Journal of Modern Physics, 2015, 6, 2239-2243
Published Online December 2015 in SciRes. http://www.scirp.org/journal/jmp
http://dx.doi.org/10.4236/jmp.2015.615229

The Spinning Period of a Free Electron and the Periods of Spin and Orbital Motions of Electron in Atomic States

Ziya Saglam[1], Mesude Saglam[2], S. Burcin Bayram[3], Tim Horton[3]

[1]Department of Physics, Aksaray University, Aksaray, Turkey
[2]Department of Physics, Ankara University, Ankara, Turkey
[3]Department of Physics, Miami University, Oxford, Ohio, USA
Email: zsaglam@aksaray.edu.tr, saglam@science.ankara.edu.tr, bayramsb@miamioh.edu, hortonts@miamioh.edu

Received 30 November 2015; accepted 26 December 2015; published 29 December 2015

Abstract

The spinning period for a free electron and the periods of spin and orbital motion of the electron in an atomic state have been calculated. We have shown that for a free electron the spinning period is: $(T_s)_{free} = 1.9\times10^{-20}$ s. But in the atomic case we show that, both the spin and the orbital periods depend on the quantum numbers n, m_l, m_s and the effective Landé-g factor, g^* which is a function of the quantum number l of the atomic state $|n,l,m_l,m_s\rangle$ given in Dirac notation. We have also calculated these periods for the ground state and some excited states—hydrogen and hydrogen-like atoms. For atomic states the approximate values of spinning period are $(T_s)_{atomic} \cong 10^{-21}$ s and the related orbital periods are: $(T_0)_{atomic} = (10^{-16} - 10^{-15})$ s. Therefore atto-second processes which are related to the pulse of 10^{-18} s will filter the orbital motion of the electron but will be long enough to detect the details of the spin motion, such as flip-flops.

Keywords

Electron Spin, Landé-g Factor, Magnetic Top Model, Spinning Period, Atto-Seconds Processes

1. Introduction

To calculate the periods of spin and orbital motions of an electron in an atomic state $|n,l,m_l,m_s\rangle$ in Dirac re-

How to cite this paper: Saglam, Z., Saglam, M., Bayram, S.B. and Horton, T. (2015) The Spinning Period of a Free Electron and the Periods of Spin and Orbital Motions of Electron in Atomic States. *Journal of Modern Physics*, **6**, 2239-2243. http://dx.doi.org/10.4236/jmp.2015.615229

presentation, we consider the total magnetic moment of an electron in the presence of a magnetic field in the z direction. The z-component of the total magnetic moment of electron is given by [1]

$$\mu_{total} = \mu_B\left(m_l + g^* m_s\right) = \mu_B m_l + \mu_B g^* m_s = \mu_l + \mu_s \quad (1)$$

where μ_B is the Bohr magneton, which is given by $\hbar e/2mc$ and g^* is the effective Landé-g factor which takes the values $g^* = l + 1 = 1,2,3,4,5$ depending on the values of the outermost electrons and $l = 0,1,2,3,4$ (corresponding to the so called s, p, d, f states respectively).

To calculate the spin period of an electron, we will use the magnetic top model which was first introduced by Barut *et al.* [2]. For calculating the period of the orbital motion we will use the current loop model [3]-[7].

2. Period of the Spinning Motion of Electron

From Equation (1) the z-component of magnetic moment associated with the spinning motion is:

$$\mu_s = \mu_B g^* m_s = \pm\frac{g^*}{2}\mu_B = \pm\frac{g^*}{2}\left(\frac{e\hbar}{2mc}\right) \quad (2)$$

To proceed further, we calculate the intrinsic magnetic moment of electron with a semiclassical, magnetic top model which was first introduced by Barut *et al.* [2].

In the magnetic top model, the spin angular momentum of electron is produced by the spinning of the electronic charge (−e) which is assumed to be uniformly distributed inside a sphere of a radius R. We denote the spin angular frequency of the rotating charged sphere by ω_s, then the magnitude of the magnetic moment of this sphere can be calculated(**Appendix I**) to be

$$\mu_{sphere} = \frac{e\omega_s R^2}{5c} \quad (3)$$

In the presence of the magnetic field $\boldsymbol{B} = B\hat{z}$, the z-component of the magnetic moment of the spinning sphere becomes:

$$\mu_z = \pm\frac{e\omega_s R^2}{5c} \quad (4)$$

If we compare Equation (2) and Equation (4) we can write:

$$\omega_s = \frac{5g^*\hbar}{4mR^2} \quad (5a)$$

$$T_s = \frac{8\pi R^2 m}{5g^*\hbar} \quad (5b)$$

where T_s is the spinning period.

Let us consider the equatorial velocity of this spinning sphere, $\upsilon = R\omega_s$. A simple relativistic argument shows that $\upsilon = c$. Therefore from Equation (5a) we can write:

$$\upsilon \equiv R\omega_s = \frac{5g^*\hbar}{4mR} = c \quad (6)$$

Which defines the radius of electron as below:

$$R = \frac{5g^*\hbar}{4mc} \quad (7)$$

For a free electron $g^* = 2$ substituting other related variables in Equation (7) gives us the radius of a free electron, R_{free}:

$$R_{free} = 9\times10^{-11}\ \text{cm} \quad (8)$$

Substitution of Equation (8) in Equation (5b) gives us the spinning period for a free electron:

$$\left(T_s\right)_{free} = 1.9\times10^{-20}\ \text{s} \quad (9)$$

which is in good agreement with the semiclassical calculation of Olszewski [8].

For an electron in an atom, we cannot calculate the radius directly from Equation (7), because we need to know the effective values of g^*. For the same reason we must take the effective values of g^* in Equation (5b) which gives us $(T_s)_{(n,l,m_l,m_s)}$ for the state $|n,l,m_l,m_s\rangle$.

In the following section we find an expression for the period of orbital motion, $(T_0)_{(n,l,m_l,m_s)}$ for the outermost electron in hydrogen and hydrogen-like atoms: which is given by Equation (14):

$$(T_0)_{(n,l,m_j,m_s)} = \frac{2\pi r_n^2 m}{\hbar |m_l + g^* m_s|}$$

When we take the ratio of the periods given in Equation (5b) and Equation (14), we find:

$$\frac{(T_s)_{(n,l,m_l,m_s)}}{(T_o)_{(n,l,m_l,m_s)}} = \frac{4|m_l + g^* m_s|}{5g^*}\frac{R^2}{r_n^2} \tag{10}$$

Substituting $R = \frac{5g^*\hbar}{4mc}$ (from Equation (7)) and $r_n = n^2 a_0$ with $a_0 = \frac{\hbar^2}{me^2}$ we get:

$$(T_s)_{(n,l,m_l,m_s)} = \frac{|m_l + g^* m_s| e^4 (g^*)}{4n^4\hbar^2 c^2}(T_o)_{(n,l,m_l,m_s)} \tag{11}$$

It is known that when there is no quantum entanglement, for a free electron, the Landé-g factor is equal to 2. For an electron in an atom the Landé-g factor is given by:

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \tag{12}$$

which varies is in range of $g_{\min} = 1 < g < 2 = g_{\max}$. Recently, Saglam *et al.* [1] showed that because of the quantum entanglements in an atom the Landé-g factor is replaced by the effective g-factor, g^* which takes the values $g^* = l+1 = 1,2,3,4,5$ depending on $l = 0,1,2,3,4$ (corresponding to the so called s, p, d, f states respectively) values of the outermost electrons together with the unfilled shells respectively. So the maximum values of the effective Landé-g factor, g^* can be as high as 5. Therefore $g^*_{\min} = 1 < g^* < g^*_{\max} = 5$.

If we calculate the effective g-factor, g^* for the ground state hydrogen atom, $|1,0,0,1/2\rangle$, we found that $g^* = 1$. For this value we calculate the period of ground state orbit and find: $(T_o)_{(1,0,0,1/2)} = 2.6\times10^{-16}$ s. Substituting this value and $n = 1$, $m_l = 0$, $m_s = 1/2$, $g^* = 1$ and other related parameters in Equation (11), we find the spinning period of electron in $|1,0,0,1/2\rangle$ state, $(T_s)_{(1,0,0,1/2)} \cong 1.48\times10^{-21}$ s. We give the values of $(T_s)_{(n,l,m_j,m_s)}$ and $(T_0)_{(n,l,m_j,m_s)}$ for the states: $|1,0,0,1/2\rangle$, $|2,1,0,1/2\rangle$, $|3,2,2,1/2\rangle$, $|4,3,1,1/2\rangle$ and $|4,3,2,1/2\rangle$ in **Table 1**.

3. Period of the Orbital Motion of Electron

From Equation (1) the *z*-component of the total magnetic moment is:

Table 1. The values of $(T_s)_{(n,l,m_j,m_s)}$ and $(T_0)_{(n,l,m_j,m_s)}$ for the states.

Spinning periods	Periods of orbital motion
$(T_s)_{(1,0,0,1/2)} \cong 1.48\times10^{-21}$ s	$(T_0)_{(1,0,0,1/2)} = 2.6\times10^{-16}$ s
$(T_s)_{(2,1,0,1/2)} \cong 7.66\times10^{-21}$ s	$(T_0)_{(2,1,0,1/2)} = 2.08\times10^{-15}$ s
$(T_s)_{(3,2,2,1/2)} \cong 1.77\times10^{-21}$ s	$(T_0)_{(3,2,2,1/2)} = 6.4\times10^{-16}$ s
$(T_s)_{(4,3,1,1/2)} \cong 1.59\times10^{-21}$ s	$(T_0)_{(4,3,1,1/2)} = 2.3\times10^{-15}$ s
$(T_s)_{(4,3,2,1/2)} \cong 1.79\times10^{-21}$ s	$(T_0)_{(4,3,2,1/2)} = 1.1\times10^{-15}$ s

$$\mu_j = \mu_{total} = \mu_B\left(m_l + g^* m_s\right) = \mu_B m_l + \mu_B g^* m_s = \mu_l + \mu_s$$

where $m_l = 0, \pm 1, \cdots, \pm l$, $m_s = \pm 1/2$, $\mu_B = \dfrac{e\hbar}{2mc}$ and $g^* = 1, 2, 3, 4, 5$.

Now we find another expression for μ_{total} in the current loop model [2]: we assume that the magnetic moment associated with the orbital motion of electron is produced by the fictitious point charge (−e) rotating in a circular orbit with the angular frequency $\omega_j = 2\pi/T_j$ and the radius r_n in *x*-*y* plane. In this model the *z*-component of the magnetic moment will be

$$\mu_{total} = \frac{IA}{c} = \frac{e\omega_j \pi r_n^2}{2\pi c} = \frac{\pi e r_n^2}{cT_{total}} \tag{13}$$

If we compare Equation (12) and Equation (13) we write:

$$\left(T_0\right)_{(n,l,m_j,m_s)} = \frac{2\pi r_n^2 m}{\hbar\left|m_l + g^* m_s\right|} \tag{14}$$

where we replace T_{total} by $\left(T_0\right)_{(n,l,m_j,m_s)}$; here the subscript (0) stands for orbital motion.

Now we can find the values of $\left(T_0\right)_{(n,l,m_j,m_s)}$ for hydrogen and hydrogen-like atoms: especially for $\left|n, n-1, m_l, m_s\right\rangle$ states we can put $r_n = n^2 a_0$ where $a_0 = \dfrac{\hbar^2}{me^2}$.

With these replacements Equation (14) becomes:

$$\left(T_0\right)_{(n,l,m_l,m_s)} = \frac{2\pi r_n^2 m}{\hbar\left|m_l + g^* m_s\right|} = \frac{2\pi\hbar^3 n^4}{me^4\left|m_l + g^* m_s\right|} \tag{15}$$

We note that the quantum number (*l*) gets involved through the effective Lande-g factor, g^* which takes the values $g^* = l + 1$.

For example, for the ground state of hydrogen atom $\left|1, 0, 0, 1/2\right\rangle$, substituting $g^* = 1$ and the other related parameter in Equation (15), we find:

$$\left(T_0\right)_{(1,0,0,1/2)} = 2.6 \times 10^{-16}\ \text{s} \tag{16}$$

Similarly for the state $\left|2, 1, 0, 1/2\right\rangle$ the corresponding period is:

$$\left(T_0\right)_{(2,1,0,1/2)} = 2.08 \times 10^{-15}\ \text{s} \tag{17}$$

where we put: $n = 2$, $l = 1$, $g^* = l + 1 = 2$ and $m_l = 0$ in Equation (15).

Acknowledgments

Authors from Miami University acknowledge financial support from the National Science Foundation (Grant No. NSF-PHY-1309571).

References

[1] Saglam, Z., Bayram, S.B. and Saglam, M. (2010) *Journal of Modern Physics*, **1**, 399-404. http://dx.doi.org/10.4236/jmp.2010.16057

[2] Barut, O.A., Bozic, M. and Maric, Z. (1992) *Annals of Physics*, **214**, 53-83. http://dx.doi.org/10.1016/0003-4916(92)90061-P

[3] Saglam, M. and Sahin, G. (2009) *Journal of Modern Physics*, *B*, **23**, 4977-4985.

[4] Saglam, M., Saglam, Z., Boyacioglu, B. and Wan, K.K. (2007) *Journal of Russian Laser Research*, **28**, 267-271. http://dx.doi.org/10.1007/s10946-007-0015-6

[5] Saglam, Z. and Boyacioglu, B. (2007) *Journal of Russian Laser Research*, **28**, 142-147. http://dx.doi.org/10.1007/s10946-007-0008-5

[6] Saglam, M., Boyacioglu, B. and Saglam, Z. (2007) *Journal of Russian Laser Research*, **28**, 377-382. http://dx.doi.org/10.1007/s10946-007-0026-3

[7] Saglam, M., Saglam, Z. and Boyacioglu, B. (2006) *Journal of the Old and New Concepts in Physics*, **3**, 181-192.

[8] Olszewski, S. (2014) *Journal of Modern Physics*, **5**, 2030-2040. http://dx.doi.org/10.4236/jmp.2014.518199

Appendix I: Calculation of the Magnetic Moment of a Spinning Charge, *Q* Distributed Uniformly inside a Sphere of Radius *R*

Let us denote the uniform charge density by ρ_0 which is related to the total charge *Q* by:

$$Q = \int_0^Q \mathrm{d}Q = \rho_o \int_0^R 4\pi r^2 \mathrm{d}r = \rho_o \left(\frac{4\pi}{3} R^3 \right) \quad \text{(A-I)}$$

where $q \equiv \mathrm{d}Q = \rho_o \left(4\pi r^2 \mathrm{d}r\right)$ is the charge of the spherical shell with the radius *r* and thickness $\mathrm{d}r$. First we want to calculate the magnetic moment of this spherical shell with the surface charge density ($\sigma = q/4\pi r^2$). Let us assume that the spinning is about z-axis with the angular frequency, ω_s. Let us consider the charge element $\mathrm{d}q$ in the area of the band with the radius ($r\sin\theta$) and the thickness ($r\mathrm{d}\theta$) in spherical coordinates:

$$\mathrm{d}q = \sigma \mathrm{d}a = \left(\frac{q}{4\pi r^2} 2\pi r^2 \sin\theta \right) \mathrm{d}\theta = \left(\frac{q}{2} \sin\theta \right) \mathrm{d}\theta \quad \text{(A-II)}$$

The current element $\mathrm{d}I$ produced by the rotating band charge with the angular frequency, ω_s will be:

$$\mathrm{d}I = \frac{\omega_s}{2\pi} \mathrm{d}q = \left(\frac{\omega_s q}{4\pi} \sin\theta \right) \mathrm{d}\theta \quad \text{(A-III)}$$

The magnetic moment element of this band current will be:

$$\mathrm{d}\mu_{band} = \frac{\mathrm{d}I}{c} \pi \left(r\sin\theta \right)^2 = \left(\frac{\omega_s q r^2}{4c} \sin^3\theta \right) \mathrm{d}\theta \quad \text{(A-IV)}$$

Integrating over the spherical shell gives us the magnetic moment of this shell, μ_{shell}:

$$\begin{aligned} \mu_{shell} &= \int \mu_{band} = \int_0^\pi \left(\frac{\omega_s q r^2}{4c} \sin^3\theta \right) \mathrm{d}\theta = \frac{\omega_s q r^2}{4c} \int_0^\pi \sin^3\theta \mathrm{d}\theta \\ &= \frac{\omega_s q r^2}{2c} \int_0^{\pi/2} \left(1-\cos^2\theta\right) \sin\theta \mathrm{d}\theta = \frac{\omega_s q r^2}{2c} \int_0^1 \left(1-x^2\right) \mathrm{d}x = \frac{\omega_s q r^2}{3c} \end{aligned} \quad \text{(A-V)}$$

If we substitude $q \equiv \mathrm{d}Q = \rho_o \left(4\pi r^2 \mathrm{d}r\right)$ in (A-V) and integrate over the spherical volume, we find the total magnetic moment of the sphere of radius *R*:

$$\mu_{sphere} = \int_0^R \mathrm{d}\mu_{sphere} = \int_0^R \frac{\omega_s r^2 \left(\rho_o 4\pi r^2 \mathrm{d}r\right)}{3c} = \frac{4\pi\omega_s\rho_o}{3c} \int_0^R r^4 \mathrm{d}r = \frac{4\pi\omega_s\rho_o}{3c} \frac{R^5}{5} \quad \text{(A-VI)}$$

Substituting (A-I) in (A-VI) we find:

$$\mu_{sphere} = \frac{Q\omega_s R^2}{5c}. \quad \text{(A-VII)}$$