0^2 - 56k_0^3/3$ . As  $k_0$  approaches unity,  $\Delta\alpha$  becomes larger, for example, when  $k_0 = 0.01$ ,  $\Delta\alpha \sim 0.0008$ , when  $k_0 = 0.1$ ,  $\Delta\alpha \sim 0.06$ .

Throughout the above derivation, the light trajectory exhibits a hyperbola orbit, and the photon trapping by objects with an extremely high density (EHD) such as black holes is not considered. The present analysis is also applicable to EHD objects when the distance between the light beam and the massive mass is much larger than the critical radius as discussed below.

For more than a century, it has been a main-stream understanding that the classical Newtonian theory yields a bending angle of  $2k_0$  for the light deflection, which is half the value of  $4k_0$  as obtained by general relativity. In fact, this is a misunderstanding, the wrong deflection angle of  $2k_0$  is due to the mistakes in solving the second-order differential equation of orbital motion with wrong boundary conditions, which has nothing to do with Newton's theory. In the conventional calculations, only one side of the deflection has been considered, with additional mistakes in the boundary conditions. If the orbital equation is solved under the correct boundary conditions, an accurate solution can be obtained.

The main-stream misunderstanding that, there exists a factor of 2 between the classical theory and general relativity for light deflection, may lead to an error of factor of two in some of the formulations related to general relativity including the field equation, when people manually input a factor of 2 to fit the misunderstanding.

The above derivation is similar to the scattering problem of charged particles in a Coulomb's field, with the same type of orbital equation and boundary conditions. When incorrect boundary conditions and asymptotic behaviors were adopted, there exists a similar mistake with only half side of the deflection considered, leading to a factor of 2 reduction for small angle deflection such as the nuclear scattering of  $\alpha$ -particles.

Critical radius for light traps or black holes: For massive objects with an extremely high density (EHD) such as neutron stars, quasars, or black holes, the light trajectory deflected by gravity in the vicinity of the massive mass with a small radius may no longer be characterized by a hyperbola orbit. To the first order approximation, a photon (or light) trap can be formed based on the classical Newton's laws of gravity and motion, under the condition that photons of an effective mass  $m_0$  rotate around a high-density mass  $M$  in a circular orbit, with  $m_0 c^2 / r = G m_0 M / r^2$ , which leads to a critical radius,

$$r_c = GM/c^2 . \quad (13)$$

For an object of mass  $M$ , with a radius smaller than the critical radius  $r_c$ , light may be trapped or captured by the gravity, as referred to as black holes for the astronomical phenomena.

The critical radius of Equation (13) as derived based on Newton's law of gravity is similar to Schwartzchild's radius using Schwartzchild's metric based on the simplifications for Einstein's field equation of space-time curvature in general relativity.

For radiations of high enough energy and traveling in certain directions, they



may escape or emit from the light traps or black holes. We can further imagine that, light trapping may also occur in the microscopic scale by some elementary particles such as electrons, as electrons have a finite mass yet with a nearly infinitely small radius, not detectable by present technologies.

In the above analyses, light waves (or photons) have been treated as an object with an infinitely small (or near but non-zero) mass, which follows the classical Newton's law of gravity. As a rough estimation,  $m_0 = h\nu_0/c^2 \sim 10^{-34} \times 10^{14}/10^{16} \sim 10^{-36}$  (kg), which is about  $10^5$  smaller than the electron mass. The near but non-zero photon effective mass is analogous to a differential (or infinitesimal) quantity in the calculus.

In the present derivation, an effective photon mass has been assumed with  $m_0 = h\nu_0/c^2$ , following the classical Newton's law of gravity, which seems contradictory to the conventional "basic principle" for photons without a mass, and also different from the perception of light deflection based on the space-time curvature of general relativity. As light deflection by the gravity has been taken as one of the key evidences differentiating general relativity from the classical Newton's law of gravity, supporting the fundamental concept of space curvature, the above assumption is among the major challenges critics might propose for the present analyses. If the above derivations could be proven free from mathematical errors or even confirmed by experimental observations, it will be worthwhile to reconsider the necessity of the related "basic principle" for the interactions between photons and gravity under certain conditions, instead of challenging a simpler and more precise derivation.

### 3.3. Gravity-Induced Frequency Shift

Based on a similar understanding as the light deflection without considering the additional gravitational field due to the symmetrical transformation, we further discuss the gravity-induced frequency shift of photons. With the same assumption for an effective mass  $m_0$ ,  $h\nu_0 = m_0c^2$ , after traveling from a distance  $r_1$  to  $r_2$  in the gravitational field, the change in the potential energy is  $Gm_0M(1/r_2 - 1/r_1)$ . Based on energy conservation, the change in the photon energy is  $h(\nu_2 - \nu_1) = Gm_0M(1/r_2 - 1/r_1)$ .

Red-shift: For photons emitting from a remote object of mass  $M$  with a radius  $R$ , when observed at a nearly infinite distance such as on the Earth, when neglecting the blue-shift caused by the observation planet (*i.e.*, the Earth) with a much smaller mass, the gravity from mass  $M$  will lower the frequency with a red shift in the wavelength, as light traveling away from the mass  $M$ ,

$$\Delta\nu/\nu_0 = (\nu - \nu_0)/\nu_0 = -Gm_0M/(Rm_0c^2) = -GM/(Rc^2) = -k_0, \quad (14)$$

where  $k_0 = GM/(Rc^2)$ , which is the same as given by general relativity, and is  $2.1227 \times 10^{-6}$  for the Sun.

In the above derivation, the photon effective mass  $m_0$  is assumed to be constant at different positions with different energies. In fact, as the photon energy



$h\nu$  changes, the corresponding effective mass  $m$  may also change according to  $mc^2 = h\nu$ , assuming a constant speed of light.

When the change in the photon effective mass is taken into account, we make the following assumption that, the differential energy follows the equation  $c^2(dm) = GmMdr(1/r)$ , and  $(\Delta m)/m = (GM/c^2)\Delta(1/r)$ . The frequency change is  $\Delta\nu/\nu_0 = \exp[GM/(rc^2)] - \exp[GM/(Rc^2)]$ . As  $r$  goes to infinite, the frequency shift is

$$\Delta\nu/\nu_0 = 1 - \exp[GM/(Rc^2)] = 1 - \exp(k_0). \quad (15)$$

When  $k_0$  is much less than unity, Equation (15) reduces to  $\Delta\nu/\nu_0 = -k_0$ , which is the same as given by general relativity. When  $k_0$  is close to or even larger than unity, with the distance from the light source, or the closest distance from the light path to the massive object, larger than the critical radius, so the photon trapping effect can be ignored, the frequency shift will be larger than the result as given by general relativity, which may contribute to the excessively large red-shift of some quasars. The above prediction is subject to further experimental confirmation. Equation (15) also yields a second critical radius for UHD objects, as  $\nu = 0$ , then  $k_0 = \ln 2$ ,  $r_{c2} = r_c / \ln 2 = 1.4427r_c$ .

Blue-shift: Similarly, when the photons are traveling towards a massive object such as the Earth, the gravity will increase the frequency with a blue shift in the wavelength

$$\Delta\nu/\nu_0 = \exp[GM_e/(R_e c^2)] - 1 = \exp(k_1) - 1, \quad (16)$$

where  $k_1 = GM_e/(R_e c^2)$ , with  $M_e$  the mass of the Earth,  $R_e$  the radius of the Earth. When  $k_1$  is much less than unity,  $\Delta\nu/\nu_0 \sim k_1$ , which is the same as given by general relativity. For photons traveling near the surface of the Earth, with a distance much smaller than the radius of the Earth, then  $\Delta\nu/\nu_0 = g\Delta H/c^2$ , with  $g$  the gravitational acceleration of the Earth, and  $\Delta H$  the travelling distance along the vertical direction.

By assuming an effective mass for photons under gravitational fields, several major phenomena including light deflection, critical radius for gravitational light trapping or black holes (*i.e.*, Schwartzchild's radius), and the gravity-induced frequency shift, can be easily explained without resorting to the complicated mathematics of space-time curvature of general relativity, yet with conclusions similar to or even more accurate than general relativity. As a result, it will be worthwhile to evaluate the necessity to adopt the space-time curvature of general relativity in order to explain the above phenomena, due to the associated complexities and inefficiencies. As to the understanding of photon effective mass, it may open up some new frontiers for the interaction between the electro-magnetic (EM) field and the gravitational field.

### 3.4. Time Delay by Gravity for Electro-Magnetic Signals

Time delay (or gravitational retardation) of optical or microwave radar signals from the Earth, with the signals reflected by other planets and retarded by gravi-

ty from a nearby massive mass (such as the Sun) along the signal path, has been referred to as the fourth classical test for the theory of general relativity [13].

The expression for the time delay as relative to the traveling time without the massive mass along the light path, has been derived from the Schwarzschild solution for an Earth-based radar signal traveling in the vicinity of the Sun, reflected by an inner planet and returning back. It has been believed that the time delay due to the curvature in the trajectory (light deflection) is much smaller than the above time retardation under the testing conditions, with the round-trip time delay given by [13],

$$\Delta t = \frac{4GM}{c^3} \left\{ \ln \left[ \frac{r_2 + (r_2^2 + d^2)^{1/2}}{-r_1 + (r_1^2 + d^2)^{1/2}} \right] - \frac{1}{2} \left[ \frac{r_2}{(r_2^2 + d^2)^{1/2}} + \frac{2r_1 + r_2}{(r_1^2 + d^2)^{1/2}} \right] \right\}, \quad (17)$$

where  $M$  is the solar mass,  $d$  is the closest distance from the light path to the center of the Sun,  $r_1$  is the distance along the line of signal flight from the Earth-based radar to the point  $S$  of the shortest distance to the Sun, and  $r_2$  is the distance along the line of flight from the planet to the point  $S$ . Under the condition that  $r_1$  and  $r_2$  are much larger than  $d$ , the above formulation reduces to  $\Delta t = \frac{4GM}{c^3} \ln \left( \frac{4r_1 r_2}{d^2} \right)$  [13] [14] [15].

In the following, we try to provide a qualitative understanding for the time delay of Equation (17), assuming its potential validity within a certain range of distances. Under the following assumptions, a similar form of expression can be derived without resorting to the space-time curvature of general relativity. First, we assume photons under the gravitational field exhibit an effective velocity  $v_e$  different from the speed of light in the vacuum. When the distance is much larger than the critical radius, as the photon approaching the massive mass  $M$ , we further assume that the photon energy can be expressed as  $m_0 c^2 = m_0 v_e^2 + Gm_0 M / r$ , where the expression for the energy conservation is different from a regular mass, and the photon energy is  $m_0 c^2$  at an infinite distance from the source of gravity. In the weak field limit,

$$v_e = c \left[ 1 - \frac{1}{2} \frac{GM}{rc^2} \right],$$

$$dt = dy/v_e = \left[ 1 + \frac{1}{2} \frac{GM}{rc^2} \right] dy/c = dy/c + \frac{1}{2} \frac{GM}{rc^3} dy, \text{ where,}$$

$r^2 = d^2 + y^2$ , with  $y$  the distance along the light path from the source (or reflector) to the closest spot  $S$ , and  $r$  the distance between the photon and the mass  $M$ .

The round-trip travel time  $t = t_0 + (GM/c^3) \left[ \left( \int_{-r_1}^0 + \int_0^{r_2} \right) dy / \sqrt{y^2 + d^2} \right]$ , corresponding to a time delay

$$\Delta t = (t - t_0) = \frac{GM}{c^3} \ln \frac{r_2 + (r_2^2 + d^2)^{1/2}}{-r_1 + (r_1^2 + d^2)^{1/2}}, \text{ which exhibits the}$$

same core factor of a logarithmic dependence on the distances as the first term in Equation (17), and reduces to  $\frac{GM}{c^3} \ln \left( \frac{4r_1 r_2}{d^2} \right)$ , when  $r_1$  and  $r_2$  are much larger than

*d*. The single-trip time delay for light traveling along the radial direction of a central mass  $M$  will be of the form  $\frac{1}{2} \frac{GM}{c^3} \ln\left(\frac{r_2}{r_1}\right)$ .

The time delay with a logarithmic dependence on the distance suffers from a fatal deficiency of non-convergence in the asymptotic behavior. The delay time increases as one of the distances, and becomes infinite as  $r_2$  goes to infinite. Whereas, a finite value of time delay as the distance becomes extremely large should be expected, since the gravitational effect will decay to zero as the distance goes to infinite. Also when the above formulation is applicable, as  $r_1$  and  $r_2$  smaller than  $d$  (where  $d$  can be on the order of the solar radius of  $10^8$  m or larger), the time delay of Equation (17) will be negative, whereas, a zero delay time should be expected as the distance close to zero.

In case the time delay caused by the gravity other than the curved trajectory indeed exists, a better convergence behavior of the delay time may be obtained by assuming a photon effective mass dependent on the distance under the gravitational field. For light traveling from a remote location to the point with a distance  $d$  from the center of the mass larger than the critical radius, the single-trip time delay is  $\Delta t = \eta_0 (GM/c^3)$ , where the constant  $\eta_0$  dependent on  $GM/(dc^2)$  is given by an integral, which is approaching zero when the distance between the source and observer is very small, or the distance  $d$  is very large, and converging to a finite value as the distance becomes very large. For a rough estimation, the delay time may be at least several times smaller than that as given by Equation (17) under the measurement conditions. The detail analysis remains an open question for future researches.

### 3.5. Gravitational Radiation

In order to facilitate the analyses of gravitational radiation, we further extend the above gravitational field to a fully symmetric form. The gravity of a moving mass will generate an extra gravitational field, similar to electrical current generating magnetic field. Under the symmetric condition, the extra gravity generated by a moving object is  $\mathbf{E}_m = (1/c^2) \mathbf{v} \times \mathbf{E}$ , which can be expressed by the curl of a vector potential,  $\mathbf{E}_m = \nabla \times \mathbf{A}$ . The time variation of the extra gravity generated by a moving object will also contribute to the gravity,  $\mathbf{E} = -\nabla \phi - \partial \mathbf{A} / \partial t$ .

Under Lorentzian transformation, the scalar potential  $\phi$  and the vector potential  $\mathbf{A}$  constitute a four dimensional vector, and  $\mathbf{E}$  and  $\mathbf{E}_m$  constitute four dimensional tensor. The related fields between a stationary coordinate and a moving coordinate can be derived by Lorentzian transformation. When  $E_{||}$ ,  $E_{m||}$ , and  $E_{\perp}$ ,  $E_{m\perp}$  represent the parallel and perpendicular components, respectively,  $E_{||} = E_{0||}$ ,  $E_{m||} = E_{m0||}$ , and  $E_{\perp} = \gamma(E_0 + \mathbf{v} \times \mathbf{E}_{m0})_{\perp}$ ,  $E_{m\perp} = \gamma(E_{m0} - \mathbf{v} \times \mathbf{E}_0/c^2)_{\perp}$ . For moving objects, the gravitational field is

$$\mathbf{E} = -\mathbf{e}_r GM (1 - v^2/c^2) r / \left[ (v^2/c^2) r^2 + (\mathbf{v} \cdot \mathbf{r})^2 / c^2 \right]^{3/2} = T(v, \theta) \mathbf{E}_0, \quad (18)$$

with  $T(v, \theta) = (1 - v^2/c^2) / \left[ 1 - (v^2/c^2) \sin^2 \theta \right]^{3/2}$ ,  $\mathbf{E}_0 = -\mathbf{e}_r (GM)/r^2$ . When

$v \ll c$ ,  $\mathbf{E}_m = \mathbf{v} \times \mathbf{E}/c^2 \ll \mathbf{E}$ . When  $\mathbf{v}$  is perpendicular to  $\mathbf{r}$ , then  $\sin \theta = 1$ ,  $E = E_0 / (1 - v^2/c^2)^{1/2} = E_0 \left( 1 + \frac{1}{2} v^2/c^2 \right)$ .

For fast moving objects with  $v \sim c$ , in the direction parallel to the speed  $v$ ,  $E \sim (1 - v^2/c^2) E \ll E_0$ , in the direction perpendicular to the speed,  $E = \gamma E_0 \gg E_0$ . The gravitational field  $E = \gamma E_0$  generated by a fast moving massive object forms an impulse-like gravitational wave (or shock wave) grazing the observer, and disappears as the massive object moves away. Strong gravitational waves can also be generated when two massive objects rotating around or merging toward each other at a high speed.

When the acceleration  $\mathbf{a}$  is considered,  $\mathbf{E}_m = GM(\mathbf{v} \times \mathbf{e}_r)/v^2/r^2 + GM(\mathbf{a} \times \mathbf{e}_r)/c^3/r$ . The first term contains  $v/c^2$ , and decreases as  $1/r^2$ , which can be ignored for slow motion. Under the centripetal force,  $(\mathbf{a} \times \mathbf{e}_r) \sim 0$ , the additional gravity due to the acceleration contains only the gradient of scalar potential,  $\mathbf{E}_2 = -\mathbf{e}_r GM(\mathbf{a} \cdot \mathbf{e}_r)/(rc^2) = -\mathbf{e}_r GMa/(rc^2)$ . For a nearly circular orbit,  $\mathbf{a} = \mathbf{e}_r GM/r^2$ ,

$$\mathbf{E} = \mathbf{E}_0 + \frac{1}{2}(v^2/c^2)\mathbf{E}_0 - \mathbf{e}_r GMa/(rc^2) = \mathbf{E}_0 + \zeta_0(v^2/c^2)\mathbf{E}_0, \tag{19}$$

with  $\zeta_0 = 3/2$ , which is the same as Equation (6).

For a binary system of an equal mass  $m_0$  rotating in a circular orbit of radius  $a_0$ , with a constant angular frequency  $\omega_0$ , the gravitational dipole radiation is zero as the two masses are always in opposite positions relative to the center of mass. The power of the gravitational quadruple radiation of the two masses is,

$$P = \frac{32}{5} Gm_0^2 a_0^4 \omega_0^6 / c^5, \tag{20}$$

which is the same as given by general relativity. Using the orbital parameters of the binary pulsar PSR1913+16 [7] [8] [9], with the elliptical orbit approximated by a circular orbit with  $a_0 = (a + p)/2$ ,  $m_0 = (m_1 + m_2)/2$ ,  $\omega_0 = 2\pi/T$ , the estimated value for the radiation power based on a circular orbit is 6.5 times smaller than the value for an elliptical orbit as given below.

Substituting the parameters of the circular orbit with an elliptical orbit, the energy of quadruple radiation per period for the two masses can be estimated as

$$\Delta W = \frac{64}{5} \pi G m_0^2 H_0^5 / (p^6 c^5), \tag{21}$$

which differs from the precise result by a constant factor, and is about two times different from the precise value for PSR1913+16. Using the elliptical orbit  $r = p/(1 + e_0 \cos \varphi)$ , with  $e_0$  the eccentricity,  $p = a(1 - e_0^2)$ , the energy for the gravitational quadruple radiation per period is

$$\Delta W = \frac{64}{5} g(e_0) \pi G \mu^2 H_0^5 / (p^6 c^5), \tag{22}$$

where  $\mu$  is the reduced mass,  $\mu = m_1 m_2 / (m_1 + m_2)$ ,  $H_0 T = 2\pi g_0(e_0) p^2$ , and  $g_0(e_0) = 1 + \frac{3}{2} e_0^2 + \frac{15}{8} e_0^4 + \dots = 1 / (1 - e_0^2)^{3/2}$ . The constant  $g(e_0)$  is dependent on

the eccentricity  $e_0$ , and estimated by  $g(e_0) = 1 + \frac{73}{24}e_0^2 + \frac{37}{96}e_0^4 + \dots$ , which is 2.21 for the PSR1913+16.

The gravitational radiation leads to a decay in the orbital period  $T$ , semi-major axis  $a$ , and the eccentricity, with  $(\Delta T/T) = \frac{3}{2}\Delta a/a$ ,

$$\Delta T/T = -3a\Delta W/(Gm_1m_2) = -\frac{192}{5}g(e_0)\pi a \frac{m_1m_2}{(m_1+m_2)^2}H_0^5/(p^6c^5) \quad (23)$$

Based on the orbital parameters for the binary pulsar PSR1913+16, the energy of the gravitational quadruple radiation per period is  $\Delta W = 2.17 \times 10^{29}$  J, corresponding to a reduction in the period  $\Delta T = -2.41 \times 10^{-12}$  s/(s), and a reduction in the semi-major axis  $\Delta a = -3.1$  mm/period, consistent with the observations [7] [8] [9].

### 3.6. Symmetrical Transformation for Other Related Phenomena

Symmetrical transformation for Doppler effect: For wave oscillations in a medium, when the source is moving at a speed  $v_1$  relative to the medium, with the observer at rest, the frequency as observed by the observer will be  $\omega_1 = \omega_0/(1 - v_1/c)$ , as in the classical Doppler effect, where  $\omega_0$  is the angular frequency of the source at rest,  $c$  is the wave propagation speed. When the observer is moving at a speed  $v_2$  relative to the medium, with the source at rest, the frequency as observed by the observer will be  $\omega_2 = (1 + v_2/c)\omega_0$ . When  $v_1 = v_2 = v$ , and much smaller than  $c$ , then  $\omega_1 = \omega_0/(1 - v/c) \sim (1 + v/c)\omega_0 = \omega_2$ .

For wave propagation in the vacuum, since there is lacking a reference to differentiate between the source and observer as which one is moving and which one is at rest, rather only a mutual relative motion of an equal velocity in an opposite direction can be designated, we can introduce a symmetric factor  $\gamma_0$  to make the two frequency shifts equal,

$$\omega_1 = \omega_0/[ (1 - v/c)\gamma_0 ] = \omega_2 = \gamma_0(1 + v/c)\omega_0, \quad (24)$$

where  $\gamma_0 = 1/\sqrt{(1 - v^2/c^2)}$ , which is the same as Lorentzian factor in special relativity. The above derivation is the same as the retarded potential for the gravity in Section 2, and is applicable to other related phenomena such as Lienard and Wiechert retarded potential for the electrical fields.

Since no other fundamental principles were needed in deriving the above frequency shift, and only the propagation distance, time, and counting the peaks (or phases) of wave oscillations are considered, the symmetric consideration between two objects with relative motion implies a more profound principle (the symmetrical principle for relative motion): under certain circumstances, the relative motion between two objects are indistinguishable and only a mutual relative speed can be designated, as a result, some of the properties for the two systems may follow the symmetrical transformation.

In order to extend the above symmetric consideration to other related phe-

nomena, we can construct a similar symmetrical transformation for the space and time coordinates  $(x, t)$ , based on which the related properties may naturally exhibit the same symmetrical feature for relative motion. For the moving system with a speed  $v$ , the coordinates can be expressed as  $t' = \gamma(t - t_0)$ ,  $x' = \gamma(x - vt)$ . In addition to the classical Galilean relation ( $\gamma = 1$ ,  $t_0 = 0$ ), there exists another relation with non-linear velocity dependence,  $\gamma = 1/\left(1 - v^2/c^2\right)^{1/2}$ ,  $t_0 = vx/c^2$ . Under this transformation, the two coordinates will be symmetrical in relative motion, and one set of the coordinates can be expressed as a function of the other set of the coordinates, with  $(x, t)$  and  $(x', t')$  following the same form of expressions. According to  $m = m_0 / \left(1 - v^2/c^2\right)^{1/2}$  as  $v \ll c$ , then  $mc^2 = m_0c^2 + mv^2/2$ , and  $\Delta E = mv^2/2 = (m - m_0)c^2 = (\Delta m)c^2$ , which correlates the amount of energy release (or absorption) with the change of mass. It is worth noting that the EM fields in Maxwell's equations exhibit a symmetrical feature between the electrical field and magnetic field, and satisfy the symmetrical transformation as the  $\gamma$  factor is included.

The above transformation is the same as Lorentzian transformation, with Lorentzian factor derived by a simple way based on the motion-induced frequency shift, without using Einstein's thinking experiment of light travel, both valid for wave propagations in any medium. Throughout the above derivations, the maximum value of light speed  $c_0$  in vacuum is only a consequence of the formulations (with divergence at this speed), but not a fundamental requirement preventing objects or fields of other kinds traveling faster than the speed of light  $c_0$ , as asserted by special relativity. It will be possible that some phenomena with a field propagation speed  $c_1$  may also satisfy the symmetrical transformation, yet with  $c_1$  larger than the speed of light  $c_0$ .

The divergence in the formulations for the speed of light also implies that, under certain conditions, light (or photons) traveling at a speed of  $c$  will not follow the symmetrical transformation, and the classical Newton's law will remain valid. When the high speed movement is considered with a speed smaller than the light, the retarded potential of Newtonian gravity based on the symmetrical transformation will be applicable to regular masses (not photons), with some of the related results similar or equivalent to those as obtained by general relativity, yet easier to understand and derive, as reflected by the anomalous precession.

Among several different kinds of fundamental laws of gravity, the general theory of relativity may approach the gravitational phenomena from a different direction of extreme conditions, which emphasizes on and may over-evaluate the symmetrical features. If Einstein's field equation of general relativity is fully symmetrical for the relative motion, it might be less effective or less efficient to solve the fully unsymmetrical phenomena related to the light, such as the light deflection by gravity, the gravity-induced frequency shift, and the critical radius of light trapping, unless the original field equation is broken in the symmetry with some simplification such as Schwartzchild's approximation, or even reduced to Newtonian theory. As a result, for the above three phenomena, New-

ton's law of gravity even without any post-Newtonian modification, will be simpler and potentially more accurate than general relativity.

In addition to two extreme situations of either fully symmetrical or entirely unsymmetrical in relative motion, with the former corresponding to two objects of the same mass and orbital parameters, and the latter corresponding to a near-zero mass with a high speed (*i.e.*, the speed of light) moving in the gravitational field of a massive mass, it is possible that some phenomena, such as a heavy mass and a light mass rotating relative to each other, may exhibit other partial symmetric features, and the symmetrical transformation may overestimate the symmetric effect for high speed movement (or the so-called relativistic effect).

### 3.7. Brief Discussions about the Understanding of Relativity

All the major conclusions of special relativity and some conclusions of general relativity have been demonstrated to be correct by the successful explanation of high speed phenomena. In this section, we provide a brief discussion about the potential complexities and confusions associated with some fundamental understandings for the theory of relativity.

Following is a century-long main stream understanding about the Michelson-Morley (M-M) experiment: "When assuming the Earth is moving relative to the light wave medium (as referred to as ether before) at a speed of  $v$ , and the speed of light is  $c$ , then 1) according to Galilean principle of motion superposition, the superposition of the two vector speeds  $v + c$  will cause the interference pattern to shift when the experimental setup is rotated by 90 degree; 2) since there is no interference pattern shift observable during the experiment, so, the experiment observations are in violation of Galilean principle of motion superposition, which implies that 3) there exists no ether, and 4) the experiment serves as one of the major evidences leading to the establishment of special relativity".

The above conventional understandings of 1)-4) about the M-M experiment may contain some confusions. It is worth mentioning that, the speed of waves only represents the propagation of wave oscillations, but not the actual shifting of medium particles in a long distance. Generally speaking, the propagation speed of waves cannot be superposed with the speed of translational movement of an object with a large displacement. Only when the medium carrying the wave is moving together with the wave, it might be feasible to consider the superposition of the two speeds in an appropriate way.

In fact, a straightforward conclusion derived from the M-M experiment is trivial: Light is a kind of wave oscillation similar to other waves. By default, Galilean superposition of motion is not for waves and translational movement. For example, if a boy carries a water gun running at a speed  $v$  in the same direction as the water shooting out of the gun with a speed  $u$ , the water shooting speed relative to the ground will be  $v + u$ , which is based on the Galilean principle of



superposition. Whereas, if the boy swims in a river at a speed  $v$ , and the water wave he generated has a propagation speed  $c$ , then the two speeds cannot be superposed. Similarly, when a boy is running at a speed  $v$  with a laser pointer in hand, the light speed of  $c$  cannot be superposed with the running speed of  $v$ , because light is a kind of wave oscillations in the vacuum carrying the EM waves. The speed of light is a native property of the vacuum (or medium), which has nothing to do with the speed of the source, unless the running speed is so high that shock wave is generated in the medium of some materials. The wave nature of light has been established and widely accepted before the M-M experiment, with the speed of light derived by Maxwell's wave equations, which is irrelevant to the speed of the source or observer. A constant speed of light has also been assumed for Doppler effect in the motion-induced frequency shift.

Also, the interpretation of M-M experiment will be the same as a double-slit interference: Since there are no changes in the optical path between the source and observe (the screen), when rotating the setup or arranging the experimental table along different directions during the double-slit interference experiments, there will be no interference pattern shift in the M-M experiment. The only difference between the two experiments is the angle between the two light beams. As to the derivations of special relativity with correct results including Lorentzian transformation, they have little correlation with the M-M experiment.

Some people may argue that the conventional understanding about the M-M experiment is based on the concept of photons, when considering the speed of photons, it will be suitable to apply the Galilean superposition principle, and the observations demonstrated the invalidity of the principle for this case.

In fact, the concept of photons originally proposed by Planck's radiation quantization is from the energy (quanta) perspective, as the light wave interacts with other matters such as electrons, the electro-magnetic field may be focusing or "collapsing" into a wavelet of energy quanta, and behave like a particle. When rotating the table in the M-M experiment, there is no change in the optical path, speed, or phase, hence there will be no changes in the interference pattern, irrelevant to the speed of the table on the Earth moving relative to whatever reference system carrying the entire experimental setup. In short, M-M experiment can be understood in a simple way by viewing light as a wave, as it should be treated in the interference experiment.

The subsequent argument about the "non-existence of ether" also turns to be confusing. As one of the sub-concepts belonging to the main concept of vacuum, it will be more convenient to define a sub-concept for "the part of the entire existence, or the part of the whole property of vacuum, as a medium carrying the electro-magnetic field", which may differ from "the subset of vacuum as a medium carrying the gravitational field". Otherwise, if one uses the word "vacuum" to represent "the being which carries the EM fields, which is a part of the whole vacuum", it will enter into a fundamental logic paradox proposed two thousand years ago as "a white house is, or is not, a house".

The concern about potential confusions also applies for the word “space curvature” adopted by general relativity. It will be more appropriate to name the curved trajectory as gravitational field curvature, similar to EM field curvature or vortex. By referring a curved motion to as “space curvature” based on a specific mechanism such as general relativity, will severely underestimate the significance of other mechanisms.

From the methodology point of view, the word “space curvature” adopted by general relativity will make the subsequent discussions and understandings for complicated phenomena more complicated and frustrated than they should be, especially when considering objects with both the gravitational and electro-magnetic interactions. When an electrical charge moves in a magnetic field along a curved path, at the same time, subject to a gravitational field, it will be confusing to name the space curvature. Also, when two objects rotating around the same massive objects at the same location with different directions, the space curvature will be of multiple values, it seems less meaningful to define a “parallel universe” or “multiple spaces”, just based on two balls running in different directions at the same location.

Some people may argue that: when general relativity mentions space curvature, the underlying meaning is the mass of the rotating object around a massive central mass is not involved in the orbital equation.

In fact, when an object of a much smaller mass  $m$  rotating around a massive mass  $M$ , the mass  $m$  is not involved in the orbital equation, which has already been reflected by Newton’s laws of gravity, as indicated by the simple expression  $(m + M)/M \sim 1$ . Also, the cancellation of the smaller mass  $m$  at both sides of the orbital equation does not imply the gravitational field will be irrelevant to the orbiting mass  $m$  under all conditions as asserted by general relativity. There may exist other phenomena with the mass  $m$  involved, unexplainable by the space curvature of general relativity.

The theory of general relativity has reflected certain features of gravity, yet, when extending some specific features to as general principles, using profound words or statements with potential underlying ambiguities and complexities, to some extent, general relativity might have made the originally simple phenomena more complicated, and also underestimated other features in violation of the general principle.

It is worth noting that, the anomalous precession can be understood by the non-linear perturbation in the orbital equation based on Newton’s law of gravity, with a similar perturbation function as the optical interference, or frequency mixing in the mixers for EM waves. The non-linear term of the additional gravitational field can be correlated with the retarded potential of EM fields proposed in the 1900’s, in combination with the symmetrical transformation similar to the EM fields.

#### 4. Conclusion

In summary, a different approach to the analyses of gravitational field is pro-

posed based on the retarded potential and symmetrical transformation for relative motion, without resorting to the space-time curvature of general relativity. Several major gravitational phenomena have been explained, including the anomalous precession of the perihelion of orbital stars or planets, the gravity-induced frequency shift, light deflection in the trajectory by the gravity, and gravitational radiations from binary pulsars, with results similar to or more accurate than those as so far obtained by general relativity.

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