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Hubble's Constant and Flat Rotation Curves of Stars: Are Dark Matter and Energy Needed?

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Abstract

Although dark energy and dark matter have not yet been detected, they are believed to comprise the majority of the universe. Observations of the flat rotation curve of galaxies may be explained by dark matter and dark energy. This article, using Newton's laws and Einstein's theory of gravitation, shows that it is possible to define a new term, called E_0, variable in time and space, of which one of its limits is the Hubble constant H_0. I show that E_0 is strongly linked to an explanation of the flat rotation curve of galaxies. This strong correlation between Hubble's constant H_0 and E_0 enables us to solve the mystery of the surplus of gravity that is stabilizing the universe.

Keywords

Hubble's Law, General Relativity, Cosmological Constant, Expanding Universe, Dark Energy

1. Introduction

Galaxies appear to rotate more quickly than allowed by the gravity of its observable matter. Logically, they should have been destroyed a long time ago, as should galaxy clusters. Because of this disagreement between theory and observations, some undetectable element is believed to be providing galaxies with the additional mass that they need to avoid breaking up. This mysterious element has been called "dark matter". Over the years, coherent theories on dark matter and dark energy have been formulated to explain the stability of galactic systems. Attempts have been made to modify Newton's laws of gravity, for example, Milgrom's MOND theory [1], which explains rather well the flat rotation curves of stars for many galaxies. However, these theories have not yet offered a sufficiently well explained source; they are rather empirical theories. In this article, using Newton's laws and Einstein's theory of general relativity [2], I show that a coherent theory is possible that can explain, at the same time, flat rotation curves, the expansion of the universe, and the acceleration of the expansion. I will define a new term, E_0, one limit of which is the Hubble constant, and I clearly show its relationship with the flat rotation curves of galaxies. This new term E_0 is universal because it applies without restriction to all celestial bodies.

2. Hubble's Constant and Defining the New Term $E_0 \approx H_0$

The Hubble constant H_0 is the constant of proportionality between the distance and speed of apparent recession of galaxies relative to any point in the observable universe. It is connected to the famous Hubble's law, which describes the expansion of the universe. Although referred to as a constant, H_0 actually varies depending on time and distance. It thus describes the growth rate of the universe at one moment and at a given distance from the point of observation. The most precise value of H_0 (from optical observations, May 2001) is 72 ± 8 $\text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$.

However, the current definition of H_0 does not relate expansion of the universe to the flat rotation curves of galaxies and galaxy clusters. I must thus extend the concept of expansion of the universe to a comparison of the gravitational acceleration for small and large distances. Thus Newtonian acceleration can be written as γ_{Newton} for small distances (less than kpc), and γ_{measured} and D_{measured} (for distances greater than kpc) are the measured acceleration and distance, respectively, between gravitationally interacting masses. We can therefore write the following relationship for a galactic system in gravitational balance:

$$\gamma_{\text{measured}} = \frac{V_{\text{measured}}^2}{D} \tag{1}$$

where V_{measured} is the measured speed of rotation. Einstein's relativity theory may also be applied to Newtonian gravitational acceleration γ_{Newton} so that the masses vary as $\frac{M}{\sqrt{1-\frac{v^2}{c^2}}}$ and the distances in the moving reference frame (for the observer) shorten according to $d\cdot\sqrt{1-\frac{v^2}{c^2}}$. In fact, Newton's second law in relativistic mechanics (special relativity) may be applicable in my calculations. Here, v represents the radial velocity between two gravitationally interacting objects (not to be confused with the rotation speed V_{measured} of planets, stars, galaxies, and galaxy clusters around the center of their respective system). This radial velocity v can also be regarded as the recessional velocity of the object of interest (galaxies, stars, planets) similar to the relationship $H_0 \cdot D$, where H_0 is the Hubble constant and D is the distance.

On the other hand, I take into account the relativistic function of acceleration γ developed by Elbeze [3] [4], in which I include the recessional rate of the universe defined by function $E_0 \cdot D$ (added at the possible radial speed v), which will be defined below (and is equivalent to the Hubble expression $H_0 \cdot D$), and obtain:

$$\gamma_{\text{Newton}} = \frac{G\cdot M}{D^2}\cdot\frac{1}{\sqrt{1-(v+Eo\cdot D)/c}}\cdot \quad \text{and} \quad V_{\text{Newton}} = \sqrt{\frac{G\cdot M}{D}\cdot\frac{1}{\sqrt{1-(v+Eo\cdot D)/c}}} \tag{2}$$

From these relationships, I can define a rate of variation of acceleration τ_γ between the Newtonian acceleration (for all distances) and the measured acceleration γ_{measured} as follows:

$$\tau_\gamma = \frac{\gamma_{\text{measured}} - \gamma_{\text{Newton}}}{\gamma_{\text{Newton}}} \tag{3}$$

I can compare Equation (3) with the Hubble constant H_0 as follows:

$$\tau_\gamma \cdot V_0 \cdot \frac{1}{D} = E_0 \approx H_0 \tag{4}$$

where E_0 is the equivalent of H_0 in this article, and V_0 is the reference radial velocity, which I will define below. By replacing accelerations γ_{measured} and γ_{Newton} with their equivalent in speed and distance in Equations (1), (2), (3), and (4), so that radial velocity is negligible in calculating the rate $E_0 \cdot D$, while applying the relativistic principle for distance and masses in radial motion (defined above and according to Newton's second law in special relativity to a radial dimension along $\boldsymbol{D}$), and with $\beta = \left(1 - \left(E_0 \cdot D/c\right)^2\right)^{-\frac{1}{2}}$, I obtain:

$$\frac{\dfrac{V_{\text{measured}}^2}{D} - \dfrac{\beta^2 \cdot G \cdot \text{Mgalaxy}}{D^2}}{\dfrac{\beta^2 \cdot G \cdot \text{Mgalaxy}}{D^2}} \cdot V_0 \cdot \frac{1}{D} = E_0 \tag{5}$$

where $E_0 \cdot D$ is the recessional speed of mass M_2 from mass M_1 (**Figure 1**).

Here, D is the distance between masses M_1 and M_2, their relative speed v (radial velocity) is negligible, and V_{measured} is the measured speed of rotation of M_2 around mass M_1. Solving Equation (4) and Equation (5) for E_0 gives two universal solutions:

$$E_0 = \frac{1}{D} \cdot \left[\pm c \cdot \sqrt{1 - \left(\frac{V_{\text{Newton}}}{V_{\text{measured}}}\right)^2 + \left(\frac{V_{\text{Newton}}}{V_{\text{measured}}}\right)^4 \left(\frac{c}{2V_0}\right)^2} - \left(\frac{V_{\text{Newton}}}{V_{\text{measured}}}\right)^2 \frac{c^2}{2V_0} \right] \tag{6}$$

We can already assume that M_1 and the gravitational constant G (from equation 2) disappear from Equation (6), because they are factors in the elements $\left(\frac{V_{\text{Newton}}}{V_{\text{measured}}}\right)$ composing it. Only speeds V_{Newton} and V_{measured}, distance D, and the reference speed V_0, which will be defined low, are present in Equation (6). Thus Equation (6) is universal, independent of M_1 and M_2, and applies to all points in the universe with no distinctions. Let us write the recessional velocities vE_0 and vH_0 as follows:

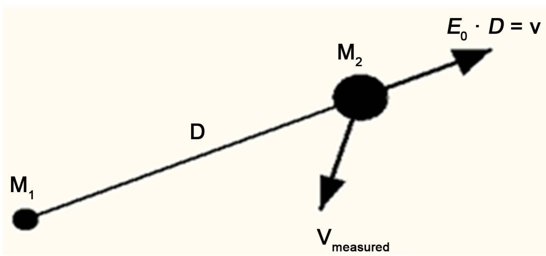

Figure 1. Measurement of the proposed recessional rate E_0 for coupled masses M_1 and M_2.

$$vE_0 = E_0 \cdot D \qquad vH_0 = H_0 \cdot D \tag{7}$$

3. Relationship between E_0 and H_0 and the Flat Rotation Curves of Galaxies

One possible solution for the speed of rotation V_{measured}, which solves Equation (6), is:

$$V_{\text{measured}} = VE_0 = \sqrt{\frac{G \cdot M_g}{D} \cdot \left[\frac{1}{\sqrt{1-(v+E_0 \cdot D)/c}}\right] \cdot \Lambda\left[Wpe \cdot \frac{D}{rh}+1\right] - \gamma D \cdot D} \tag{8}$$

where γ_D is the acceleration of the expansion of the universe (which is here regarded as small compared with gravitational acceleration γ for distances < 1 Mpc); G is Newton's gravitational constant; M_g is the mass of the planet, star, galaxy, or galaxy cluster; D and v are the distance and relative radial speed, respectively, between masses M_1 and M_2 (**Figure 1**); $E_0 \cdot D$ is the recessional velocity of the object of interest; rh is the atomic radius of hydrogen; and the two functions Λ (value between 1 and 2) and Wpe, which is on the order of 10^{-32} depending on the nature of the celestial body (e.g., galaxy, star), are defined in **Appendix 1** and **Appendix 2**, respectively. By replacing $V_{\text{measured}} = VE_0$ (Equation (8)) in Equation (6), the term $\sqrt{\frac{G \cdot M}{D} \cdot \frac{1}{\sqrt{1-(v+E_0 \cdot D)/c}}}$ from Equation (8) (which defines the Newtonian part of VE_0) disappears before the factors $\left(\frac{V_{\text{Newton}}}{V_{\text{measured}}}\right)$, so that with

$$a(D) = \frac{1}{\sqrt{\Lambda \cdot \left(Wpe \cdot \frac{D}{rh}+1\right)}}$$

then Equation (5) becomes

$$E_0 = \frac{1}{D} \cdot \left[\pm c \cdot \sqrt{1-a(D)^2 + a(D)^4\left(\frac{c}{2V_0}\right)^2} - a(D)^2 \frac{c^2}{2V_0}\right] \tag{9}$$

I can apply Equation (8) (e.g., to our galaxy) in the case of gravitational balance of the Milky Way, and assume the acceleration of the expansion of the universe to be equal to γ_D (defined low), here γ_D is the acceleration in $\text{m} \cdot \text{s}^{-2}$, we obtain the **Figures 2-4**.

In **Figure 2**, the position of our Sun is at approximately 8.2 kpc and its speed is $230\ \text{km} \cdot \text{s}^{-1}$. The speed returns to its Newtonian form to 2.5 Mpc. In my calculations, only the mass of the bulb is considered; I do not take account of the mass of the galactic gas clouds, which in any case do not influence the effective existence of the flat rotation curves of the galaxy, but modify only the form of the curve speeds. I apply Equation (9) to define E_0 (the proposed new expansion rate) in relation to the Hubble constant H_0 (Hubble expansion rate) (**Figure 3**).

Figure 3 shows four terms: vE_0 and vH_0, the proposed expansion rate (E_0), and the Hubble constant (H_0). Values of E_0 and H_0 are given in units of km·s^{-1}/Mpc; in this article, H_0 is equal to 73.0 ± 1.75 km·s^{-1}/Mpc for a distance of $D \approx 20$ Mpc [5]

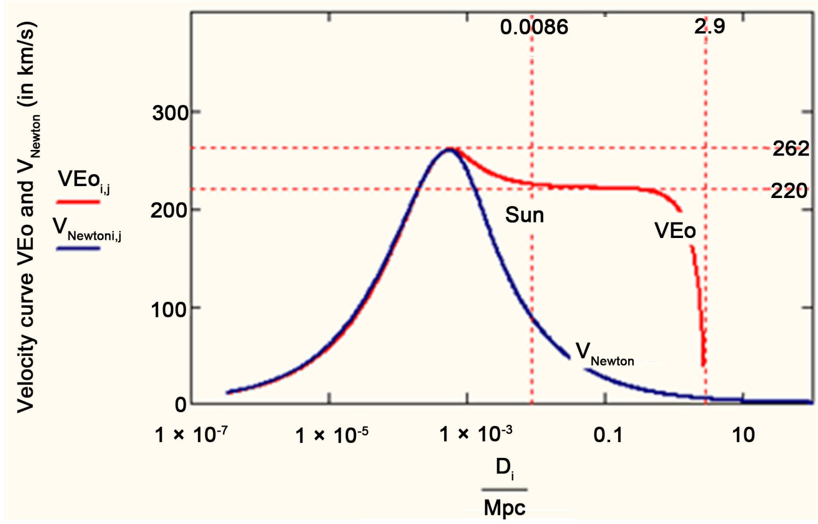

Figure 2. Flat velocity curve of the Milky way galaxy.

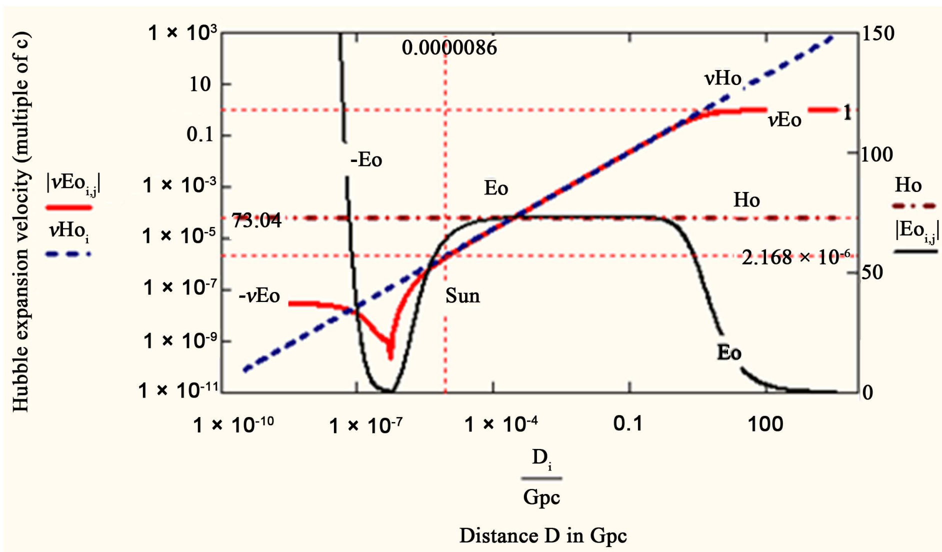

Figure 3. Relationship between the Hubble constant and E_0.

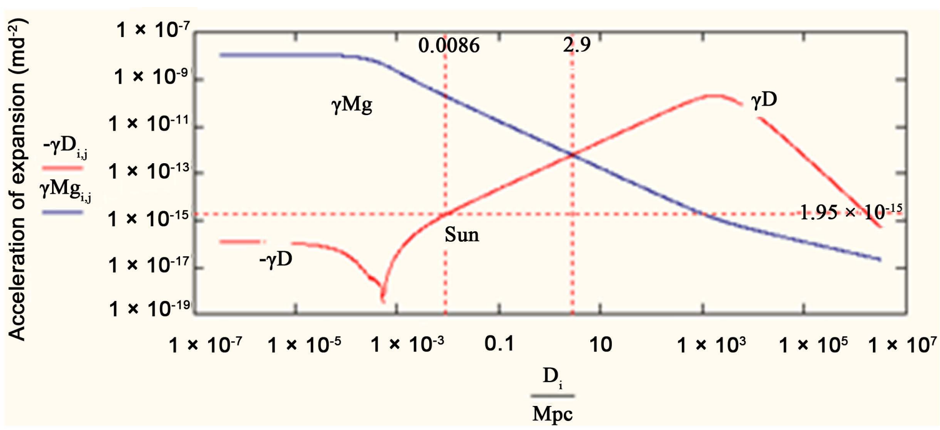

Figure 4. Acceleration of a celestial body of mass M_g and acceleration of expansion of the universe γ_D.

[6], and E_0 for a large distance ($10\ \text{Mpc} \le D \le 50\ \text{Mpc}$) is equal to 73.036 ± 0.1 km·s^{-1}/Mpc. I stated the position of the Sun in the Milky Way, which is approximately equal to 8.6 pc (0.0000068 Gpc) from the galactic center, with a rotation speed of 230 km·s^{-1} and an expansion rate of $2.168 \cdot 10^{-6}\ \text{c} = 0.650\ \text{km} \cdot \text{s}^{-1}$.

In **Figure 3**, we can see the difference between H_0 and E_0. For H_0, the speed vH_0 represents a straight line with a fixed slope, whereas for E_0, the slope is variable all along distance *D*. At large distances, the expansion rate vE_0 is positive (repulsion, *i.e.*, galaxies move apart) and tends toward the speed of light *c* (thus conforming to Einstein's relativity theory); at a smaller distance it becomes negative (attraction) and almost constant, for part of the curve after the reversal since the distance $0.5\ \text{kpc} \to 0\ \text{kpc}$. This part of the curve is at small distances less than $0.5\ \text{kpc}$, particularly regarding the solar system and its planets for which the expansion function E_0 also applies. I can also plot the curve of the acceleration of the expansion of the universe γ_D, taking the derivative of $E_0 \cdot D$ with respect to time as

$$E_0 \cdot D \cdot \frac{\mathrm{d}(E_0 \cdot D)}{\mathrm{d}D} = \gamma_D$$

where $\mathrm{d}D = V \cdot \mathrm{d}t = E_0 \cdot D \cdot \mathrm{d}t$ (**Figure 4**).

In **Figure 4**, we can see three areas of acceleration: a zone from 0 to 0.5 pc where there is a constant rate of negative expansion (of $10^{-16}\ \text{m} \cdot \text{s}^{-2}$, attraction); a zone between 0.5 pc and 2 Gpc where there is an accelerated rate of positive expansion (repulsion); and a zone between 2 Gpc and above 3 Tpc where there is a decelerated rate of expansion (always repulsive). Finally, the acceleration γ_D produced by the expansion of the universe is much less than the acceleration γM_g produced by mass M_g up to the value of 2.9 Mpc for the Milky Way, and is negligible for our solar system. It seems that beyond 2.9 Mpc, other celestial bodies cannot be orbiting the Milky Way!

4. Definition of the Value of the Speed of Reference V_0 in Our Solar System

I can rewrite an approximation of Equation (8) by neglecting the acceleration of the universe for the Earth (**Figure 4**) and by setting $v = 0$ as follows (here for our solar system):

$$V_{\text{measured}} = VE_0 = V_{\text{earth}} = \sqrt{\frac{G \cdot Mg}{D} \cdot \left(\frac{1}{\sqrt{1 - \frac{E_0 \cdot D}{c}}} \right) \cdot \Lambda \left[Wpe \cdot \frac{D}{rh} + 1 \right]} \quad (10)$$

This reference speed V_0 will be calculated at the level of our solar system. This has two principal advantages: we know the precise Earth-Sun distance, and the flat expansion velocities are relatively stable (here attractive to the Sun; see **Figure 3**). Using universal Equations (8), (9), and (10), which also apply to our solar system, I deduce the value of $V_{\text{ref}} = V_0$, depending on the position and speed of rotation of the Earth around the Sun ($D = R_{\text{earth-sun}}$ and $V_{\text{earth}} = VE_0$, respectively). By comparing the value of E_0, which is given by Equation (10), with the value in Equation (9), I get:

$$V_0 = c \frac{\frac{V_{\text{earth}}^2}{G \cdot Mg} - \frac{\Lambda}{D}\left(Wpr \cdot \frac{D}{rH} + 1\right)}{\Lambda \cdot \left(Wpr \cdot \frac{D}{rH} + 1\right)} \cdot Y \cdot D \tag{11}$$

In this relationship the value of Y is obtained by replacing the value of V_0 in Equation (9) by $Y \cdot E_0 \cdot D$. Indeed, the V_0 value is similar at the speed of expansion of the universe, and this fact answers the classical formula of Hubble. The function Y is a constant dependent on the zone of space considered. For a distance D (about the distances of the solar system) $Y \approx 10$. Finally, the calculation of V_0 in the solar system gives $V_0 = 283.644\ \text{m} \cdot \text{s}^{-1}$. To obtain this value, I have to check the equation

$$V_0 \left(V_{\text{ref}}\right) / V_{\text{ref}} = 1$$

where $V_0 = Y \cdot E_0 \cdot D$ (Figure 5).

The term $\left(n_0/n\right)^5$ depends on the nature of the star, galaxy, or planet in question; here for our solar system $n = 2.455388002$ and $n_0 = 2$, and for the Milky Way $n = 2$ and $n_0 = 2$. The values of n and no define the function Λ. Thus, the reference speed V_0 will have a value dependent on the nature of the considered system, or:

$$V_0 = V_{\text{ref}} \cdot \left(\frac{n}{n_0}\right)^5 \quad \text{with } V_{\text{ref}} = 101.7\ \text{m} \cdot \text{s}^{-1} \tag{12}$$

For the solar system: $V_0\left(\text{Sun}\right) = V_{\text{ref}} \cdot \left(\frac{n\left(=2.455388002\right)}{n_0\left(=2\right)}\right)^5 = 283.644\ \text{m} \cdot \text{s}^{-1}$.

In the Milky Way galaxy: $V_0 = V_{\text{ref}} \cdot \left(\frac{n\left(=2\right)}{n_0\left(=2\right)}\right)^5 = 101.7\ \text{m} \cdot \text{s}^{-1}$.

Thus, V_{ref} is a universal speed of reference for all gravitational systems. This value V_{ref} is thus directly related to the value of the expansion rate of the universe E_0, as well as to the flat rotation curves of galaxies. Figure 6 shows the point where the distance of the Sun from the center of the Milky Way and the ratio $V_0/V_{\text{ref}} = \left(n/n_0\right)^5$

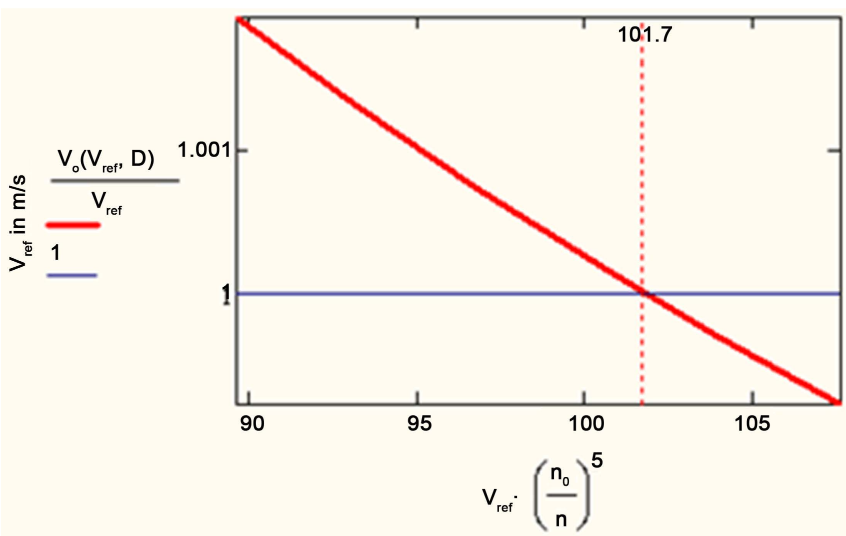

Figure 5. Ratio V_0/V_{ref}.

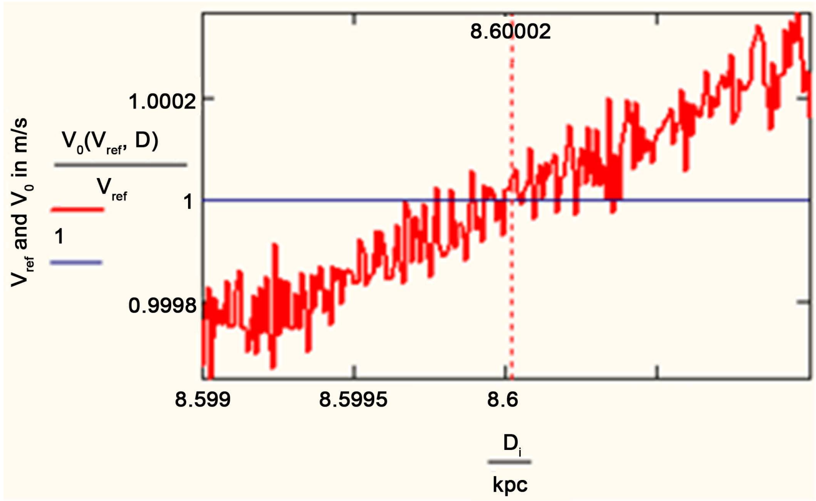

Figure 6. Distance from Sun to center of Milky Way for $V_0/V_{\text{ref}} = 1$.

coincide. This equality requires the conditions $\frac{V_0(V_{\text{ref}})}{V_{\text{ref}}} = 1$ for $n_0 = n$ and D_{sun} to the center of the Milky Way equal to $8.60002\ \text{kpc}$.

5. Conclusion

I have shown in this article that it is possible to link the concept of expansion of the universe through a new term E_0 (which is comparable to the Hubble constant H_0), and the flat rotation curves of galaxies, with one limit according to galactic mass. For the Milky Way, the limit (end of the non-Newtonian rotation curves) is around 2.9 Mpc. This is possible using both the widely accepted gravitational theories of Newton and Einstein. I suggest a solution (Equations (8) and (10)) for the expression of the flat rotation speed of galaxies (or other galactic objects) which is derived from the universal relation of expansion of the universe E_0 (Equation (9)). This solution gives excellent results. The first and second derivatives of the value of E_0 (Equation (6)) are a function of time, and show that the universe is in acceleration until about $3\ \text{Mpc}$, then in deceleration until approximately $3\ \text{Tpc}$. This is new information about the universe. Thus, we can legitimately ask whether dark matter is necessary to explain the flat rotation curves of galaxies? And because of this accelerating expansion, could it be that dark energy does not exist? In this article, I have shown that the basic concept of the expansion of the universe also applies to the solar system like with planets and their satellites. Based on this, the Hubble constant H_0 and of Elbeze E_0 could be measured with more accuracy. The results presented in this article may solve some current issues such as the energy density of galaxies, their masses, and may help to solve the mystery of the formation of the universe and the presence of filaments of material connecting the clusters and super clusters of galaxies of the current standard model, all without the need to involve hypothetical dark matter and dark energy.

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Appendix 1: Calculation of Λ

Let $n = n_0 = 2$, a relative coefficient taken as a reference for our galaxy, the Milky Way. Any other object in the universe will also have a number n that varies according to the object's characteristics such as its mass distribution, its linear density σ, and the shape of its flat rotation curves. For the Sun $n = 2.455388002$. In this article, the distribution of the masses is comparable to that of a thin disk (the spherical distribution does not change the final results) of apparent mass equal to MD:

$$MD = \int_0^D 2\pi r \cdot \sigma_0 \cdot \mathrm{e}^{-\frac{r}{R_0}} \cdot \mathrm{d}r \tag{A1.1}$$

where D is the distance from the center of the object of interest (e.g., galaxy, star) and R_0 is the radius of the central core, where for a galactic disk such as that of the Milky Way, R_0 is generally taken as equal to $0.15\ \text{kpc}$, for the sun and the earth their radius. The density σ_0 is equal to $\sigma_0 = \text{Mass of the body}/\text{kpc}^2$.

The value of Λ is defined as $\Lambda = \frac{MD}{M'D}$, where $M'D$ is the hidden value of the galactic mass near to its actual value for the distance D being considered. Let this mass be equal to

$$M'D = \int_0^D 2\pi r \cdot \alpha\sigma_0 \cdot \mathrm{e}^{\frac{r}{\beta \cdot R_0}} \cdot \mathrm{d}r. \tag{A1.2}$$

I define the coefficients α and β as being equal to

$$\alpha = \frac{a}{n} \quad \text{here } a = 2.3 \quad \text{and} \quad \beta = 2 \cdot \left(\frac{n_0}{n}\right)^3 \tag{A1.3}$$

and finally obtain Λ as a function of distance $D: \Lambda = \frac{MD}{M'D}$.

Appendix 2: Calculation of *Wpe*

In this article, the value of Wpe for our purposes is

$$Wpe(rH, rH_0, Ep) = 5.42 \cdot 10^{-30}. \tag{A2.1}$$

Here, Wpe is a function of at least three parameters: rH is the atomic radius of hydrogen; $rH_0 = rH \cdot n^2$ is the average atomic radius in the galaxy (or other object of interest) in terms of the orbital radius of hydrogen, where n is defined in **Appendix 1**; and Ep is the electron rest energy.

The relationship giving Wpe is written as follows:

$$Wpe = \frac{1}{ze} - \frac{1}{zp}$$

$$ze = \sqrt{1 + \frac{E\Delta p \cdot xp + E\Delta e \cdot xe}{Ee}}$$

$$zp = \sqrt{1 + \frac{E\Delta p \cdot xp + E\Delta e \cdot xe}{Ep}}$$

$$xe = \left[\frac{2\lambda e}{\Gamma\Delta e} \cdot \frac{rH}{rH_0}\right]^3 \qquad xp = \left[\frac{2\lambda p}{\Gamma\Delta p} \cdot \frac{rH}{rH_0}\right]^3$$

The value $(E\Delta p \cdot xp + E\Delta e \cdot xe)$ is always very small compared with Ep and Ee (proton and electron rest energy), and we can write:

$$Wpe = \frac{1}{ze} - \frac{1}{zp} \approx \frac{(E\Delta p \cdot xp + E\Delta e \cdot xe)^2 \cdot (Ee^2 - Ep^2)}{2Ee^2 Ep^2} \quad \text{(A2.2)}$$

where Ee and Ep are the electron and proton rest energies, respectively; λe and λp are modified wavelengths of the electron and the proton; $\Gamma\Delta e$, $E\Delta e$ and $\Gamma\Delta p$, $E\Delta p$ are the radial and volumetric interaction energy between the electron and the proton in the hydrogen atom. These values define the volume and the binding energy of the electromagnetic reaction between the electron and the proton, and ze and zp are the energy charge (similar to electric charge) of the proton and electron in the hydrogen atom as follow table:

Energy of electron and proton	λe and λp	$\Gamma\Delta e$ and $\Gamma\Delta p$	$E\Delta e$ and $E\Delta p$
$Ee = 8.18711 \cdot 10^{-14} J$	$2.82 \cdot 10^{-15}$ m	$7.3 \cdot 10^{-13}$ m	$3.76 \cdot 10^{-20}$ J
$Ep = 1.50328 \cdot 10^{-10} J$	$1.53 \cdot 10^{-18}$ m	$1.7 \cdot 10^{-14}$ m	$8.86 \cdot 10^{-22}$ J

By using the values in table, we obtain a value for our galaxy the Milky Way of $Wpe = 5.42 \cdot 10^{-30}$. For a more detailed explanation of these values, the reader may wish to refer to the book by Elbeze [7].

In fact, for a first reading of this appendix, we can neglect the values of the above table and consider only the value of $Wpe = 5.42 \cdot 10^{-30}$ as being a constant of integration.