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The Golden Ratio

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Abstract

The Lorentz transformation (if $x = ct$) is the same the golden ratio:
$\frac{t'}{t} = \frac{1 - v/c}{\sqrt{1 - v^2/c^2}} = \frac{\sqrt{5} - 1}{2} = 0.618$.

Keywords

Lorentz Transformation, Relativity, Golden Ratio

1. Introduction

So, the aim of this work is to derive the golden ratio connected to Lorentz transformation. Usually the golden ratio is derived from the a rectangle [1] but book [2] derives the golden ratio from a right-angled triangle. This makes possible to connect Lorentz transformation with the golden ratio. The Lorentz factor formula can be the easiest explained from the right-angled triangle. The formula $\sqrt{c^2 - v^2}$ is in fact the formula of one of the right-angled triangle side:

1) $\sqrt{c^2 - v^2} = c\sqrt{1 - \left(v^2/c^2\right)}$

It can be seen from this formula also that "c" is the hypotenuse of the right-angled triangle. At the same time "$c > v$" is one criterion of Lorentz factor formula as well. That means, that the hypotenuse of the right-angled triangle is "c", the other sides is "v" and $\sqrt{c^2 - v^2}$. This allows that from so defined right-angled triangle to explain the formula of Lorentz transformation too and the golden ratio formula as well. For the derivation of the equations high school mathematics is used. That's the reason why the equations are more transparent and easier to understand because they are simply.

2. We Introduce First the (t'/t) Formula

The derivation of Lorentz transformation can be found at the end of the book marked by [3] in the Appendix I. The derivation of Lorentz transformation equations starts from the fact that we measure two light-rays move face each other, in a standing and in a moving coordinate system. So the travelled distance of the light-ray in the standing coordinate system is $x = ct$. Naturally in the movable coordinate system the travelled distance of the light-ray is $x' = ct'$. Here it can be seen that Lorentz's starting point were a few assumptions, (see [3] Appendix I) and from these simple assumptions got the formula named after him. After that Einstein substituted in the Lorentz transformation equation t', the equation of $x = ct$. This derivation can be found in the book marked number [3] on the page 32. Now I show this derivation.

2) So, $x = ct$ and $x/c = t$:

$$t' = \frac{t - \left(v/c^2\right)x}{\sqrt{1-\left(v^2/c^2\right)}} = \frac{t - \left(v/c\right)\left(x/c\right)}{\sqrt{1-\left(v^2/c^2\right)}} = \frac{t - \left(v/c\right)t}{\sqrt{1-\left(v^2/c^2\right)}} = t\frac{1-\left(v/c\right)}{\sqrt{1-\left(v^2/c^2\right)}}$$

3) Equation (t'/t):

$$\frac{t'}{t} = \frac{1-\left(v/c\right)}{\sqrt{1-\left(v^2/c^2\right)}}$$

We are going to prove that the equation 3) is the golden ratio.

2.1. Now We Derive the Other Form of the Equation (3)

Before we start with the derivation of the golden ratio, we transform the Equation (3).

4) In further derivation we shall use the following equations from the Appendix I of the book [3]:

$$a = \frac{1}{\sqrt{1-\left(v^2/c^2\right)}}; \quad b = \frac{\left(v/c\right)}{\sqrt{1-\left(v^2/c^2\right)}}$$

5) In further derivation we shall use the following equations from the [4]:

$$\xi = \operatorname{artanh}\left(v/c\right)$$

($-1.0 < \left(v/c\right) < 1.0$ See the **Figure 1**).

6) From the [4] we take also the following equations:

$$\cosh\xi = \frac{1}{\sqrt{1-\left(v^2/c^2\right)}}; \quad \sinh\xi = \frac{\left(v/c\right)}{\sqrt{1-\left(v^2/c^2\right)}}$$

7) We shall transform the Equation (3) and take the Equation (4) and Equation(5) and Equation (6) into account:

$$\frac{t'}{t} = \frac{1-\left(v/c\right)}{\sqrt{1-\left(v^2/c^2\right)}} = \frac{1}{\sqrt{1-\left(v^2/c^2\right)}} - \frac{\left(v/c\right)}{\sqrt{1-\left(v^2/c^2\right)}} = a - b = \cosh\xi - \sinh\xi = \mathrm{e}^{-\xi}$$

8) So:

$$\frac{t'}{t} = \frac{1-\left(v/c\right)}{\sqrt{1-\left(v^2/c^2\right)}} = \mathrm{e}^{-\xi} = \mathrm{e}^{-\operatorname{artanh}\left(v/c\right)} = \frac{1}{\mathrm{e}^{\operatorname{artanh}\left(v/c\right)}}$$

The Equation (8) can be found in [5].

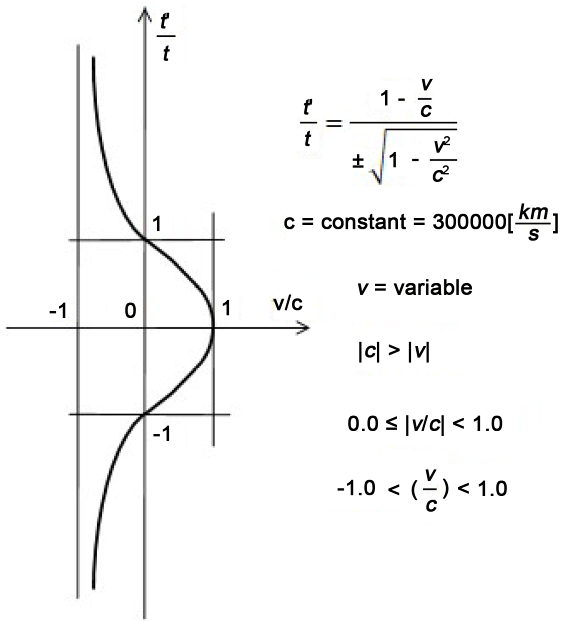

Figure 1. The Equation (8).

2.2. Derivation of the Golden Ratio

We mentioned in the introduction, the formula of golden ratio, we are going to derive it from a right-angled triangle [2]. (Note: **Figure 2** can be found in [2]).

9) See **Figure 2**: $F = 2v = \sqrt{c^2 - v^2}$

$$c^2 - v^2 = (2v)^2 \to c^2 - v^2 = 4v^2 \to c^2 = 5v^2 \to v = c\frac{1}{\sqrt{5}}$$

10) The golden ratio:

$$\frac{OE}{OA} = \frac{c-v}{F} = \frac{c-v}{2v} = \frac{c - c\frac{1}{\sqrt{5}}}{2c\left[\frac{1}{\sqrt{5}}\right]} = \frac{\sqrt{5}-1}{2}$$

11) See Equation (9):

$$v = c\frac{1}{\sqrt{5}} \to \frac{v}{c} = \frac{1}{\sqrt{5}}$$

12) Now we transform the Equation (10) so that it should be equal with Equation (8). We will take into account that:

$$F = 2v = \sqrt{c^2 - v^2}$$

$$\frac{OE}{OA} = \frac{c-v}{F} = \frac{c-v}{2v} = \frac{c(1 - v/c)}{c\sqrt{1 - v^2/c^2}} = \frac{1 - v/c}{\sqrt{1 - v^2/c^2}} = \frac{\sqrt{5}-1}{2} = 0.618$$

13) Of course, the Equation (12) is identical with Equation (8):

$$\frac{t'}{t} = \frac{1 - (v/c)}{\sqrt{1 - (v^2/c^2)}} = e^{-\xi} = e^{-\operatorname{artanh}(v/c)} = \frac{1}{e^{\operatorname{artanh}(v/c)}} = \frac{\sqrt{5}-1}{2}$$

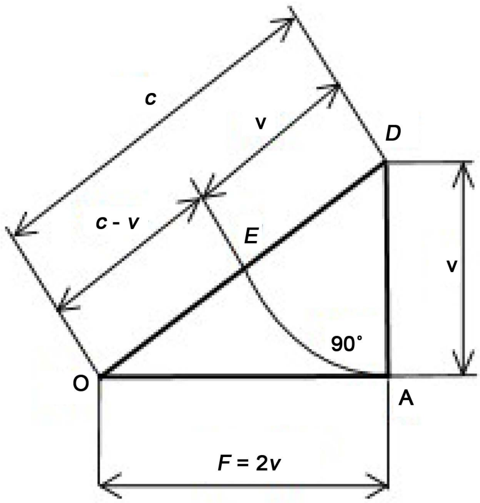

Figure 2. In the book [2] the golden ratio = OE/OA, and $F = 2v$.

3. Conclusion

It can be seen in the Equation (13) that t'/t equation can be expressed in more ways. But every equation is the equation of golden ratio. Naturally the golden ratio appears only then if $(v/c) = 1/\sqrt{5}$.

14) The Equation (13):

$$\frac{\sqrt{5}-1}{2} = \frac{1}{\frac{\sqrt{5}+1}{2}} = e^{-\xi} = e^{-\operatorname{artanh}(v/c)} = 0.618$$

15) So:

$$\frac{\sqrt{5}+1}{2} = e^{\xi} = e^{\operatorname{artanh}(v/c)} = 1.61803$$

4. Discussion

Naturally the golden ratio often appears in physics. Hardy suggest to use the golden ratio (See E. Naschi work, references [6]. Lorentz transformation equation is the same as suggested by Hardy. It would be good to prove in other way the connection of two equations. That's the reason why the two equations are not unambiguous:

16) The Hardy equation [6], and the Equation (13):

$$\Phi = \text{Golden ratio} = e^{-\operatorname{artanh}\left(\frac{1}{\sqrt{5}}\right)}$$

17) But still it is interesting the Equation (11) trivial solution:

$$\text{equation (9)}: v = c\frac{1}{\sqrt{5}} \to c = 300000\,[\text{km/s}] \text{ and } v = \left(\frac{300000}{\sqrt{5}}\right)[\text{km/s}]$$

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