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A Possible Alternative to the Accelerating Universe III

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Abstract

This work extends the author's two previous works (2015), *Journal of Modern Physics*, **6**, 78-87, and 1360-1370, by obtaining the index of refraction *n* of the dark energy for additional values of the cosmological density parameters, and for the two methods of obtaining *n*: least squares fit, and electromagnetic theory. Comparison of the alternative model with the accelerating universe for the new values of the density parameters and *n* is given in two tables. The new values for *n* are used to obtain a range of ages for the Einstein de Sitter (EdS) universe. It is shown that the EdS universe must be older than the comparison accelerating universe. This requirement is met for the Planck 2015 value of the Hubble constant, corrected for the speed of light reduction by *n*. A supporting measurement as well as a disagreeing measurement is also discussed. Possible support from a stellar age determination is also discussed. It is shown that the expression obtained earlier for the increased apparent magnitude of the SNe Ia provides as good a fit for a closed universe with $\Omega(tot) = 1.005$, as it does for the flat EdS universe. Comparison is presented in a third table. An upper bound on Ω_Λ is given for a closed universe that eventually collapses back on itself that is too small for the value needed for the accelerating universe.

Keywords

Dark Energy, Speed of Light, Age of Universe, Closed Universe

1. Introduction

In two previous works by the author [1] [2], hereafter referred to as I and II, it was shown that it is possible to explain the diminished brightness of the Type Ia supernovae (SNe Ia) found by Perlmutter *et al.* [3] [4], Riess *et al.* [5], and Schmidt *et al.* [6], and the increased distance to the "standard ruler" of the baryon acoustic oscillations (BAO) determined by Anderson *et al.* [7] [8], by assuming that the speed of light through the dark energy of intergalactic space has been reduced to c/n, where n is the index of refraction of the dark energy. Thus, in this alternative model, the dark energy no longer has associated with its energy-momentum source tensor a negative pressure that causes the expansion of the universe to accelerate (for a review see, e.g., Wang [9]), but instead has an index of refraction greater than unity. It was also assumed that the dark energy is another phase of dark matter, and that the phase transformation started to take place at about redshift $z = 1.65 \pm 0.15$, as discussed in Riess *et al.* [10], where there appears to be a supernova not exhibiting acceleration, and in the proposed alternative model, is where the dark energy started to appear as a consequence of the expansion cooling of the dark matter that was present in intergalactic space. Since that expansion cooling did not take place for the dark matter associated with the galaxies, because it is the space between the galaxies that expands, not the galaxies themselves, hence within the galaxies, n remains unity, and the speed of light is c. Since galaxies do not have a sharp boundary, with the dark matter halos extending well beyond the central luminous baryonic regions, there will be a transition region where n changes from unity to its intergalactic value, but for simplicity this is ignored at this stage of the study.

The purpose of this work is to extend the previous work in I and II by making use of additional values of the cosmological density parameters to obtain new values for n, and to use these new values to obtain additional estimates of the age of the universe according to the alternative model.

Also, since the comparisons in I and II have been between the ΛCDM accelerating universe, and only the flat Einstein de Sitter universe, for completeness, an additional comparison is made here with a closed universe.

In Section 2, the new values of n will be obtained by using both the least squares method that was introduced in I, and the square root method based on electromagnetic theory that was introduced in II. Both methods will be based on the values of the cosmological density parameters from the Planck satellite work of Abe *et al.* [11]. A comparison with the prediction for the increased distance to the SNe Ia, and the BAO standard ruler for different values of the cosmological density parameters is presented in **Table 1** and **Table 2**. In Section 3, the range of values for n obtained in Section 2 is used to obtain a range of ages for the universe. All of these ages are greater than or equal to 14 Gyr, and hence exceed that for the accelerating universe of 13.8 ± 0.1 Gyr as given in [11]. Significantly, a qualitative argument is presented that shows that the age of the comparison Einstein de Sitter universe has to be greater than that of the accelerating universe, as was already found quantitatively in II. Possible empirical supports, as well as a possible empirical objection to the alternative model, are also discussed. In Section 4, as mentioned above, instead of comparing the accelerating universe with the flat Einstein de Sitter universe, the comparison is made with a closed universe. In I it was briefly noted that the expression that was found there for the increase in the apparent magnitude of the SNe Ia did not require the universe to be flat, and in this section, this is demonstrated for a closed university with a total density parameter,

$\Omega = 1.005$, which is the one sigma upper bound given in [11]. It is also shown that fo- this case, both the least squares value of *n*, and the age of the closed universe, are the same (to three places) as that for the Einstein-de Sitter universe. In Table 3, a comparison is made between the closed universe and the flat universe for their percentage fit with the accelerating universe. An upper bound on Ω_Λ for a closed universe that eventually collapses back on itself is also derived and discussed. In Section 5, there are concluding remarks.

2. Additional Determinations of *n*

In I, where the cosmological density parameters were given by $\Omega_m = 0.30$, $\Omega_\Lambda = 0.70$, the least squares value of *n* was found to be $n = 1.49$. In II, upon setting $\Omega_\Lambda = \Omega_{de}$ (where the subscript "de" refers to dark energy which in the alternative model has the same energy density as the cosmological term, but, in contrast, has negligible stress), and using the electromagnetic relation, $n = (KK_\mu)^{1/2}$, where *K* is the dielectric constant, and K_μ is the relative permeability of the dark energy, and the assumption that $KK_\mu = \Omega_{de}/\Omega_m$ so that one has another method of determining *n* given by $n = (\Omega_{de}/\Omega_m)^{1/2}$, it was found for the above density parameters that $n = 1.53$. In what follows, it will be convenient to denote the least squares value for *n* as $n(ls)$, and to denote the value for *n* obtained from the square root as $n(sr)$. It was also found in II that $n(ls) = 1.47$ and $n(sr) = 1.46$, upon employing the one sigma upper limit of $\Omega_m = 0.308 \pm 0.012$ given in [11], so that $\Omega_m = 0.32$, and $\Omega_\Lambda = 0.68$. In this section, values of $n(ls)$ and $n(sr)$ will be obtained for the mean value $\Omega_m = 0.308$, and the one sigma lower limit $\Omega_m = 0.296$, with $\Omega_m + \Omega_\Lambda = 1$ as before, so that $\Omega_\Lambda = 0.692$, and $\Omega_\Lambda = 0.704$, respectively.

For the reader's convenience, the analysis leading to $n(ls)$ will next be briefly reviewed. As derived in Section 6 of I, and briefly recapitulated in II, $X_\Lambda(z)$ is proportional to the distance in the accelerating universe out to an object at redshift *z*, while $X_m(z)$ is proportional to the distance for the Einstein de Sitter universe. (See also the discussion in Section 4 below, where more details are given.) The general expression for $X(z)$ is

$$X(z) = \int_0^z dz'/E(z'), \tag{1}$$

with

$$E(z) \equiv H(z)/H_0 = (\Omega_m)^{1/2}\sqrt{(1+z)^3 + (\Omega_\Lambda/\Omega_m)}, \tag{2}$$

where $H(z) \equiv \dot{a}/a$ is the Hubble parameter, and H_0, its value at the present epoch, the Hubble constant, and where *a* is the FLRW expansion parameter, and also $a(z) = a_0/(1+z)$. From (1) and (2), as given in I and II, it follows that

$$X_\Lambda(z) = (\Omega_m)^{-1/2}\int_0^z dz'\Big/\sqrt{(1+z)^3 + (\Omega_\Lambda/\Omega_m)}, \tag{3}$$

so that for the values $\Omega_m = 0.308, \Omega_\Lambda = 0.692$, one has

$$X_\Lambda = (0.308)^{-1/2}\int_0^z dz'\Big/\sqrt{(1+z')^3 + 2.247}. \tag{4}$$

As noted in I and II, this integral has to be evaluated numerically. In contrast, for the Einstein de Sitter universe, with $\Omega_m = 1, \Omega_\Lambda = 0$, one has the same value for $X_m(z)$ as was found in I and II, for which the integral is immediate, and given by

$$X_m(z) = \int_0^z dz'(1+z')^{-3/2} = 2\left(1-(1+z)^{-1/2}\right). \tag{5}$$

As in I and II, the difference of the logarithmic proportional distances between the accelerating universe and the Einstein de Sitter universe is given by

$$\log X_\Lambda(z) - \log X_m(z) = \log\left(X_\Lambda(z)/X_m(z)\right) = d, \tag{6}$$

where

$$d = \log\left(1+(n-1)\ln(1+z)\right) \tag{7}$$

is the prediction of the alternative model under the assumption (for which there is as yet no theoretical foundation) that the light speed has been reduced to c/n when traveling through the dark energy of intergalactic space, as shown in I, Section 4, page 82, and where the increase in apparent magnitude δm is shown to be given by $\delta m = 5d$. For the percentage comparisons, it is sufficient to work with d rather than δm. For $\Omega_m = 0.308$, $\Omega_\Lambda = 0.692$ it was found for the redshift range, $0 \le z \le 1.0$, that $n(ls) = 1.48$. Another value for n is obtained under the assumption that it is of electromagnetic origin, and as discussed in II, and above, so that $n(sr) = (KK_\mu)^{1/2} = (\Omega_{de}/\Omega_m)^{1/2}$. Hence, for $\Omega_m = 0.308, \Omega_{de} = \Omega_\Lambda = 0.692$, one has $n(sr) = 1.50$. In Table 1, a comparison is given for the percentage disagreement with the accelerating universe for $n(ls)$ and $n(sr) = 1.50.$, this is analogous to Table 1 in II, and for just $n(ls)$, to Table 3 in I.

Table 1. Comparison of $\log\left(X_\Lambda(z)/X_m(z)\right)$ with $d = \log\left(1+(n-1)\ln(1+z)\right)$ for $n(ls) = 1.48$ and $n(sr) = 1.50$, $\Delta \equiv d - \log\left(X_\Lambda(z)/X_m(z)\right)$, and for brevity, $R \equiv \log\left(X_\Lambda/X_m\right)$, and the arguments for R, X_Λ, X_m are omitted. $\Omega_m = 0.308, \Omega_\Lambda = 0.692$, $\Omega_\Lambda/\Omega_m = 2.247$.

z	$\log(X_\Lambda/X_m)$	$d(1.48)$	$\Delta(1.48)$	$\Delta(1.48)/R\%$	$d(1.50)$	$\Delta(1.50)$	$\Delta(1.50)/R\%$
0.1	0.0208	0.0194	−0.0014	−6.7	0.0202	−0.0006	−2.9
0.2	0.0387	0.0364	−0.0023	−5.9	0.0379	−0.0008	−2.1
0.3	0.0539	0.0515	−0.0024	−4.5	0.0535	−0.0004	−0.7
0.4	0.0671	0.0650	−0.0021	−3.1	0.0675	0.0004	0.6
0.5	0.0786	0.0772	−0.0014	−1.8	0.0802	0.0016	2.0
0.6	0.0885	0.0883	−0.0002	−0.2	0.0917	0.0032	3.6
0.7	0.0972	0.0985	0.0013	1.3	0.1022	0.0050	5.1
0.8	0.1048	0.1079	0.0031	3.0	0.1119	0.0071	6.8
0.9	0.1115	0.1166	0.0051	4.6	0.1209	0.0094	8.4
1.0	0.1175	0.1247	0.0072	6.1	0.1292	0.0117	10.0

In Table 2, the percentage disagreement results are given for $\Omega_m = 0.296$, and $\Omega_\Lambda = 0.704$. It was found that for this case, $n(ls) = 1.50$ and $n(sr) = 1.54$. Hence with Table 1 and Table 2 here, and Table 1 in II, the totality of cases are presented for $\Omega_m = 0.308 \pm 0.012$, along with the corresponding values for Ω_Λ.

Table 2. Comparison of $\log(X_\Lambda(z)/X_m(z))$ with $d = \log(1+(n-1)\ln(1+z))$ for $n(ls) = 1.50$ and $n(sr) = 1.54$, $\Delta \equiv d - \log(X_\Lambda(z)/X_m(z))$, and for brevity, $R \equiv \log(X_\Lambda/X_m)$, , and the arguments for R, X_Λ, X_m are omitted. $\Omega_m = 0.296, \Omega_\Lambda = 0.704$, $\Omega_\Lambda/\Omega_m = 2.378$.

z	$\log(X_\Lambda/X_m)$	$d\,(1.50)$	$\Delta(1.50)$	$\Delta(1.50)/R\,\%$	$d\,(1.54)$	$\Delta(1.54)$	$\Delta(1.54)/R\,\%$
0.1	0.0210	0.0202	−0.0008	−3.8	0.0218	0.0008	3.8
0.2	0.0394	0.0379	−0.0015	−3.8	0.0408	0.0014	3.6
0.3	0.0551	0.0535	−0.0016	−2.9	0.0575	0.0024	4.4
0.4	0.0686	0.0675	−0.0011	−1.6	0.0725	0.0039	5.7
0.5	0.0804	0.0802	−0.0002	−0.2	0.0860	0.0056	7.0
0.6	0.0906	0.0917	0.0011	1.2	0.0982	0.0076	8.4
0.7	0.0996	0.1022	0.0026	2.6	0.1094	0.0098	9.8
0.8	0.1074	0.1119	0.0045	4.2	0.1197	0.0123	11.5
0.9	0.1144	0.1209	0.0065	5.7	0.1292	0.0148	12.9
1.0	0.1206	0.1292	0.0086	7.1	0.1381	0.0175	14.5

It is clear from the last column in Table 2, in which, particularly at $z = 0.5$, the disagreement is 7.0%, that $n(sr) = 1.54$ is ruled out. However, this need not mean that the square root method of obtaining n is at fault, rather it could be that the problem is with the values of the density parameters, *i.e.*, $\Omega_m = 0.296, \Omega_\Lambda = 0704$. If one assumes that $n(sr) = \sqrt{\Omega_{de}/\Omega_m}$ yields a valid determination of n for, say, redshifts, $0 \le z \le 0.7$, then it follows that the large percentage disagreement for $n(sr) = 1.54$ is indicative that the above values of the cosmological density parameters are ruled out instead. If one demands that the percentage disagreement not exceed |2%|, for $z = 0.5$, then from Table 1, $\Omega_m = 0.308$ provides a lower bound for Ω_m, while for an upper bound, it was found that for $\Omega_m = 0.317$ the disagreement at $z = 0.5$ is −1.8%, while for $\Omega_m \ge 0.318$, the disagreement is less than −2%, it follows that $\Omega_m = 0.317$ provides an upper bound. Hence, allowing $\pm 2\%$ disagreement, the requirement that the square root method is valid, can be seen as predicting

$$0.308 \le \Omega_m \le 0.317 . \tag{8}$$

Also, since for $\Omega_m = 0.317$, $\Omega_{de} = \Omega_\Lambda = 0.693$, one has that $n(sr) = 1.47$. The corresponding range of values of n, that include least squares values as well as square root values is given by $n = 1.48^{+0.02}_{-0.01}$. However, in calculating $n(ls)$, for $\Omega_m = 0.308$, if one restricts the redshifts to $z \le 0.7$, one finds that $n(ls) = 1.49$. Thus a possibly better estimate of n is given by

$$n = 1.49^{+0.01}_{-0.02}. \tag{9}$$

Indeed, for n = 1.49, at z = 0.5, one finds from **Table 1** that $\Delta = 0.0001$, and hence $\Delta/R = 0.1\%$.

It will be noticed from the tables that lower values of n fit better for higher values of z, while higher values of n fit better for lower values of z. Interestingly, the disagreement for the lower values of z, *i.e.* $z \le 0.3$, suggest that if the alternative, model is correct, it predicts a slightly brighter SNe Ia than the accelerating ΛCDM universe for this lower range of redshifts. As noted in I, this could provide another means for choosing between the two theories. On the other hand, the percentage disagreements for, say, $z > 0.7$, are probably an indication that the simplifying assumption that the index of refraction is constant for these higher values of z is not valid, and that at these redshifts, and higher, the dark matter has not fully transformed into dark energy. Thus a future, more accurate model should assume that $n = n(z)$, with the requirement that $n(z)$ approach unity for sufficiently high z. As noted above, at $z = 1.65 \pm 0.15$ [10], the universe no longer seems to be accelerating, and in the present model, this should be the approximate redshift where the intergalactic dark matter began to make a phase transition into dark energy, and the index of refraction started to increase from unity. But one will need to know more about the properties of dark matter and dark energy to go further. Indeed, it will be noticed that in this work, no assumptions have been made about the particle nature of the dark matter, nor the dynamical nature of the phase transition into dark energy. At this stage of the investigation, the model is purely phenomenological, and it may be possible to determine $n(z)$ phenomenologically, which could then be helpful in probing the nature of the dark matter, and the proposed phase transition into dark energy.

3. Age of the Universe

In II it was found that for $n = 1.46$, the age of the Einstein de Sitter universe, with $H_0 = 67.8 \pm 0.9\ \mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$ [11], which was rounded to 68 $\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$, was 14.0 Gyr, which was obtained in II, Section 5, (17), and is equivalent to (11) below. In this section, a range of ages for the Einstein de Sitter universe based on the range of values of n given in Section 2 will be presented here, and discussed. The above age of 14.0 Gyr, and those that will be obtained below, are obviously greater than the age of the accelerating universe of 13.8 ± 0.1 Gyr, given in [11], and hence it is of interest to present a qualitative argument to show why the age of the Einstein de Sitter universe in the alternative model is necessarily greater than the age of the accelerating universe. To accomplish this, it is convenient to divide the expansion of the universe into three eras, based on the different speed of light in these three eras, as discussed below: the first era is the expansion from the big bang to the time when the universe started to accelerate. At that time, the expansion parameter would be somewhat larger for the accelerating universe than that for the Einstein de Sitter universe, since while both universes had been decelerating, the former's deceleration would have diminished more rapidly to zero. Call this value of the expansion parameter for the accelerating universe a'. Therefore the time for the Einstein de Sitter universe to expand to a' would be greater than that for the accelerating universe, call this time difference Δt_1, with $\Delta t_1 > 0$. The second era is the interval of time in which the universe expanded from a' to its size at redshift $z = 0.5$, call this value of the expansion parameter a''. It would require a longer amount of time for the Einstein de Sitter universe to expand from a' to a'' than for the accelerating universe, so that the former would require an additional time Δt_2 with $\Delta t_2 > 0$. The third era would be the expansion from $z = 0.5$ to the present epoch $z = 0$, and again the accelerating universe would reach the present value of the expansion parameter a_0 in a shorter time than the decelerating universe which would require an additional time $\Delta t_3 > 0$. During this last era, the speed of light according to the alternative model is c/n, and hence the increased length of time the light takes to travel from the SNe Ia to earth, over what it would be if it traveled at speed c, leads to the increase in time needed for the decelerating Einstein de Sitter universe to expand the extra distance needed to explain the increase in apparent magnitude of the SNe Ia at $z = 0.5$, over what it would have been if light had traveled with speed c. On the other hand, during the second epoch, the speed of light through the dark energy of intergalactic space is gradually changing from c to c/n; however, the details of this change, together with the postulated phase change of dark matter into the dark energy of intergalactic space is left to future studies. In any case, it is clear that the sum $\Delta t_1 + \Delta t_2 + \Delta t_3$ leads to a greater total time back to the big bang for the Einstein de Sitter universe than for the accelerating universe, as was already found in the particular case considered above, for n 1.46; but, as demonstrated above, the result is quite general.

Now, as was discussed in II, because the determination of the Hubble constant involves the first order Doppler effect for the light that has traveled through intergalactic space, and since the distances involved are for $z < 0.5$ in which, according to the model, the speed of light is c/n, one has to correct the Doppler expression to allow for this, so that it becomes $\lambda_0 = \lambda\left(1 + \left(nv/c\right)\right)$, where λ_0 is the wavelength observed at the present epoch, λ is the wavelength in the rest frame of the receding galaxy, and v is the Hubble flow recession velocity of the galaxy. With the red shift $z \equiv \left(\lambda_0 - \lambda\right)/\lambda$, one has for $n = 1$, $cz = v$, and the basic discovery of Hubble is that $v = H_0 D$, where D is the proper distance to the galaxy at z, so that the standard expression is $cz = H_0 D$, from which one determines the Hubble constant as $H_0 = cz/D$. But when one takes into account the reduced speed of light, the new corrected value of the Hubble constant, denoted by H_0^*, is given by

$$H_0^* = cz/nD = H_0/n. \tag{10}$$

Since the age of the Einstein de Sitter universe, taken to be from the big bang to the present epoch, and denoted by T_0, is given by $\left(2/3\right)\left(\dot{a}/a\right)_0^{-1}$, for $n = 1$, this becomes $T_0 = 2H_0^{-1}/3$. However, as remarked in II, for $n \neq 1$, as is the case here, this has to be corrected to an age T_0^* given by

$$T_0^* = \frac{2}{3}\left(H_0^*\right)^{-1} = \frac{2}{3}nH_0^{-1}. \tag{11}$$

Since (11) is indifferent to whether one works with $n(ls)$, or $n(sr)$, upon introducing the value of n in (9), and with $H_0^{-1} = 14.4 \pm 0.2\ \text{Gyr}$ based on H_0 = 67.8 ± 0.9 km·s^{-1}·Mpc^{-1} [11] one has that

$$T_0^* = 14.3 \pm 0.3\ \text{Gyr}. \tag{12}$$

This clearly exceeds the age of the accelerating universe of $13.8 \pm 0.1\ \text{Gyr}$, as was expected on the basis of the qualitative analysis given above. In this case, $\Delta t_1 + \Delta t_2 + \Delta t_3$ = 0.5 ± 0.3 Gyr.

With regard to this revision of the Hubble constant, it should be pointed out that since the light from, say, the Cepheid variables that is used in determining H_0, passes through the host galaxy as well as our own Galaxy, where in both cases the speed of light is c, this has the consequence that the effective speed of light for the entire path is greater than c/n. However a rough estimate indicates that a correction for this effect would be less than 0.5 percent, and in view of the sizes of the other uncertainties, it can be ignored at this stage of the study.

An interesting hint of possible support for the alternative model comes from the fairly recent study of the star HD 140283 by Bond *et al.* [12], who describe it as a subgiant with low metallicity in the solar neighborhood, at approximately 100 lyr from earth. They found that when all uncertainties are included the star's age is 14.46 ± 0.8 Gyr. They emphasized that this age was not in disagreement with the age of the universe when allowance was made for the uncertainty in the star's age. At that time, 2013, no other age than that of the accelerating universe (which was then given as 13.77 ± 0.06 Gyr) was available for comparison, but clearly the mean age they found for the star puts its age closer to T_0^* than to that of the accelerating universe. However, until the substantial uncertainty in the star's age has been reduced, obviously no firm conclusion can be drawn. But it is noteworthy that, in addition to the divergent lensing possibility described in Section 4 of II, the difference in the ages of the universe for the two models is yet another avenue of approach for deciding between the two models.

Quite recently there has appeared the latest finding of Riess *et al.* [13] that yields a 2.4% determination of the local value of the Hubble constant of $73.24 \pm 1.74\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. This leads to a Hubble time to three places given by $H_0^{-1} = 13.4 \pm 0.3\ \text{Gyr}$. Hence, according to the alternative model, since the age of the universe, as given in (11) is $(2/3)nH_0^{-1}$, upon introducing the largest value of n obtained from (9), *i.e.* $n = 1.50$, one obtains an age T_0^* for the Einstein de Sitter universe of $13.4 \pm 0.3\ \text{Gyr}$. Since, as was pointed out earlier in this section, the age of the Einstein de Sitter universe has to be greater than that of the accelerating universe, hence greater than $13.8 \pm 0.1\ \text{Gyr}$, their value for H_0 is possibly in conflict with the alternative model. On the other hand, the possible conflict is less than with the 3.3% determination by Riess *et al.* [14] of 2011,for which $H_0 = 73.8 \pm 2.4\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. Interestingly, Cheng and Huang [15], on the basis of their BAO studies, found that $H_0 = 68.11 \pm 0.86\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ which is in excellent agreement with the Planck [11] finding that $H_0 = 67.8 \pm 0.9$ km·s^{-1}·Mpc^{-1}, the value used here. They discussed their disagreement with the 2011 value of H_0 found by Riess *et al.* [14]. Since the alternative model also disagrees with this value given in [14], and to a lesser degree with their more recent value [13], this could be a sign that the alternative model is possibly on the right track, and has predictive powers. However, as noted above, the large uncertainties surrounding the various values make it impossible to draw scientifically valid conclusions as to agreement or disagreement.

It has undoubtedly been noticed that the value of n is very nearly—if not exactly—the reciprocal of the two-thirds factor relating the age of the Einstein de Sitter universe to the Hubble time. At this writing, there is no explanation for this unexpected relation, and it is left to future studies to find a possible solution.

4. Comparison with a Closed Universe

As mentioned in I, the expression that was derived therein Section 4, p.82, for the increased apparent magnitude of the SNe Ia given by

$$\delta m = 5\log\left(1+(n-1)\ln(1+z)\right), \tag{13}$$

does not just hold for the flat Einstein de Sitter universe, it is applicable to any isotropic universe for which the expansion parameter satisfies $a = a_0/(1+z)$, which is a generic relationship for FLRW expanding universes. In view of the long standing interest in closed universes, and the author's work on a closed pulsating universe [16] [17], it is appropriate to determine what values of n emerge from comparison with a closed universe. As pointed out in [11], the equivalent cosmological density parameter for curvature differs from zero by an amount given by $\Omega_k = \pm 0.005$. However, it should be emphasized that this value represents a measurement uncertainty, and the true value of the curvature parameter could be much smaller, and even zero, as predicted by the inflationary models of Guth [18] [19], Linde [20], and Albrecht and Steinhardt [21]. This possible curvature contribution to Ω in the author's approach is written differently, and will now be derived.

The standard line element for a general, homogeneous, isotropic, time-orthogonal, FLRW universe, in isotropic coordinates, can be written as

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a(t)^2\left(1+\left(kr^2/4\right)\right)^{-2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,\ (i,j=1,2,3) \tag{14}$$

with $r^2 = \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$, and $k = 1, 0, -1$ for a closed, flat, and open universe, respectively. The Einstein field equation for the energy density $G_0^0 = -\kappa T_0^0$, in the absence of CMB radiation, neutrino background, and the cosmological term, reduces to

$$\dot{a}^2 - \left(8\pi G\rho a^2/3\right) = -kc^2. \tag{15}$$

For the Einstein de Sitter universe $k = 0$, while in this section $k = 1$. Hence this equation can be rewritten as

$$1+\frac{c^2}{\dot{a}^2} = \frac{8\pi G\rho a^2}{3\dot{a}^2}. \tag{16}$$

Since $\rho_0 a_0^3 = \rho a^3$ from the covariant conservation law, and since $a = a_0/(1+z)$, the above becomes

$$1+\frac{c^2}{\dot{a}^2}=\frac{8\pi G\rho_0(1+z)^3 a^2}{3\dot{a}^2}. \tag{17}$$

Hence, at the present epoch, $z = 0$, with $a = a_0, \dot{a} = \dot{a}_0$, and $H_0 \equiv \dot{a}_0/a_0$, one has

$$1+\frac{c^2}{\dot{a}_0^2}=\frac{8\pi G\rho_0}{3H_0^2}=\frac{\rho_0}{\rho_c}, \tag{18}$$

where $\rho_c \equiv 3H_0^2/8\pi G$. And since $\Omega_m \equiv \rho_0/\rho_c$, in contrast with the Einstein de Sitter universe, for which $\Omega_m = 1$, one has instead

$$\Omega_m = 1+\frac{c^2}{\dot{a}_0^2}. \tag{19}$$

One now proceeds as in I and II. From (17), and the relations leading to (19), one has

$$\frac{H}{H_0}=\frac{\dot{a}}{H_0 a}=\sqrt{\frac{8\pi G\rho_0(1+z)^3}{3H_0^2}-\frac{c^2(1+z)^2}{a_0^2H_0^2}}. \tag{20}$$

Then from $\Omega_m \equiv \rho_0/\rho_c = 8\pi G\rho_0/3H_0^2$, and the definition $E(z) \equiv H(z)/H_0$, upon simplification, (20), becomes

$$E(z)=(\Omega_m)^{1/2}(1+z)\sqrt{(1+z)-\left(c^2/\dot{a}_0^2\Omega_m\right)}. \tag{21}$$

Upon replacing Ω_m by $\left(1+\left(c^2/\dot{a}_0^2\right)\right)$ from (19), the above may be rewritten as

$$E(z)=\left(1+\left(c^2/\dot{a}_0^2\right)\right)^{1/2}(1+z)\sqrt{(1+z)-\left(c^2/\dot{a}_0^2\left(1+\left(c^2/\dot{a}_0^2\right)\right)\right)}. \tag{22}$$

The proportional distance function X(z), as previously mentioned, is defined as

$$\mathrm{X}(z)\equiv\int_0^z \mathrm{d}z'/E(z'). \tag{23}$$

Now $\mathrm{X}_\Lambda(z)$ will be the same as previously used in comparison with the flat universe, here, on the other hand, it will be compared with the new form for $\mathrm{X}_m(z)$ which, from (22), is given by

$$\mathrm{X}_m(z)=\left(1+\left(c^2/\dot{a}_0^2\right)\right)^{-1/2}\int_0^z\frac{\mathrm{d}z'}{(1+z')\sqrt{(1+z')-\left(c^2/\dot{a}_0^2\left(1+\left(c^2/\dot{a}_0^2\right)\right)\right)}}. \tag{24}$$

Upon denoting the ratio $c^2/\dot{a}_0^2$ by f, the above expression can be written as

$$\mathrm{X}_m(z)=(1+f)^{-1/2}\int_0^z\frac{\mathrm{d}z'}{(1+z')\sqrt{(1+z)-f(1+f)^{-1}}}. \tag{25}$$

After combining the factor $2f^{-1/2}(1+f)^{1/2}$ that emerges from the integral with the pre-factor in (25), the above integration yields

$$\mathrm{X}_m(z)=2f^{-1/2}\left\{\tan^{-1}\sqrt{\frac{(1+z)-f(1+f)^{-1}}{f(1+f)^{-1}}}-\tan^{-1}f^{-1/2}\right\}. \tag{26}$$

The following cosmological density parameters, $\Omega_m = 0.308, \Omega_\Lambda = 0.692$ were chosen for the accelerating universe to obtain X_Λ for comparison with the closed universe. Thus, the results presented in Table 3 below will be similar to that of Table 1, except X_m will now be that for a closed universe. However, in Table 1, in comparison with the flat universe, $n(ls) = 1.48$ was used, and so, before making the comparison, it is necessary to determine $n(ls)$ for the closed universe. For $f \equiv c^2/\dot{a}_0^2 = 0.005$, it was found $n(ls)$ had the same value as for the flat universe, *i.e.*, 1.48. In Table 3, the last column is taken from the fifth column in Table 1.

Table 3. Comparison of $\log(X_\Lambda(z)/X_m(z))$, where $X_m(z)$ is for a closed universe, with the logarithmic distance correction $d = \log(1+(n-1)\ln(1+z))$ for $n = 1.48$, $\Delta \equiv d - \log(X_\Lambda/X_m)$. For brevity, $R \equiv \log(X_\Lambda/X_m)$, and the arguments for Δ, R, X_Λ, X_m are omitted.

z	$\log(X_\Lambda/X_m)$	d	Δ	$\Delta/R\%$	$\Delta/R\%(flat)$
0.1	0.0208	0.0194	−0.0014	−6.7	−6.7
0.2	0.0387	0.0364	−0.0023	−5.9	−5.9
0.3	0.0540	0.0515	−0.0025	−4.6	−4.5
0.4	0.0673	0.0650	−0.0023	−3.4	−3.1
0.5	0.0787	0.0772	−0.0015	−1.9	−1.8
0.6	0.0887	0.0883	−0.0004	−0.5	−0.2
0.7	0.0974	0.0985	0.0011	1.1	1.3
0.8	0.1050	0.1079	0.0029	2.8	3.0
0.9	0.1118	0.1166	0.0048	4.3	4.6
1.0	0.1178	0.1247	0.0069	5.9	6.1

As can be seen from the table, the fit for the closed universe is very nearly the same as for the flat Einstein de Sitter universe. Moreover, as will be shown next, the age of the closed universe is very nearly the same as for the flat universe for this value of Ω_m that is so close to unity.

It is convenient to rewrite (15) as

$$\frac{\dot{a}^2}{2} - \frac{GM}{a} = -\frac{c^2}{2}, \tag{27}$$

where $M \equiv (4\pi/3)\rho a^3$, although, to be sure, for a closed, spherical universe, the actual mass is actually $2\pi^2 \rho a^3$. However, in the following, one never has to make explicit use of the mass, and because of the obvious similarity to Newtonian mechanics, (27) is simpler to work with, since the solution to the differential equation is well known, and is given by the parametric equations for a cycloid

$$a = (GM/c^2)(1 - \cos\varphi), \tag{28}$$

$$t = \left(GM/c^3\right)\left(\varphi - \sin\varphi\right). \tag{29}$$

From these two equations one obtains an expression for $\dot{a}$ given by

$$\dot{a} = c\sin\varphi/\left(1-\cos\varphi\right), \tag{30}$$

from which, together with (28), one obtains the Hubble parameter $H \equiv \dot{a}/a$ in the form

$$H = \frac{c^3\sin\varphi}{GM\left(1-\cos\varphi\right)^2}. \tag{31}$$

For $\varphi \ll 1$, from (31), $t = GM\varphi^3/6c^3$, and $H = 4c^3/GM\varphi^3$, hence

$$tH = 2/3, \tag{32}$$

which is the same relation that holds for the Einstein de Sitter universe. For the larger value of φ that is needed here, one expands t and H to the next higher order terms. After combining (29) and (31), one has

$$tH = \frac{\sin\varphi\left(\varphi-\sin\varphi\right)}{\left(1-\cos\varphi\right)^2}. \tag{33}$$

After setting $\sin\varphi = \varphi - \varphi^3/6 + \varphi^5/120$, and $\varphi - \sin\varphi = \varphi^3/6 + \varphi^5/120$, upon neglecting higher order terms, and then taking the product, the numerator in (33) becomes $\left(\varphi^4/6\right)\left(1-\left(13\varphi^2/60\right)\right)$, after further neglect of higher order terms. After approximating the denominator as $\left(\varphi^4/4\right)\left(1-\left(\varphi^2/12\right)\right)$, and upon further neglect of higher order terms, one finds

$$tH = \frac{2}{3}\left(1-\frac{\varphi^2}{20}\right). \tag{34}$$

To determine φ in terms of the departure ε of Ω_m from its value for the flat Einstein de Sitter universe, one uses from (19) that $\Omega_m = 1 + c^2/\dot{a}^2$, so that $\varepsilon = c^2/\dot{a}^2$, and one will determine φ in terms of ε. From (30), since $c^2/\dot{a}^2 = \left(1-\cos\varphi\right)^2/\sin^2\varphi$, one obtains

$$\Omega_m = \frac{2}{1+\cos\varphi}. \tag{35}$$

After substituting the approximation $\cos\varphi = 1 - \varphi^2/2 + \varphi^4/24$ in (35) and expanding upon further neglect of higher order terms, one obtains

$$\Omega_m = 1 + \varphi^2/4 + \varphi^4/24. \tag{36}$$

Upon setting $\Omega_m = 1+\varepsilon$ in (36), there results a quadratic equation for φ^2 given by

$$\varphi^4 + 6\varphi^2 - 24\varepsilon = 0. \tag{37}$$

The positive root for φ^2 to first order in ε is $\varphi^2 = 4\varepsilon$. Hence, for this level of approximation, (34) becomes

$$tH = \frac{2}{3}\left(1-\frac{\varepsilon}{5}\right). \tag{38}$$

With $\varepsilon = 0.005$, one has $t = 0.666H^{-1}$, and hence for this case, as well as for closed universes of even lesser curvature, the age difference between a closed universe and the Einstein de Sitter universe is negligible. Some further observations concerning the two models are of interest.

Although for the closed universe, from (30), $\dot{a} > c$, for $\varphi < \pi/2$, this does not violate c being the limiting speed, since $\dot{a}$ is analogous to the Minkowski speed in special relativity, which can be arbitrarily large, since the speed of light determined in this way is infinite. In other words, the coordinate time t in the FRLW line element is analogous to a proper time; for further discussion, see that by the author in [22]. Also, it should be noted that the solution for $a(t)$ for the Einstein de Sitter universe, *i.e.*, $a(t) \propto t^{2/3}$, which is usually described as having the time run $0 \le t \le \infty$, also holds for $t < 0$, so one may think of the time running for the complete solution as $-\infty \le t \le \infty$. In the negative branch, for which the time runs $-\infty \le t < 0$, one has a universe collapsing from infinity to a cusp at $t = 0$, the big bang, where ρ becomes infinite. It is therefore appropriate to see the full Einstein de Sitter universe as a limiting case of a closed cyclic universe in which the amplitude of the cycle has become infinite as well as the period. Thus, instead of there being infinitely many finite cycles of finite amplitude, there is one infinitely long cycle of infinite amplitude. The infinite cycle is split into two halves of a cycle, the first half being the branch descending from infinity, and the second half, the traditional expanding branch rising to infinity. Since the amplitude of the cycle in the closed universe is proportional to the total mass of the universe as given in (28), the flat Einstein de Sitter universe may be thought of as a limiting case of a closed universe with infinite mass and infinite radius of curvature as discussed in Einstein [23], Silberstein [24], and the author [25]. Also, because of the time-reversibility of the solution, it reads the same, whether one goes from the negative branch to the positive branch or vice-versa. Since, on physical grounds, it is unlikely that before the big bang the universe collapsed in from infinity, but rather collapsed in from a finite size, the cyclic, closed universe seems preferable. The problem of the cusp in $a(t)$ and the singularity in $\rho(t)$ at $t = 0$ can be dealt with classically [26], but a discussion of this lies outside the scope of this work.

Finally, in I it was pointed out that there is a fundamental difficulty with the cosmological term Λ, based on a generalization of Newton's first law, that led to the conclusion that Λ vanishes. It is therefore of interest to briefly recapitulate and extend an argument based on a closed universe that sets an upper bound on Λ, assumed non-negative, which would make it too small to lead to the universe accelerating now, as is claimed. The finding arose, nearly a decade before the accelerating universe was proposed, inresponse to work by Weinberg [27] [28], who found, using the weak anthropic principle [29], an upper bound on Λ that was based on assuming that the universe is flat, and does not collapse back on itself, that proved to be sufficiently large as to include the current value of Λ. Following Weinberg's work, the author was able to show that for a closed universe that does collapse back on itself, one obtains a much smaller bound [30]; this was further discussed and extended in [31] which was referenced in I.

To show this here, note that with a cosmological term present, the field Equation (15) for a closed universe becomes

$$\dot{a}^2 - (8\pi G/3)\rho a^2 - (\Lambda c^2 a^2/3) = -c^2 . \quad (39)$$

Either from the field equation for stress, or by differentiating the above equation, taking into account that ρa^3 is a constant, the equation of motion becomes,

$$\ddot{a} = -(4\pi G/3)\rho a + (\Lambda c^2 a/3) . \quad (40)$$

Under the assumption that the universe does collapse back on itself after reaching the value of the expansion parameter when it stops increasing and starts to recoil denoted by a_r, with $\dot{a}_r = 0$, and $\ddot{a}_r < 0$, it follows from (40) that $\Lambda c^2 < 4\pi G\rho_r$ From the conservation of mass-energy from the contracted Bianchi identities, one has that $\rho_r a_r^3 = \rho_0 a_0^3$, and hence, upon introducing the cosmological term's mass density (rather than energy density) defined as $\rho_\Lambda \equiv \Lambda c^2/8\pi G$, the inequality becomes

$$\rho_\Lambda < (\rho_0/2)(a_0/a_r)^3 . \quad (41)$$

After dividing both sides of the inequality by ρ_c, which was not done in [30] [31], one has

$$\Omega_\Lambda < (\Omega_m/2)(a_0/a_r)^3 . \quad (42)$$

Thus, under these circumstances, Ω_Λ would clearly be too small to account for the accelerating universe, for which $\Omega_\Lambda > \Omega_m$. However, because of the above assumptions, for which there is as yet no empirical support, the result is obviously not conclusive, although it is consistent with the rejection of the cosmological term in the proposed alternative to the accelerating universe.

5. Concluding Remarks

It is clear from the work in the preceding sections, as well as that in I and II, that the proposed alternative model, based on the slowing down of light by the dark energy in intergalactic space, can explain the diminished brightness of the SNe Ia, and the increased distance to the "standard ruler" of the BAO, as well as can the accelerating universe, that is based on attributing a negative pressure to the dark energy, such as displayed by the cosmological term. However, the crucial test for the alternative model will be for astronomers to determine through suitable observations, such as the one described in Section 4 of II, whether in fact the speed of light in intergalactic space for, say, $z \le 0.7$ is c/n, with $n \approx 1.50$. If eventual astronomical observation should show that this is indeed the case, it will then prove theoretically challenging to obtain a general expression for n as a function of redshift, and to show further that n does not exhibit any evidence of dispersion in the optical range, as found by the SNe Ia studies.

Finally, it follows from the discussion in Section 3 that the alternative model is predicting a greater age for the universe than that predicted by the accelerating universe. As was further discussed there, this has bearing on the maximum age of stars, and the value of the Hubble constant, so that their more accurate determination should provide two other areas to test the model astronomically.

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