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Signature of Gravitational Waves in Stellar Spectroscopy

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Abstract

The possible detection of gravitational waves by interferometric observations of distant light sources is studied. It is shown that a gravitational wave affects the interferometric pattern of stellar light in a particular way. Michelson and Hanbury Brown-Twiss interferometers are considered, and it is shown that the latter is the most adequate for such a detection.

Keywords

Gravitational Waves, Spectroscopy

1. Introduction

A gravitational wave (GW) could be detected indirectly by its interaction with the light emitted by astronomical objects. Thus, for instance, the passage of a GW produces a time delay in the signal received from distant sources (Estabrook and Wahlquist [1]). Similarly, the presence of a stochastic background of GWs can be inferred from a statistical analysis of pulsar timing (Hellings and Downs [2]). GWs can also interact with the polarization of electromagnetic waves (Hacyan [3] [4]).

In this paper, we study the effect of GWs on the interferometry of stellar light. Two basic types of interferometric devices used in astronomy are considered: the Michelson (see, e.g., [5]) and the Hanbury Brown-Twiss [6] interferometers. The former uses the interference between two signals, and the latter uses the interference between intensities of light. An intensity interferometer has, in general, some advantages over a Michelson interferometer. It will be shown in the following that the passage of a GW could be more easily detected by intensity interferometry.

Section 2 of the present paper is devoted to the analysis of an electromagnetic wave in the presence of a plane fronted GW. The analysis is based on previous works (Hacyan [3] [4]) in which the form of the electromagnetic

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field is deduced using a short-wave length approximation. A general formula for the correlation of electric fields is obtained and the result is applied to interferometric analysis in Section 3; particular cases are worked out.

2. The Electromagnetic Field

The metric of a plane GW in the weak field limit is

$$\mathrm{d}s^2 = -2\mathrm{d}u\mathrm{d}v + (1+f)\mathrm{d}x^2 + (1-f)\mathrm{d}y^2 + 2g\mathrm{d}x\mathrm{d}y, \tag{1}$$

where the two degrees of polarization of the GW are given by the potentials $f(u)$ and $g(u)$, which are functions of u only. The relation with Minkowski coordinates t and z is

$$u = \frac{1}{\sqrt{2}}(t-z), \qquad v = \frac{1}{\sqrt{2}}(t+z).$$

In the following, quadratic and higher order terms in f and g are neglected, and we set $c = 1$.

The direction of a light ray in the absence of a GW is k, with $|k| = \omega$, the frequency of the (monochromatic) wave. We set

$$k = \omega(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta),$$

thus defining the angles θ and ϕ. In the following, it will be convenient to define the functions

$$F(u;\phi) = f(u)\cos 2\phi + g(u)\sin 2\phi, \tag{2}$$

$$G(u;\phi) = -f(u)\sin 2\phi + g(u)\cos 2\phi. \tag{3}$$

In the short-wave length approximation, the electromagnetic potential is taken as

$$A_\mu = a_\mu \mathrm{e}^{iS},$$

where S is the eikonal function satisfying the equation $g^{\mu\nu}S_{,\mu}S_{,\nu} = 0$. Then, $K_\alpha \equiv S_{,\alpha}$ is a null-vector defining the direction of propagation of the electromagnetic wave, and a_α is a four-vector such that $a_\mu K^\mu = 0$.

The electromagnetic vector is [4]

$$E_\alpha = i\left[\Omega a_\alpha + (a_\beta t^\beta)K_\alpha\right]\mathrm{e}^{iS}, \tag{4}$$

where t^α is a time-like four-vector and $\Omega = -K_\mu t^\mu$ is the frequency measured by a detector with t^α tangent to its world-line. Choosing $t^\alpha = (1/\sqrt{2})(1,1,0,0)$, it follows that

$$\Omega(u;\theta,\phi) = \omega\left[1 - \frac{1}{2}(1+\cos\theta)F(u;\phi)\right], \tag{5}$$

and the eikonal function is

$$S(x;k) = x\cdot k + \frac{\omega}{\sqrt{2}}(1+\cos\theta)\int F(u';\phi)\,\mathrm{d}u'.$$

As in Ref. [4], for a plane wave we use a gauge such that $a_v = 0$, which is equivalent to

$$(\boldsymbol{n}_{gw} - \omega^{-1}\boldsymbol{k})\cdot\boldsymbol{a} = 0,$$

where $\boldsymbol{n}_{gw}$ is the unit vector in the direction of propagation of the GW.

The four vector a_α depends on the coordinate u through the functions $f(u)$ and $g(u)$. With the gauge $a_v = 0$, a particular solution is [4]

$$
\begin{aligned}
a_u &= \frac{1}{k_v}\left[k^x a_x + k^y a_y\right] \\
a_v &= 0 \\
a_x &= \left(1+\frac{1}{2} f(u)\right)\bar{a}_x + \frac{1}{2} g(u)\bar{a}_y \\
a_y &= \frac{1}{2} g(u)\bar{a}_x + \left(1-\frac{1}{2} f(u)\right)\bar{a}_y
\end{aligned} \tag{6}
$$

where $\bar{a}_x$ and $\bar{a}_y$ are constants defining an electromagnetic plane wave in the absence of GWs.

Let us use a tetrad $e^{\alpha}_{(a)}$ such that $e^{\alpha}_{(a)} e^{\beta}_{(b)} g_{\alpha\beta} = \eta_{(ab)}$, where $\eta_{(ab)} = \mathrm{diag}(1,1,1,-1)$ is the Minkowski matrix. Then, if $\partial_{(a)} = e^{\alpha}_{(a)} \partial_{\alpha}$, the tetrad is defined by

$$
\begin{aligned}
\partial_{(1)} &= \left(1-\frac{1}{2} f(u)\right)\partial_x - \frac{1}{2} g(u)\partial_y \\
\partial_{(2)} &= -\frac{1}{2} g(u)\partial_x + \left(1+\frac{1}{2} f(u)\right)\partial_y \\
\partial_{(3)} &= \frac{1}{\sqrt{2}}\left(-\partial_u + \partial_v\right) \\
\partial_{(4)} &= \frac{1}{\sqrt{2}}\left(\partial_u + \partial_v\right).
\end{aligned} \tag{7}
$$

Accordingly the tetrad components of a_α and K_α are

$$
a_{(n)} = \left(\bar{a}_x, \bar{a}_y, -a_{(4)}, a_{(4)}\right), \tag{8}
$$

and

$$
K_{(n)} = \begin{pmatrix}
\left(1-\frac{1}{2} f(u)\right)k_x - \frac{1}{2} g(u) k_y \\
-\frac{1}{2} g(u) k_x + \left(1+\frac{1}{2} f(u)\right)k_y \\
\frac{1}{\sqrt{2}}\left[-k_u\left(1-F(u)\right)+k_v\right] \\
\frac{1}{\sqrt{2}}\left[k_u\left(1-F(u)\right)+k_v\right]
\end{pmatrix}. \tag{9}
$$

Notice in particular that $K_{(4)} = -\Omega$, and $\eta^{(mn)} K_{(m)} K_{(n)} = 0$, as it should be.

The electric field in tetrad components is

$$
E_{(n)} = i\left(-K_{(4)} a_{(n)} + a_{(4)} K_{(n)}\right)\mathrm{e}^{iS}, \tag{10}
$$

and of course $E_{(4)} = 0$.

Correlations

For an electromagnetic plane wave with wave vector $K_\alpha(u)$, we find after some lengthy but straightforward algebra (keeping only terms of first order)

$$
\begin{aligned}
\eta^{(mn)} E^{*}_{(m)}(x;k) E_{(n)}(x';k) = \Big\{ & S_0\left[1-\frac{1}{2}(1+\cos\theta)\left(F(u;\phi)+F(u';\phi)\right)\right] \\
& + iS_3(1+\cos\theta)\left(G(u;\phi)-G(u';\phi)\right)\Big\}\mathrm{e}^{i\left(S(x';k)-S(x;k)\right)},
\end{aligned} \tag{11}
$$

where

$$S_0 = \omega^2 \left(|\bar{a}_x|^2 + |\bar{a}_y|^2 \right),$$

$$iS_3 = \omega^2 \left(\bar{a}_y^* \bar{a}_x - \bar{a}_x^* \bar{a}_y \right)$$

are Stokes parameters ($S_3 = 0$ for linear and $|S_3| = S_0$ for circular polarizations).

3. Interferometry

Consider two detectors with space-time coordinates x_1 and x_2, each receiving two plane electromagnetic waves with wave-vectors k_1 and k_2, and use the shorthand notation

$$\begin{aligned} &\eta^{(mn)} E^*_{(m)} \left(x_a; k_j \right) E_{(n)} \left(x_b; k_j \right) \\ &\equiv E^\dagger{}_{a;j} E_{b;j} \equiv S_0 \exp \left\{ \Re_{aj} + i\Im_{aj} + \Re_{bj} - i\Im_{bj} \right\}, \end{aligned} \tag{12}$$

where

$$\Re_{aj} = -\frac{1}{2} \left(1 + \cos\theta_j \right) F \left(u_a; \phi_j \right), \tag{13}$$

$$\Im_{aj} = -S \left(x_a; k_j \right) + \frac{S_3}{S_0} ((1 + \cos\theta_j) G \left(u_a; \phi_j \right), \tag{14}$$

the subindexes *a*, *b* and *j* refer to the labels 1 and 2 of *x* and *k*.

A Michelson interferometer permits to measure the average intensity

$$\begin{aligned} \langle I \rangle \equiv &\left\langle E^\dagger{}_{1;1} E_{1;1} + E^\dagger{}_{1;2} E_{1;2} + E^\dagger{}_{2;1} E_{2;1} + E^\dagger{}_{2;2} E_{2;2} \right\rangle \\ &+ \left\langle E^\dagger{}_{1;1} E_{2;1} + E^\dagger{}_{2;1} E_{1;1} + E^\dagger{}_{1;2} E_{2;2} + E^\dagger{}_{2;2} E_{1;2} \right\rangle, \end{aligned} \tag{15}$$

where the second term is the interference term.

A Hanbury Brown-Twiss interferometer permits to measure the interference between intensities:

$$\begin{aligned} \langle I_1 I_2 \rangle \equiv &\left\langle \left(E^\dagger{}_{1;1} E_{1;1} + E^\dagger{}_{1;2} E_{1;2} \right) \left(E^\dagger{}_{2;1} E_{2;1} + E^\dagger{}_{2;2} E_{2;2} \right) \right\rangle \\ &+ \left\langle E^\dagger{}_{1;1} E_{1;2} E^\dagger{}_{2;2} E_{2;1} + E^\dagger{}_{1;2} E_{1;1} E^\dagger{}_{2;1} E_{2;2} \right\rangle, \end{aligned} \tag{16}$$

where the second term is the interference between the two intensities.

Define

$$\Re_\pm = \Re_{11} \pm \Re_{12} + \Re_{21} \pm \Re_{22}$$

$$\Im_\pm = \Im_{11} - \Im_{21} \pm \Im_{12} \mp \Im_{22}.$$

With this notation, we have for a Michelson interferometer:

$$\langle I \rangle = 2S_0 \left[\left(2 + \Re_+ \right) \left(1 + \cos\left(\Im_+/2 \right) \cos\left(\Im_-/2 \right) \right) - \Re_- \sin\left(\Im_+/2 \right) \sin\left(\Im_-/2 \right) \right], \tag{17}$$

and for a Hanbury Brown-Twiss interferometer:

$$\langle I_1 I_2 \rangle = 2S_0^2 \left(1 + \Re_+ \right) \left(2 + \cos\Im_- \right). \tag{18}$$

Define also the complex functions

$$h(u) = f(u) + ig(u)$$

and

$$\Theta_i = \mathrm{e}^{-2i\phi_i} \cos^2 \frac{\theta_i}{2}.$$

Then

$$\Re_{\pm} = -\Re e\left\{\left[h(u_1)+h(u_2)\right](\Theta_1 \pm \Theta_2)\right\}, \tag{19}$$

and

$$\Im_{\pm} = (x_2 - x_1)\cdot(k_1 \pm k_2) + \Re e\left\{\left[\sqrt{2}\,\omega\int_{u_1}^{u_2} h(u')\,\mathrm{d}u' - 2i\frac{S_3}{S_0}\left(h(u_1)-h(u_2)\right)\right](\Theta_1 \pm \Theta_2)\right\}. \tag{20}$$

In the absence of GWs, $\Re_{\pm} = 0$, and

$$\Im_{+} \to -2\omega(t_2 - t_1) + (\boldsymbol{r}_2 - \boldsymbol{r}_1)\cdot(\boldsymbol{k}_1 + \boldsymbol{k}_2)$$

$$\Im_{-} \to (\boldsymbol{r}_2 - \boldsymbol{r}_1)\cdot(\boldsymbol{k}_1 - \boldsymbol{k}_2),$$

implying that $\langle I_1 I_2\rangle$ is time independent. It thus follows that the time variation of $\langle I_1 I_2\rangle$ is due entirely to the presence of a GW. This time dependence can be made explicit setting

$$\Im_{+} = -2\omega\Delta t + \Delta\boldsymbol{r}\cdot(\boldsymbol{k}_1 + \boldsymbol{k}_2) + \Delta\Im_{+}$$

$$\Im_{-} = -\Delta\boldsymbol{r}\cdot\Delta\boldsymbol{k} + \Delta\Im_{-}$$

where $\Delta t = t_2 - t_1$, $\boldsymbol{\Delta} r = \boldsymbol{r}_2 - \boldsymbol{r}_1$, $\boldsymbol{\Delta k} = \boldsymbol{k}_2 - \boldsymbol{k}_1$, and $\Delta\Im_{\pm}$ are small terms due to the GW. This implies that the terms $\Re_{\pm}$ and $\Delta\Im_{\pm}$ are of first order in the potentials *f* and *g* of the GW.

It should be noticed that the field correlation $\langle I\rangle$ contains terms such as $\cos\left[\boldsymbol{\Delta x}\cdot(\boldsymbol{k}_1 + \boldsymbol{k}_2)\right]$, which are highly oscillatory and hinder a precise measurement with a Michelson interferometer. On the other hand, such terms do not appear in the correlation of the intensities:

$$\langle I_1 I_2\rangle = 2S_0^2\left[(1+\Re_{+})\left[2+\cos(\boldsymbol{\Delta x}\cdot\Delta\boldsymbol{k})\right] + \Delta\Im_{-}\sin(\boldsymbol{\Delta x}\cdot\Delta\boldsymbol{k})\right]. \tag{21}$$

The time dependence is included only in the terms $\Re_{+}$ and $\Delta\Im_{-}$, which are entirely due to the passage of the GW. The term with $\Delta x\cdot(k_1 + k_2)$ is not present in this last formula.

3.1. Temporal Coherence

As a particular application of the above formulas, we can calculate the temporal coherence of a single signal in the presence of a GW. This can be obtained setting $x_1 = (t, r)$, $x_2 = (t+\tau, r)$, and $k_1 = k_2 \equiv k$. Then $\Re_{-} = 0 = \Im_{-}$ and accordingly

$$\langle I\rangle = 2S_0\,(2+\Re_{+})\left(1+\cos(\Im_{+}/2)\right) \tag{22}$$

and

$$\langle I_1 I_2\rangle = 4\,S_0^2\,(1+\Re_{+}). \tag{23}$$

Explicitly, in this particular case,

$$\Re_{+}(t,\tau) = -2\Re e\left\{\left[h(t+\tau)+h(t)\right]\Theta\right\}, \tag{24}$$

which is the only relevant term for the time correlation of the intensity correlation, and is entirely due to the GW.

3.2. Sinusoidal Waves and Pulses

In the particular case of a sinusoidal monochromatic GW of frequency ω_{gw}, we can set

$$h(u) = h_0 \mathrm{e}^{i\omega_{gw}u}, \tag{25}$$

where $h_0 \equiv |h_0|\mathrm{e}^{i\alpha}$ is a complex constant and α a constant phase.

As for a pulse of GW, it can be approximated by a delta function: $h(u) = h_0 \delta(u - u_0)$. In this case, only $\Im_\pm$ is changed after $u > u_0$. We have

$$\Im_\pm (u > u_0) = \Im_\pm (u < u_0) + \sqrt{2}\omega \Re e \{ h_0 (\Theta_1 \pm \Theta_2) \} w(u_0; u_1, u_2), \tag{26}$$

where $w(u_0; u_1, u_2)$ is a function such that $w = 1$ if $u_1 < u_0 < u_2$ and $w = 0$ otherwise. Thus, a pulse of gravitational wave would produce a change both in $\langle I \rangle$ and $\langle I_1 I_2 \rangle$.

4. Conclusion

The main conclusion from the present results is that the passage of a GW produces a *time-dependent* perturbation in the intensity interference of a distant light sources, an interference which would otherwise have a static pattern. Thus, a time variation of $\langle I_1 I_2 \rangle$ will denote the passage of a gravitational wave. A similar effect would be more difficult to observe with $\langle I \rangle$, a direct signal interferometer, due to the presence of highly oscillating terms, as shown above.

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