

The Masses of P_c^* (4380) and P_c^* (4450) as Di-Hadronic States

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Abstract

The masses of the recently reported by LHCb two pentaquark charmonium states P_c^* (4380) and P_c^* (4450) which are suggested to possess pentaquark configuration (uudc \bar{c}) have been estimated considering a di-hadronic state consisting of a meson $c\bar{c}$ and a baryon (uud). The binding energies of the states have been estimated with a van der Waals type of molecular interaction between the hadrons. A spin interaction has also been considered. Masses of these two states are well reproduced with the aforesaid molecular interaction which indicates that the multi-quarks P_c^* (4380) and P_c^* (4450) can be well described as meson-baryon bound states.

Keywords

Pentaquark, Di-Hadronic Molecule, Molecular Interaction

1. Introduction

The existence of pentaquark charmonium states with the decay of $\Lambda_b^0 (\Lambda_b^0 \rightarrow J/\psi K^- p)$ has been reported by LHCb [1] recently. The intermediate states have been identified as P_c^* (4380) and P_c^* (4450). The states are identified as sum of two up quarks, one down quark, one charm quark and one anti-charm quark with spin $\frac{3}{2}$ and $\frac{5}{2}$ respectively. The identification of these pentaquark states is exciting and will give new impetus to the study of the properties and dynamics of multi-quark states [2]. The exotics remain less known and less unders-

tood compared to the properties of mesons ($q\bar{q}$) and baryons (qqq) which are well studied both in theory and experiment. The properties and dynamics of exotic states like tetraquark, pentaquark, hexaquark states are yet to be studied and it is well understood that they cannot be described in the framework of conventional quark model. A number of models like the quark model [3] [4], bag model [5] [6], and non-relativistic potential models [7] [8] have been used to study the multi-quark systems. The description of the multi-quark states as di-hadronic states considering them as a bound state of a meson and a baryon is one of the useful candidates for studying the properties of such systems.

2. Method

In the present work pentaquark states P_c^* (4380) and P_c^* (4450) are described as di-hadronic molecules consisting of a meson and a baryon assuming a van der Waals type of molecular interaction acting between the constituent hadrons [9] [10]. P_c^* (4380) state of spin $\frac{3}{2}$ is assumed to have configuration as proton-charmonium state as $(uudc\bar{c})$ whereas P_c^* (4450) state of spin $\frac{5}{2}$ has been described as $\Delta - J/\psi$ state. A spin interaction has also been considered. Masses of P_c^* (4380) and P_c^* (4450) as di-hadronic molecule have been estimated using the mass formula.

Assuming the pentaquark states as meson-baryon system the mass formula for the low-lying di-hadronic molecule runs as:

$$M_{\text{Total}} = M_1 + M_2 + E_{BE} + E_{SD} \quad (1)$$

where M_1, M_2 represent the masses of the constituent hadrons respectively, E_{BE} represents the binding energy of the di-hadronic system and E_{SD} represents the spin-dependent term.

The binding energy can be expressed as:

$$E_{BE} = \langle \Psi(r) | V(r) | \Psi(r) \rangle \quad (2)$$

where r is the radius parameter of the di-hadronic molecule and $V(r)$ is the di-hadronic molecular potential which is expressed as [9] [10]:

$$V(r) = \frac{-k_{\text{mol}}}{r} e^{-C^2 r^2/2} \quad (3)$$

where k_{mol} [9] [10] is the residual strength of the strong interaction molecular coupling and C is the effective colour screening of the confined gluons. It may be mentioned that the residual interaction of the confined gluon is considered similar to van der Waals interaction and is assumed to be due to asymptotic expression ($r_{12} \rightarrow \infty$) of the residual confined one-gluon exchange interaction with strength k_{mol} [9] [10].

$\Psi(r)$ is the wave function of the di-hadronic state. To estimate E_{BE} we have used the wave functions for the ground state of the hadronic molecule from statistical model which runs as: [11] [12]

$$|\Psi(r)|^2 = \frac{315}{64\pi r_{12}^2} (r_{12} - r)^{\frac{3}{2}} \theta(r_{12} - r) \quad (4)$$

$$|\Psi(r)|^2 = \frac{8}{\pi^2 r_{12}^6} (r_{12}^2 - r^2)^{\frac{3}{2}} \theta(r_{12} - r) \quad (5)$$

corresponding to the linear type of background potential and harmonic type of background potential respectively [11] [12]. r_{12} is the radius of the hadronic molecule and $\theta(r_{12} - r)$ is usual step function. With $r_{12} = r_1 + r_2$, where r_1 and r_2 represent the individual radii of the hadrons constituting the molecule respectively and using Equations (2), (3) and (4) we get E_{BE} as:

$$E_{BE} = \frac{2.25k_{\text{mol}}}{r_{12}} 2F_2 \left[(1, 1.5), (2.25, 2.75), -\beta \right] \quad (6)$$

$$E_{BE} = \frac{16k_{\text{mol}}}{\pi r_{12}^5 C^5} \left[-6C + 2r_{12}^2 C^3 + \frac{3e^{-\beta} (2\pi)^{\frac{1}{2}} \text{Erfi} \left[\frac{1}{\beta^2} \right]}{r_{12}} \right] \quad (7)$$

where $\beta = C^2 r_{12}^2 / 2$, $C = 50$ MeV [13] and $k_{\text{mol}} = 0.59$ and 0.65 [14] corresponding to linear and harmonic type of background potentials respectively. The radius of the corresponding baryons have been estimated by adjusting the value of charge radii [15] [16] against the form factor of corresponding baryon [17] [18]. The radius of “p” and “ Δ ” are obtained as 7.59 GeV^{-1} and 5.977 GeV^{-1} respectively. The radius of $c\bar{c}$ has been used from [19] as $r(J/\psi) = 2.005 \text{ GeV}^{-1}$.

The spin hyperfine interaction can be expressed as [20]:

$$E_{SD} = \frac{8}{9} \frac{\alpha_s}{M_1 M_2} \mathbf{S}_1 \cdot \mathbf{S}_2 |\Psi(0)|^2 \quad (8)$$

where M_1 and M_2 are the masses of the constituent hadrons in the di-hadronic molecule, α_s is the strong interaction constant, S_1 and S_2 are the spins of the hadrons involved, $|\Psi(0)|^2$ is the di-hadronic wave function at the origin. With $\alpha_s = 0.59$ [21] the E_{SD} has been estimated subsequently using the relation (8). The masses of P_c^* (4380), P_c^* (4450) have been estimated using the Equation (1) with mass of the respective meson (M_1) and baryon (M_2) [15] [22] and displayed at the **Table 1**.

3. Discussions

In the present work we have estimated masses of particles P_c^* (4380) $\left(\frac{3}{2}\right)$ and P_c^* (4450) $\left(\frac{5}{2}\right)$ considering them as di-hadronic (meson-baryon) molecules. The masses have been obtained as 4171 MeV and 4492 MeV for P_c^* (4380) and 4168 MeV and 4191 MeV for P_c^* (4450) with the input of two wave functions from the statistical model. The results are found to be in good agreement with the experiment. We have observed that the mass of P_c^* (4450) state is well reproduced where as for the P_c^* (4380) has somewhat smaller (~ 200 MeV) value which may be attributed to the uncertainty in the radius parameter used. It is interesting to note here that the pentaquarks P_c^* $\left(\frac{3}{2}\right)$, P_c^* $\left(\frac{5}{2}\right)$ are well described in the di-hadronic molecules with a weak van der Waals type of interaction between them. It is also pertinent to point out that the statistical model wave function is also very successful in describing the hadrons. The pentaquark state is one of the leading candidates for the study of the multiquark state. The description of pentaquark as diquark-diquark-antiquark state has been done by a number of authors [3] [4]. It may be mentioned that recently some predictions of diquark model for hidden charm pentaquarks P_c^* (4450) and P_c^* (4380) have been proposed by Li *et al.* [23]. They have found that in the SU(3) limit, for U-spin related decay modes the ratio of the decay rates of Cabibbo suppressed to Cabibbo allowed decay channels is given by $\frac{|V_{cd}|^2}{|V_{cs}|^2}$. The present investigation shows that they are also described well in the framework of di-hadronic states.

Table 1. Binding energies and masses of pentaquark charmonium states as meson-baryon states.

Particle	State	With $ \Psi(0) ^2$ from (4)		With $ \Psi(0) ^2$ from (5)		
		$E_{BE} + E_{SD}$ (MeV)	M (MeV)	$E_{BE} + E_{SD}$ (MeV)	M (MeV)	M_{exp} (MeV) [1]
P_c^* (4380) (spin $\frac{3}{2}$)	p - J/ ψ	136	4171	133	4168	$4380 \pm 8 \pm 29$
P_c^* (4450) (spin $\frac{5}{2}$)	Δ - J/ ψ	163	4492	162	4491	$4449.8 \pm 1.7 \pm 2.5$

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