

# Income Inequality Generates Chaos

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## Abstract

The present paper is an attempt to bridge the gulf between economics and econophysics. That is, constructing a chaos-based theoretical model, we show the behavior of the goal-driven agents exhibits the behavior of the purpose-free agents. Main conclusion is: economy becomes chaos if 1) capital gain of the middle class people is large enough for them to consume eight times as much as their income gain and 2) market for the middle class people is large enough and number of the middle class people is 16 times as large as the amount of products made by one producer.

## Keywords

Income Inequality, Econophysics, Chaos, Logistic Map, Maximization of Profits

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## 1. Introduction

Studies on income inequality, which progressed with masterpieces being series of empirical researches by Thomas Piketty and his colleagues ([1]-[8] etc.), have entered a new stage since [9] showed the appropriate income distribution by developing a theoretical framework to complement the empirical researches and bridge the wide conceptual gulf that exists between economics, which assumes rational agents who maximize their utility or profit, and econophysics, which assumes purpose-free agents who act randomly with near zero intelligence as [10] showed. More precisely, [9] explained the goal-driven behavior of agents, by constructing a theoretical model of purpose-free agents.

The present paper is another attempt to bridge the gulf between economics and econophysics. That is, we construct a chaos-based theoretical model where, in contrast to [9], the behavior of the goal-driven agents exhibits the behavior of the purpose-free agents.

## 2. Basic Model

Let us consider an intertemporal economy that consists of one representative

firm and two classes of people, namely, working class people (*i.e.*, laborers) and middle class people (*i.e.*, consumers). The firm inputs the labor of the working class people to sell the products to the middle class people whose income source is the dividend from the firm and the capital gain from their assets. As in [11], we assume that consumers (*i.e.*, middle class people), who purchase either 0 or 1 unit of products, are heterogeneous in that each of these consumers has different willingness-to-pay for the products, to generate the demand curve as aggregate of such demands. In the following, letting subscript  $t$  denote the period, we specify the aggregate demand function in period  $t$  as  $p_t = a_t - b_t x_t$ , where  $p_t$  is the product's price,  $x_t$  is the aggregate demand for the product,  $a_t$  is the highest willingness-to-pay among the middle class people and  $b_t$  is a positive parameter that expresses the distribution of the willingness-to-pay of the middle class people. We assume the willingness-to-pay of each consumer relates positively to her/his income in period  $t$  so that the shape of the aggregate demand curve reflects the income distribution. In the following, also for the simplicity of the analysis, we assume that each consumer's willingness-to-pay is equal to  $\theta \times 100\%$  of her/his income, and letting  $c$  be a positive constant, we specify the relationship between  $a_t$  and  $b_t$  as

$$\frac{a_t}{b_t} = c \quad (1)$$

by assuming that the income distribution of the middle class people is constant over time.

As for the decision making of the firm, we assume she/he determines the wage rate to maximize her/his profit in each period, anticipating, as in the efficiency wage model ([12] [13] and so on), that higher wage rate pulls up the efficiency of the labor, and hence the quality and the price of the product. Letting  $w_t$  and  $p_t$  denote the wage rate and the price of the product in period  $t$ , we specify the relationship between  $w_t$  and  $p_t$  as  $p_t = A_t w_t$ , where  $A_t$  is a positive variable that expresses the efficiency of the labor in period  $t$ , which we assume to go down if the supply of the product (which is equal to the demand for the product)  $x_t$  increases. We also assume that  $x_t$  relates positively to the highest willingness-to-pay  $a_t$ , so that  $A_t$  relates positively to  $a_t$ . In the following, letting  $\eta$  be a positive parameter, we specify the negative relationship between  $a_t$  and  $A_t$  as

$$A_t = \frac{\eta}{a_t} \quad (2)$$

Finally, if we specify that one unit of production requires  $\phi$  units of labor, the firm's profit in period  $t$ ,  $\pi_t = (p_t - \phi w_t)x_t$  is expressed as

$$\pi_t = \left( \frac{\eta}{a_t} - \phi \right) w_t \left( a_t - \frac{\eta w_t}{a_t} \right) \frac{c}{a_t} \quad (3)$$

Since the firm's control variable is  $w_t$ , we have the firm's first order condition for the profit maximization as  $\frac{d\pi_t}{dw_t} = \left( \frac{\eta}{a_t} - \phi \right) \left( a_t - \frac{2\eta w_t}{a_t} \right) \frac{c}{a_t} = 0$ , to obtain the

profit maximizing wage rate in period  $t$  as

$$w_t = \frac{a_t^2}{2\eta} \tag{4}$$

By substituting (4) into (3), we have the firm's maximum profit in period  $t$  as

$$\pi_t = \frac{ca_t}{4} \left( 1 - \frac{\phi}{\eta} a_t \right) \tag{5}$$

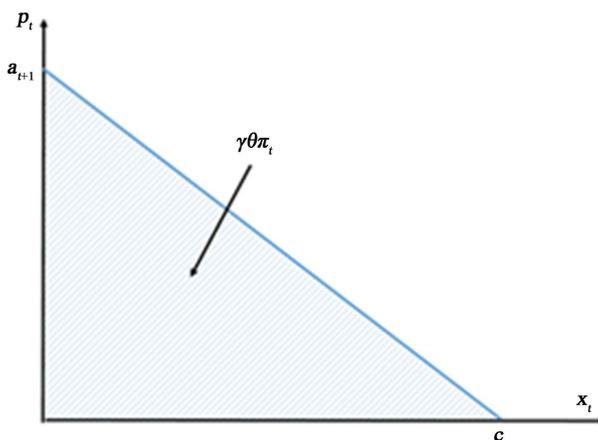
from which we can see that increase in the highest willingness-to-pay in period  $t$  ( $a_t$ ) has two effects, that is, profit enhancing effect caused by price increase (which is expressed by the term,  $\frac{ca_t}{4}$  and profit reducing effect caused by wage increase (which is expressed by the term,  $\left( 1 - \frac{\phi}{\eta} a_t \right)$ ). Now the model is complete.

### 3. Fixed Inequality and the Chaos

Based on the above analysis, this section shows that fixed income inequality among the middle class people, as well as the immobility between the two classes (*i.e.*, the middle class and the labor class), generates chaos.

For this purpose, let us assume that the firm's profit in each period is allocated to the middle class people so as to maintain the distribution of the income, which is equivalent with the distribution of the willingness-to-pay. Assuming that total income of the middle class people (*i.e.*, sum of the dividend from the firm plus the capital gain from their assets) is  $\gamma$  times as large as their total income gain (which is equal to  $\pi_t$ ), total willingness-to-pay of the middle class people is  $\gamma\theta\pi_t$ , since, as mentioned above, each consumer's willingness-to-pay is equal to  $\theta \times 100\%$  of her/his income. Thus, aggregate demand curve in period  $t + 1$  is depicted as a triangle as in **Figure 1**, with its area and the width of the bottom being  $\gamma\theta\pi_t$  and  $c$ , which establishes the following equation:

$$\frac{ca_{t+1}}{2} = \gamma\theta\pi_t \tag{6}$$



**Figure 1.** Aggregate demand curve in period  $t + 1$ .

This relationship between  $\pi_t$  and  $a_t$  combined with (5) generates the dynamics of  $a_t$  as

$$a_{t+1} = \frac{\gamma\theta a_t}{2} \left( 1 - \frac{\phi}{\eta} a_t \right) \tag{7}$$

Thus, by assuming  $\gamma\theta = 8$  and  $\frac{\phi}{\eta} = 1$ , and substituting these into (7), we obtain

$$a_{t+1} = 4a_t(1 - a_t) \tag{8}$$

which is the logistic map [14] demonstrated to generate chaos. **Figure 2** is the logistic map of Equation (8) that shows the unpredictable cobwebbing behavior of  $a_t$ .

In this case, since supremum of  $a_t$  is 1,  $\eta$  is the infimum value that one unit of wage generates. So that, we have the following proposition.

*Proposition 1: Intertemporal fluctuation of the market becomes chaos if 1) the capital gain of the middle class people is large enough for them to consume eight times as much as their income gain and 2) the infimum value that one unit of wage generates is the same as the amount of products made by one producer.*

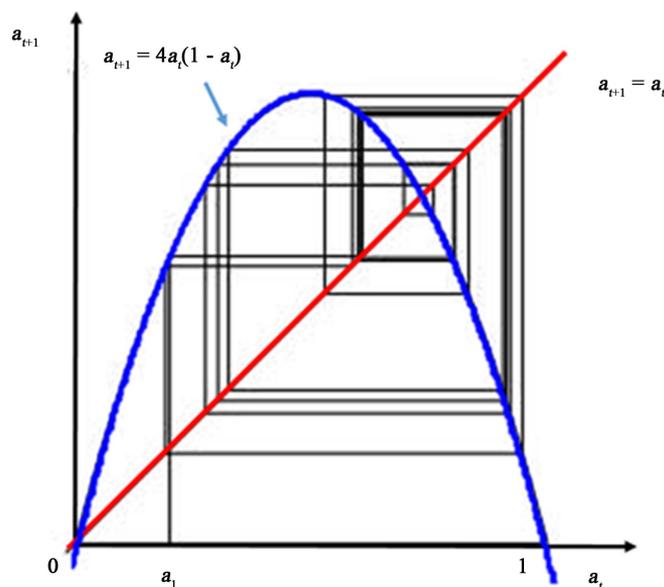
If we combine (8) with (6), we have the dynamics of  $\pi_t$  as

$$\pi_{t+1} = 4\pi_t \left( 1 - \frac{16}{c} \pi_t \right) \tag{9}$$

Thus, by assuming  $c = 16$  and substituting these into (9), we obtain

$$\pi_{t+1} = 4\pi_t(1 - \pi_t) \tag{10}$$

which is also the logistic map that generates chaos whose behavior is shown virtually the same as in **Figure 2**.



**Figure 2.** Logistic map of  $a_t$ .

*Proposition 2: Economy becomes chaos if 1) the capital gain of the middle class people is large enough for them to consume eight times as much as their income gain and 2) market for the middle class people is large enough number of the middle class people is 16 times as large as the amount of products made by one producer.*

#### 4. Conclusions

The present paper attempted to bridge the gulf between economics and econophysics by constructing a chaos-based theoretical model where the behavior of the goal-driven agents exhibits the behavior of the purpose-free agents.

Main conclusion is: economy becomes chaos if 1) the capital gain of the middle class people is large enough for them to consume eight times as much as their income gain and 2) market for the middle class people is large enough and number of the middle class people is 16 times as large as the amount of products made by one producer.

We truly hope this research note will contribute to the progress of studies on the income inequality and the chaos.

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