Journal of Modern Physics, 2015, 6, 1721-1736
Published Online September 2015 in SciRes. http://www.scirp.org/journal/jmp
http://dx.doi.org/10.4236/jmp.2015.612174

Local Particle-Ghost Symmetry

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Received 16 June 2015; accepted 27 September 2015; published 30 September 2015

Abstract

We study the quantization of systems with local particle-ghost symmetries. The systems contain ordinary particles including gauge bosons and their counterparts obeying different statistics. The particle-ghost symmetries are new type of fermionic symmetries between ordinary particles and their ghost partners, different from the space-time supersymmetry and the BRST symmetry. There is a possibility that they are useful to explain phenomena of elementary particles at a more fundamental level, by extension of our systems. We show that our systems are formulated consistently or subsidiary conditions on states guarantee the unitarity of systems, as the first step towards the construction of a realistic fundamental theory.

Keywords

Fermionic Symmetry, BRST Quantization, Quartet Mechanism

1. Introduction

Graded Lie algebras or Lie superalgebras have been frequently used to formulate theories and construct models in particle physics. Typical examples are supersymmetry (SUSY) [1]-[4] and BRST symmetry [5]-[7].

The space-time SUSY [8] [9] is a symmetry between ordinary particles with integer spin and those with half-integer spin, and the generators called supercharges are space-time spinors that obey the anti-commutation relations [10] [11].

The BRST symmetry is a symmetry concerning unphysical modes in gauge fields and abnormal fields called Faddeev-Popov ghost fields [12]. Though both gauge fields and abnormal fields contain negative norm states, theories become unitary on the physical subspace, thanks to the BRST invariance [13] [14]. The BRST and anti-BRST charges are anti-commuting space-time scalars.

Recently, models that contain both ordinary particles with a positive norm and their counterparts obeying different statistics have been constructed and those features have been studied [15]-[18]. Models have fermionic symmetries different from the space-time SUSY and the BRST symmetry. We refer to this type of novel symmetries as "particle-ghost symmetries".

The particle-ghost symmetries have been introduced as global symmetries, but we do not need to restrict them to the global ones. Rather, it would be meaningful to examine systems with local particle-ghost symmetries from following reasons. It is known that any global continuous symmetries can be broken down by the effect of quantum gravity such as a wormhole [19]. Then, it is expected that a fundamental theory possesses local symmetries, and global continuous symmetries can appear as accidental ones in lower-energy scale. In the system with global particle-ghost symmetries, the unitarity holds by imposing subsidiary conditions on states by hand. In contrast, there is a possibility that the conditions are realized as remnants of local symmetries in a specific situation. Hence, it is interesting to investigate features of particle-ghost symmetries more closely and widely, and to apply them on a microscopic theory beyond the standard model.

In this paper, we study the quantization of systems with local particle-ghost symmetries. The systems contain ordinary particles including gauge bosons and their counterparts obeying different statistics. We show that our systems are formulated consistently or subsidiary conditions on states guarantee the unitarity of systems, as the first step towards the construction of a realistic fundamental theory. The conditions can be originated from constraints in case that gauge fields have no dynamical degrees of freedom.

The contents of this paper are as follows. We construct models with local fermionic symmetries in Section 2, and carry out the quantization of the system containing scalar and gauge fields in Section 3. Section 4 is devoted to conclusions and discussions on applications of particle-ghost symmetries. In the Appendix Section, we study the system that gauge fields are auxiliary ones.

2. Systems with Local Fermionic Symmetries

2.1. Scalar Fields with Local Fermionic Symmetries

Recently, the system described by the following Lagrangian density has been studied [15]-[18],

$$\mathcal{L}_{\varphi,c_\varphi} = \partial_\mu \varphi^\dagger \partial^\mu \varphi - m^2 \varphi^\dagger \varphi + \partial_\mu c_\varphi^\dagger \partial^\mu c_\varphi - m^2 c_\varphi^\dagger c_\varphi, \tag{1}$$

where φ is an ordinary complex scalar field and c_φ is the fermionic counterpart obeying the anti-commutation relations. The system has a global $OSp(2|2)$ symmetry that consists of $U(1)$ and fermionic symmetries. The unitarity holds by imposing suitable subsidiary conditions relating the conserved charges on states.

Starting from (1), the model with the local $OSp(2|2)$ symmetry is constructed by introducing gauge fields. The resultant Lagrangian density is given by

$$\begin{aligned}
&\mathcal{L} = \mathcal{L}_{\mathrm{M}} + \mathcal{L}_{\mathrm{G}} \\
&\mathcal{L}_{\mathrm{M}} = \left\{\left(\partial_\mu - igA_\mu - igB_\mu\right)\varphi^\dagger - gC_\mu^- c_\varphi^\dagger\right\}\left\{\left(\partial^\mu + igA^\mu + igB^\mu\right)\varphi + gC^{+\mu} c_\varphi\right\} \\
&\quad + \left\{\left(\partial_\mu - igA_\mu + igB_\mu\right)c_\varphi^\dagger - gC_\mu^+ \varphi^\dagger\right\}\left\{\left(\partial^\mu + igA^\mu - igB^\mu\right)c_\varphi - gC^{-\mu}\varphi\right\} \\
&\quad - m^2\varphi^\dagger\varphi - m^2 c_\varphi^\dagger c_\varphi,
\end{aligned} \tag{2}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{G}} = &-\left\{\partial_\mu A_\nu - \partial_\nu A_\mu + ig\left(C_\mu^+ C_\nu^- - C_\nu^+ C_\mu^-\right)\right\}\left\{\partial^\mu B^\nu - \partial^\nu B^\mu\right\} \\
&- \frac{1}{2}\left\{\partial_\mu C_\nu^+ - \partial_\nu C_\mu^+ + 2ig\left(B_\mu C_\nu^+ - B_\nu C_\mu^+\right)\right\} \\
&\cdot \left\{\partial^\mu C^{-\nu} - \partial^\nu C^{-\mu} - 2ig\left(B^\mu C^{-\nu} - B^\nu C^{-\mu}\right)\right\},
\end{aligned} \tag{3}$$

where A_μ and B_μ are the gauge fields relating the (diagonal) $U(1)$ symmetries, C_μ^+ and C_μ^- are gauge fields relating the fermionic symmetries, and g is the gauge coupling constant. The quantized fields of $C_\mu^\pm$ obey the anti-commutation relations.

The $\mathcal{L}$ is invariant under the local $U(1)$ transformations,

$$\begin{aligned}
&\delta_A \varphi = -i\epsilon\varphi,\ \delta_A \varphi^\dagger = i\epsilon\varphi^\dagger,\ \delta_A c_\varphi = -i\epsilon c_\varphi,\ \delta_A c_\varphi^\dagger = i\epsilon c_\varphi^\dagger, \\
&\delta_A A_\mu = \frac{1}{g}\partial_\mu \epsilon,\ \delta_A B_\mu = 0,\ \delta_A C_\mu^+ = 0,\ \delta_A C_\mu^- = 0,
\end{aligned} \tag{4}$$

$$\delta_{\mathrm{B}}\varphi=-i\xi\varphi,\ \delta_{\mathrm{B}}\varphi^{\dagger}=i\xi\varphi^{\dagger},\ \delta_{\mathrm{B}}c_{\varphi}=i\xi c_{\varphi},\ \delta_{\mathrm{B}}c_{\varphi}^{\dagger}=-i\xi c_{\varphi}^{\dagger},$$
$$\delta_{\mathrm{B}}A_{\mu}=0,\ \delta_{\mathrm{B}}B_{\mu}=\frac{1}{g}\partial_{\mu}\xi,\ \delta_{\mathrm{B}}C_{\mu}^{+}=-2i\xi C_{\mu}^{+},\ \delta_{\mathrm{B}}C_{\mu}^{-}=2i\xi C_{\mu}^{-} \tag{5}$$

and the local fermionic transformations,

$$\delta_{\mathrm{F}}\varphi=-\zeta c_{\varphi},\ \delta_{\mathrm{F}}\varphi^{\dagger}=0,\ \delta_{\mathrm{F}}c_{\varphi}=0,\ \delta_{\mathrm{F}}c_{\varphi}^{\dagger}=\zeta\varphi^{\dagger},$$
$$\delta_{\mathrm{F}}A_{\mu}=-i\zeta C_{\mu}^{-},\ \delta_{\mathrm{F}}B_{\mu}=0,\ \delta_{\mathrm{F}}C_{\mu}^{+}=2i\zeta B_{\mu}+\frac{1}{g}\partial_{\mu}\zeta,\ \delta_{\mathrm{F}}C_{\mu}^{-}=0, \tag{6}$$

$$\delta_{\mathrm{F}}^{\dagger}\varphi=0,\ \delta_{\mathrm{F}}^{\dagger}\varphi^{\dagger}=\zeta^{\dagger}c_{\varphi}^{\dagger},\ \delta_{\mathrm{F}}^{\dagger}c_{\varphi}=\zeta^{\dagger}\varphi,\ \delta_{\mathrm{F}}^{\dagger}c_{\varphi}^{\dagger}=0,$$
$$\delta_{\mathrm{F}}^{\dagger}A_{\mu}=-i\zeta^{\dagger}C_{\mu}^{+},\ \delta_{\mathrm{F}}^{\dagger}B_{\mu}=0,\ \delta_{\mathrm{F}}^{\dagger}C_{\mu}^{+}=0,\ \delta_{\mathrm{F}}^{\dagger}C_{\mu}^{-}=-2i\zeta^{\dagger}B_{\mu}+\frac{1}{g}\partial_{\mu}\zeta^{\dagger}, \tag{7}$$

where ϵ and ξ are infinitesimal real functions of x, and ζ and $\zeta^{\dagger}$ are Grassmann-valued functions of x.

The $\mathcal{L}_{\mathrm{M}}$ and $\mathcal{L}_{\mathrm{G}}$ are simply written as

$$\mathcal{L}_{\mathrm{M}}=\left(D_{\mu}\Phi\right)^{\dagger}\left(D^{\mu}\Phi\right)-m^{2}\Phi^{\dagger}\Phi \quad \text{and} \quad \mathcal{L}_{\mathrm{G}}=-\frac{1}{4}\mathrm{Str}\left(F_{\mu\nu}F^{\mu\nu}\right), \tag{8}$$

respectively. In $\mathcal{L}_{\mathrm{M}}$, D_{μ} and Φ are the covariant derivative and the doublet of fermionic transformation defined by

$$D_{\mu}\equiv\begin{pmatrix}\partial_{\mu}+igA_{\mu}+igB_{\mu} & gC_{\mu}^{+}\\ -gC_{\mu}^{-} & \partial_{\mu}+igA_{\mu}-igB_{\mu}\end{pmatrix} \quad \text{and} \quad \Phi\equiv\begin{pmatrix}\varphi\\ c_{\varphi}\end{pmatrix}, \tag{9}$$

respectively. In $\mathcal{L}_{\mathrm{G}}$, Str is the supertrace defined by $\mathrm{Str}M=a-d$ where $\boldsymbol{M}$ is the 2 × 2 matrix given by

$$\boldsymbol{M}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}. \tag{10}$$

The $F_{\mu\nu}$ is defined by

$$F_{\mu\nu}\equiv\frac{1}{ig}\left[D_{\mu},D_{\nu}\right]=\begin{pmatrix}A_{\mu\nu}+B_{\mu\nu} & -iC_{\mu\nu}^{+}\\ iC_{\mu\nu}^{-} & A_{\mu\nu}-B_{\mu\nu}\end{pmatrix}, \tag{11}$$

where $A_{\mu\nu}$, $B_{\mu\nu}$, $C_{\mu\nu}^{+}$ and $C_{\mu\nu}^{-}$ are the field strengths given by

$$A_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}+ig\left(C_{\mu}^{+}C_{\nu}^{-}-C_{\nu}^{+}C_{\mu}^{-}\right), \tag{12}$$

$$B_{\mu\nu}=\partial_{\mu}B_{\nu}-\partial_{\nu}B_{\mu}, \tag{13}$$

$$C_{\mu\nu}^{+}=\partial_{\mu}C_{\nu}^{+}-\partial_{\nu}C_{\mu}^{+}+2ig\left(B_{\mu}C_{\nu}^{+}-B_{\nu}C_{\mu}^{+}\right), \tag{14}$$

$$C_{\mu\nu}^{-}=\partial_{\mu}C_{\nu}^{-}-\partial_{\nu}C_{\mu}^{-}-2ig\left(B_{\mu}C_{\nu}^{-}-B_{\nu}C_{\mu}^{-}\right). \tag{15}$$

Under the transformations (4)-(7), the field strengths are transformed as

$$\delta_{A}A_{\mu\nu}=0,\ \delta_{A}B_{\mu\nu}=0,\ \delta_{A}C_{\mu\nu}^{+}=0,\ \delta_{A}C_{\mu\nu}^{-}=0, \tag{16}$$

$$\delta_{\mathrm{B}}A_{\mu\nu}=0,\ \delta_{\mathrm{B}}B_{\mu\nu}=0,\ \delta_{\mathrm{B}}C_{\mu\nu}^{+}=-2i\xi C_{\mu\nu}^{+},\ \delta_{\mathrm{B}}C_{\mu\nu}^{-}=2i\xi C_{\mu\nu}^{-}, \tag{17}$$

$$\delta_{\mathrm{F}}A_{\mu\nu}=-i\zeta C_{\mu\nu}^{-},\ \delta_{\mathrm{F}}B_{\mu\nu}=0,\ \delta_{\mathrm{F}}C_{\mu\nu}^{+}=2i\zeta B_{\mu\nu},\ \delta_{\mathrm{F}}C_{\mu\nu}^{-}=0, \tag{18}$$

$$\delta_{\rm F}^{\dagger} A_{\mu\nu} = -i\zeta^{\dagger} C_{\mu\nu}^{+},\ \delta_{\rm F}^{\dagger} B_{\mu\nu} = 0,\ \delta_{\rm F}^{\dagger} C_{\mu\nu}^{+} = 0,\ \delta_{\rm F}^{\dagger} C_{\mu\nu}^{-} = -2i\zeta^{\dagger} B_{\mu\nu}. \tag{19}$$

Using the global fermionic transformations,

$$\begin{aligned} &\tilde{\boldsymbol{\delta}}_{\rm F}\varphi = -c_{\varphi},\ \tilde{\boldsymbol{\delta}}_{\rm F}\varphi^{\dagger} = 0,\ \tilde{\boldsymbol{\delta}}_{\rm F}c_{\varphi} = 0,\ \tilde{\boldsymbol{\delta}}_{\rm F}c_{\varphi}^{\dagger} = \varphi^{\dagger}, \\ &\tilde{\boldsymbol{\delta}}_{\rm F}A_{\mu} = -iC_{\mu}^{-},\ \tilde{\boldsymbol{\delta}}_{\rm F}B_{\mu} = 0,\ \tilde{\boldsymbol{\delta}}_{\rm F}C_{\mu}^{+} = 2iB_{\mu},\ \tilde{\boldsymbol{\delta}}_{\rm F}C_{\mu}^{-} = 0, \end{aligned} \tag{20}$$

$$\begin{aligned} &\tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}\varphi = 0,\ \tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}\varphi^{\dagger} = c_{\varphi}^{\dagger},\ \tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}c_{\varphi} = \varphi,\ \tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}c_{\varphi}^{\dagger} = 0, \\ &\tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}A_{\mu} = -iC_{\mu}^{+},\ \tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}B_{\mu} = 0,\ \tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}C_{\mu}^{+} = 0,\ \tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}C_{\mu}^{-} = -2iB_{\mu}, \end{aligned} \tag{21}$$

$\mathcal{L}$ is rewritten as

$$\mathcal{L} = \tilde{\boldsymbol{\delta}}_{\rm F}\tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}\mathcal{L}_{\varphi,A} = -\tilde{\boldsymbol{\delta}}_{\rm F}^{\dagger}\tilde{\boldsymbol{\delta}}_{\rm F}\mathcal{L}_{\varphi,A}, \tag{22}$$

where $\mathcal{L}_{\varphi,A}$ is given by

$$\begin{aligned} \mathcal{L}_{\varphi,A} = &\left\{\left(\partial_{\mu} - igA_{\mu} - igB_{\mu}\right)\varphi^{\dagger} - gC_{\mu}^{-}c_{\varphi}^{\dagger}\right\}\left\{\left(\partial^{\mu} + igA^{\mu} + igB^{\mu}\right)\varphi + gC^{+\mu}c_{\varphi}\right\} \\ &- m^{2}\varphi^{\dagger}\varphi - \frac{1}{4}A_{\mu\nu}A^{\mu\nu}. \end{aligned} \tag{23}$$

2.2. Spinor Fields with Local Fermionic Symmetries

For spinor fields, we consider the Lagrangian density,

$$\mathcal{L}_{\psi,c_{\psi}} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + i\bar{c}_{\psi}\gamma^{\mu}\partial_{\mu}c_{\psi} - m\bar{c}_{\psi}c_{\psi}, \tag{24}$$

where ψ is an ordinary spinor field and c_{ψ} is its bosonic counterpart obeying commutation relations. This system also has global $U(1)$ and fermionic symmetries, and the unitarity holds by imposing suitable subsidiary conditions on states.

Starting from (24), the Lagrangian density with local symmetries is constructed as

$$\begin{aligned} &\mathcal{L}^{sp} = \mathcal{L}_{\rm M}^{sp} + \mathcal{L}_{\rm G}, \\ &\mathcal{L}_{\rm M}^{sp} = i\bar{\psi}\gamma^{\mu}\left\{\left(\partial_{\mu} + igA_{\mu} + igB_{\mu}\right)\psi + gC_{\mu}^{+}c_{\psi}\right\} - m\bar{\psi}\psi \\ &\qquad + i\bar{c}_{\psi}\gamma^{\mu}\left\{\left(\partial_{\mu} + igA_{\mu} - igB_{\mu}\right)c_{\psi} - gC_{\mu}^{-}\psi\right\} - m\bar{c}_{\psi}c_{\psi}, \end{aligned} \tag{25}$$

where $\mathcal{L}_{\rm G}$ is given by (3), $\bar{\psi} \equiv \psi^{\dagger}\gamma^{0}$, $\bar{c}_{\psi} \equiv c_{\psi}^{\dagger}\gamma^{0}$ and γ^{μ} are the γ matrices satisfying $\left\{\gamma^{\mu},\gamma^{\nu}\right\} = 2\eta^{\mu\nu}$. The $\mathcal{L}_{\rm M}^{sp}$ is rewritten as

$$\mathcal{L}_{\rm M}^{sp} = i\bar{\Psi}\Gamma^{\mu}D_{\mu}\Psi - m\bar{\Psi}\Psi, \tag{26}$$

where Γ^{μ} and Ψ are the extension of γ-matrices and the doublet of fermionic transformation defined by

$$\Gamma^{\mu} \equiv \begin{pmatrix} \gamma^{\mu} & 0 \\ 0 & \gamma^{\mu} \end{pmatrix} \quad \text{and} \quad \Psi \equiv \begin{pmatrix} \psi \\ c_{\psi} \end{pmatrix}, \tag{27}$$

respectively.

The $\mathcal{L}^{sp}$ is invariant under the local $U(1)$ transformations,

$$\begin{aligned} &\delta_{A}\psi = -i\epsilon\psi,\ \delta_{A}\psi^{\dagger} = i\epsilon\psi^{\dagger},\ \delta_{A}c_{\psi} = -i\epsilon c_{\psi},\ \delta_{A}c_{\psi}^{\dagger} = i\epsilon c_{\psi}^{\dagger}, \\ &\delta_{A}A_{\mu} = \frac{1}{g}\partial_{\mu}\epsilon,\ \delta_{A}B_{\mu} = 0,\ \delta_{A}C_{\mu}^{+} = 0,\ \delta_{A}C_{\mu}^{-} = 0, \end{aligned} \tag{28}$$

$$\delta_B\psi = -i\xi\psi,\ \delta_B\psi^\dagger = i\xi\psi^\dagger,\ \delta_B c_\psi = i\xi c_\psi,\ \delta_B c_\psi^\dagger = -i\xi c_\psi^\dagger,$$
$$\delta_B A_\mu = 0,\ \delta_B B_\mu = \frac{1}{g}\partial_\mu\xi,\ \delta_B C_\mu^+ = -2i\xi C_\mu^+,\ \delta_B C_\mu^- = 2i\xi C_\mu^- \tag{29}$$

and the local fermionic transformations,

$$\delta_F\psi = -\zeta c_\psi,\ \delta_F\psi^\dagger = 0,\ \delta_F c_\psi = 0,\ \delta_F c_\psi^\dagger = -\zeta\psi^\dagger,$$
$$\delta_F A_\mu = -i\zeta C_\mu^-,\ \delta_F B_\mu = 0,\ \delta_F C_\mu^+ = 2i\zeta B_\mu + \frac{1}{g}\partial_\mu\zeta,\ \delta_F C_\mu^- = 0, \tag{30}$$

$$\delta_F^\dagger\psi = 0,\ \delta_F^\dagger\psi^\dagger = -\zeta^\dagger c_\psi^\dagger,\ \delta_F^\dagger c_\psi = \zeta^\dagger\psi,\ \delta_F^\dagger c_\psi^\dagger = 0,$$
$$\delta_F^\dagger A_\mu = -i\zeta^\dagger C_\mu^+,\ \delta_F^\dagger B_\mu = 0,\ \delta_F^\dagger C_\mu^+ = 0,\ \delta_F^\dagger C_\mu^- = -2i\zeta^\dagger B_\mu + \frac{1}{g}\partial_\mu\zeta^\dagger, \tag{31}$$

where ϵ and ξ are infinitesimal real functions of x, and ζ and $\zeta^\dagger$ are Grassmann-valued functions of x.

Using the global fermionic transformations,

$$\tilde{\boldsymbol{\delta}}_F\psi = -c_\psi,\ \tilde{\boldsymbol{\delta}}_F\psi^\dagger = 0,\ \tilde{\boldsymbol{\delta}}_F c_\psi = 0,\ \tilde{\boldsymbol{\delta}}_F c_\psi^\dagger = -\psi^\dagger,$$
$$\tilde{\boldsymbol{\delta}}_F A_\mu = -iC_\mu^-,\ \tilde{\boldsymbol{\delta}}_F B_\mu = 0,\ \tilde{\boldsymbol{\delta}}_F C_\mu^+ = 2iB_\mu,\ \tilde{\boldsymbol{\delta}}_F C_\mu^- = 0, \tag{32}$$

$$\tilde{\boldsymbol{\delta}}_F^\dagger\psi = 0,\ \tilde{\boldsymbol{\delta}}_F^\dagger\psi^\dagger = -c_\psi^\dagger,\ \tilde{\boldsymbol{\delta}}_F^\dagger c_\psi = \psi,\ \tilde{\boldsymbol{\delta}}_F^\dagger c_\psi^\dagger = 0,$$
$$\tilde{\boldsymbol{\delta}}_F^\dagger A_\mu = -iC_\mu^+,\ \tilde{\boldsymbol{\delta}}_F^\dagger B_\mu = 0,\ \tilde{\boldsymbol{\delta}}_F^\dagger C_\mu^+ = 0,\ \tilde{\boldsymbol{\delta}}_F^\dagger C_\mu^- = -2iB_\mu, \tag{33}$$

$\mathcal{L}^{sp}$ is rewritten as

$$\mathcal{L}^{sp} = \tilde{\boldsymbol{\delta}}_F\tilde{\boldsymbol{\delta}}_F^\dagger\mathcal{L}_{\psi,A} = -\tilde{\boldsymbol{\delta}}_F^\dagger\tilde{\boldsymbol{\delta}}_F\mathcal{L}_{\psi,A}, \tag{34}$$

where $\mathcal{L}_{\psi,A}$ is given by

$$\mathcal{L}_{\psi,A} = i\bar{\psi}\gamma^\mu\left\{\left(\partial_\mu + igqA_\mu + igB_\mu\right)\psi + gC_\mu^+ c_\psi\right\} - m\bar{\psi}\psi - \frac{1}{4}A_{\mu\nu}A^{\mu\nu}. \tag{35}$$

3. Quantization

We carry out the quantization of the system with scalar and gauge fields described by $\mathcal{L} = \mathcal{L}_M + \mathcal{L}_G$.

3.1. Canonical Quantization

Based on the formulation with the property that *the hermitian conjugate of canonical momentum for a variable is just the canonical momentum for the hermitian conjugate of the variable* [16], the conjugate momenta are given by

$$\pi \equiv \left(\frac{\partial\mathcal{L}}{\partial\dot{\varphi}}\right)_R = \left(\partial_0 - igA_0 - igB_0\right)\varphi^\dagger - gC_0^- c_\varphi^\dagger, \tag{36}$$

$$\pi^\dagger \equiv \left(\frac{\partial\mathcal{L}}{\partial\dot{\varphi}^\dagger}\right)_L = \left(\partial_0 + igA_0 + igB_0\right)\varphi + gC_0^+ c_\varphi, \tag{37}$$

$$\pi_{c_\varphi} \equiv \left(\frac{\partial\mathcal{L}}{\partial\dot{c}_\varphi}\right)_R = \left(\partial_0 - igA_0 + igB_0\right)c_\varphi^\dagger - gC_0^+\varphi^\dagger, \tag{38}$$

$$\pi_{c_\varphi}^\dagger \equiv \left(\frac{\partial\mathcal{L}}{\partial\dot{c}_\varphi^\dagger}\right)_L = \left(\partial_0 + igA_0 - igB_0\right)c_\varphi - gC_0^-\varphi, \tag{39}$$

$$\Pi_A^{\mu} \equiv \left(\frac{\partial \mathcal{L}}{\partial \dot{A}_{\mu}}\right)_{\mathrm{L}} = 2B^{\mu 0},\ \Pi_B^{\mu} \equiv \left(\frac{\partial \mathcal{L}}{\partial \dot{B}_{\mu}}\right)_{\mathrm{R}} = 2A^{\mu 0}, \tag{40}$$

$$\Pi_C^{+\mu} \equiv \left(\frac{\partial \mathcal{L}}{\partial \dot{C}_{\mu}^{+}}\right)_{\mathrm{L}} = C^{-\mu 0},\ \Pi_C^{-\mu} \equiv \left(\frac{\partial \mathcal{L}}{\partial \dot{C}_{\mu}^{-}}\right)_{\mathrm{R}} = C^{+\mu 0}, \tag{41}$$

where $\dot{\mathcal{O}} = \partial \mathcal{O}/\partial t$, and R and L stand for the right-differentiation and the left-differentiation, respectively. From (40) and (41), we obtain the primary constraints,

$$\Pi_A^0 = 0,\ \Pi_B^0 = 0,\ \Pi_C^{+0} = 0,\ \Pi_C^{-0} = 0. \tag{42}$$

Using the Legendre transformation, the Hamiltonian density is obtained as

$$\begin{aligned}
\mathcal{H} &= \pi\dot{\varphi} + \dot{\varphi}^{\dagger}\pi^{\dagger} + \pi_{c_\varphi}\dot{c}_{\varphi} + \dot{c}_{\varphi}^{\dagger}\pi_{c_\varphi}^{\dagger} + \dot{A}_{\mu}\Pi_A^{\mu} + \Pi_B^{\mu}\dot{B}_{\mu} + \dot{C}_{\mu}^{+}\Pi_C^{+\mu} + \Pi_C^{-\mu}\dot{C}_{\mu}^{-} - \mathcal{L} \\
&\quad + \lambda_A\Pi_A^0 + \Pi_B^0\lambda_B + \lambda_C^{+}\Pi_C^{+0} + \Pi_C^{-0}\lambda_C^{-} \\
&= \pi\pi^{\dagger} + \pi_{c_\varphi}\pi_{c_\varphi}^{\dagger} + (D_i\Phi)^{\dagger}(D^i\Phi) + m^2\Phi^{\dagger}\Phi - igA_0\left(\pi\varphi - \varphi^{\dagger}\pi^{\dagger} + \pi_{c_\varphi}c_\varphi - c_\varphi^{\dagger}\pi_{c_\varphi}^{\dagger}\right) \\
&\quad - igB_0\left(\pi\varphi - \varphi^{\dagger}\pi^{\dagger} - \pi_{c_\varphi}c_\varphi + c_\varphi^{\dagger}\pi_{c_\varphi}^{\dagger} + 2C_i^{+}\Pi_C^{+i} - 2\Pi_C^{-i}C_i^{-}\right) \\
&\quad - gC_0^{+}\left(\pi c_\varphi - \varphi^{\dagger}\pi_{c_\varphi}^{\dagger} + iC_i^{-}\Pi_A^{i} - 2iB_i\Pi_C^{+i}\right) - g\left(c_\varphi^{\dagger}\pi^{\dagger} - \pi_{c_\varphi}\varphi - iC_i^{+}\Pi_A^{i} + 2iB_i\Pi_C^{-i}\right)C_0^{-} \\
&\quad + \Pi_{Ai}\Pi_B^{i} + A_{ij}B^{ij} + \partial_i A_0\,\Pi_A^{i} + \Pi_B^{i}\partial_i B_0 + \Pi_{Ci}^{+}\Pi_C^{-i} + \frac{1}{2}C_{ij}C^{ij} + \partial_i C_0^{+}\,\Pi_C^{+i} + \Pi_C^{-i}\partial_i C_0^{-} \\
&\quad + \lambda_A\Pi_A^0 + \Pi_B^0\lambda_B + \lambda_C^{+}\Pi_C^{+0} + \Pi_C^{-0}\lambda_C^{-},
\end{aligned} \tag{43}$$

where Roman indices i and j denote the spatial components and run from 1 to 3, λ_A, λ_B, λ_C^{+} and λ_C^{-} are Lagrange multipliers, and $\dot{A}_0 + \lambda_A$, $\dot{B}_0 + \lambda_B$, $\dot{C}_0^{+} + \lambda_C^{+}$ and $\dot{C}_0^{-} + \lambda_C^{-}$ are rewritten as λ_A, λ_B, λ_C^{+} and λ_C^{-} in the final expression.

Secondary constraints are obtained as follows,

$$\frac{\mathrm{d}\Pi_A^0}{\mathrm{d}t} = \left\{\Pi_A^0, H\right\}_{\mathrm{PB}} = ig\left(\pi\varphi - \varphi^{\dagger}\pi^{\dagger} + \pi_{c_\varphi}c_\varphi - c_\varphi^{\dagger}\pi_{c_\varphi}^{\dagger}\right) + \partial_i\Pi_A^{i} = 0. \tag{44}$$

$$\frac{\mathrm{d}\Pi_B^0}{\mathrm{d}t} = \left\{\Pi_B^0, H\right\}_{\mathrm{PB}} = ig\left(\pi\varphi - \varphi^{\dagger}\pi^{\dagger} - \pi_{c_\varphi}c_\varphi + c_\varphi^{\dagger}\pi_{c_\varphi}^{\dagger} + 2C_i^{+}\Pi_C^{+i} - 2\Pi_C^{-i}C_i^{-}\right) + \partial_i\Pi_B^{i} = 0, \tag{45}$$

$$\frac{\mathrm{d}\Pi_C^{+0}}{\mathrm{d}t} = \left\{\Pi_C^{+0}, H\right\}_{\mathrm{PB}} = g\left(\pi c_\varphi - \varphi^{\dagger}\pi_{c_\varphi}^{\dagger} + iC_i^{-}\Pi_A^{i} - 2iB_i\Pi_C^{+i}\right) + \partial_i\Pi_C^{+i} = 0, \tag{46}$$

$$\frac{\mathrm{d}\Pi_C^{-0}}{\mathrm{d}t} = \left\{\Pi_C^{-0}, H\right\}_{\mathrm{PB}} = g\left(c_\varphi^{\dagger}\pi^{\dagger} - \pi_{c_\varphi}\varphi - iC_i^{+}\Pi_A^{i} + 2iB_i\Pi_C^{-i}\right) + \partial_i\Pi_C^{-i} = 0, \tag{47}$$

where H is the Hamiltonian $H = \int \mathcal{H}\mathrm{d}^3x$ and $\{A, B\}_{\mathrm{PB}}$ is the Poisson bracket. The Poisson bracket for the system with canonical variables (Q_k, P_k) and $(Q_k^{\dagger}, P_k^{\dagger})$ is defined by [16]

$$\{f, g\}_{\mathrm{PB}} \equiv \sum_k \left[\left(\frac{\partial f}{\partial Q_k}\right)_{\mathrm{R}}\left(\frac{\partial g}{\partial P_k}\right)_{\mathrm{L}} - (-)^{|Q_k|}\left(\frac{\partial f}{\partial P_k}\right)_{\mathrm{R}}\left(\frac{\partial g}{\partial Q_k}\right)_{\mathrm{L}} + (-)^{|Q_k|}\left(\frac{\partial f}{\partial Q_k^{\dagger}}\right)_{\mathrm{R}}\left(\frac{\partial g}{\partial P_k^{\dagger}}\right)_{\mathrm{L}} - \left(\frac{\partial f}{\partial P_k^{\dagger}}\right)_{\mathrm{R}}\left(\frac{\partial g}{\partial Q_k^{\dagger}}\right)_{\mathrm{L}}\right], \tag{48}$$

where $|Q_k|$ is the number representing the Grassmann parity of Q_k, *i.e.*, $|Q_k| = 1$ for the Grassmann odd Q_k and $|Q_k| = 0$ for the Grassmann even Q_k. There appear no other constraints, and all constraints are first class ones and generate local transformations.

We take the gauge fixing conditions,

$$A^0 = 0,\ B^0 = 0,\ C^{+0} = 0,\ C^{-0} = 0,\ \partial_i A^i = 0,\ \partial_i B^i = 0,\ \partial_i C^{+i} = 0,\ \partial_i C^{-i} = 0. \tag{49}$$

The system is quantized by regarding variables as operators and imposing the following relations on the canonical pairs,

$$\left[\varphi(\boldsymbol{x},t),\pi(\boldsymbol{y},t)\right] = i\delta^3(\boldsymbol{x}-\boldsymbol{y}),\ \left[\varphi^\dagger(\boldsymbol{x},t),\pi^\dagger(\boldsymbol{y},t)\right] = i\delta^3(\boldsymbol{x}-\boldsymbol{y}), \tag{50}$$

$$\left\{c_\varphi(\boldsymbol{x},t),\pi_{c_\varphi}(\boldsymbol{y},t)\right\} = i\delta^3(\boldsymbol{x}-\boldsymbol{y}),\ \left\{c_\varphi^\dagger(\boldsymbol{x},t),\pi_{c_\varphi}^\dagger(\boldsymbol{y},t)\right\} = -i\delta^3(\boldsymbol{x}-\boldsymbol{y}), \tag{51}$$

$$\left[A_i(\boldsymbol{x},t),\Pi_A^j(\boldsymbol{y},t)\right] = i\left(\delta_i^j - \frac{\partial_i\partial^j}{\Delta}\right)\delta^3(\boldsymbol{x}-\boldsymbol{y}), \tag{52}$$

$$\left[B_i(\boldsymbol{x},t),\Pi_B^j(\boldsymbol{y},t)\right] = i\left(\delta_i^j - \frac{\partial_i\partial^j}{\Delta}\right)\delta^3(\boldsymbol{x}-\boldsymbol{y}), \tag{53}$$

$$\left\{C_i^+(\boldsymbol{x},t),\Pi_C^{+j}(\boldsymbol{y},t)\right\} = -i\left(\delta_i^j - \frac{\partial_i\partial^j}{\Delta}\right)\delta^3(\boldsymbol{x}-\boldsymbol{y}), \tag{54}$$

$$\left\{C_i^-(\boldsymbol{x},t),\Pi_C^{-j}(\boldsymbol{y},t)\right\} = i\left(\delta_i^j - \frac{\partial_i\partial^j}{\Delta}\right)\delta^3(\boldsymbol{x}-\boldsymbol{y}), \tag{55}$$

where $\left[\mathcal{O}_1,\mathcal{O}_2\right] \equiv \mathcal{O}_1\mathcal{O}_2 - \mathcal{O}_2\mathcal{O}_1$, $\left\{\mathcal{O}_1,\mathcal{O}_2\right\} \equiv \mathcal{O}_1\mathcal{O}_2 + \mathcal{O}_2\mathcal{O}_1$, and only the non-vanishing ones are denoted. Here, we define the Dirac bracket using the first class constraints and the gauge fixing conditions, and replace the bracket with the commutator or the anti-commutator.

On the reduced phase space, the conserved $U(1)$ charges N_A and N_B and the conserved fermionic charges $Q_{\rm F}$ and $Q_{\rm F}^\dagger$ are constructed as

$$N_A = -i\int \mathrm{d}^3x\left(\pi\varphi - \varphi^\dagger\pi^\dagger + \pi_{c_\varphi}c_\varphi - c_\varphi^\dagger\pi_{c_\varphi}^\dagger\right), \tag{56}$$

$$N_B = -i\int \mathrm{d}^3x\left(\pi\varphi - \varphi^\dagger\pi^\dagger - \pi_{c_\varphi}c_\varphi + c_\varphi^\dagger\pi_{c_\varphi}^\dagger + 2C_i^+\Pi_C^{+i} - 2\Pi_C^{-i}C_i^-\right), \tag{57}$$

$$Q_{\rm F} = -\int \mathrm{d}^3x\left(\pi c_\varphi - \varphi^\dagger\pi_{c_\varphi}^\dagger + iC_i^-\Pi_A^i - 2iB_i\Pi_C^{+i}\right), \tag{58}$$

$$Q_{\rm F}^\dagger = -\int \mathrm{d}^3x\left(c_\varphi^\dagger\pi^\dagger - \pi_{c_\varphi}\varphi - iC_i^+\Pi_A^i + 2iB_i\Pi_C^{-i}\right). \tag{59}$$

The following algebraic relations hold:

$$\begin{aligned}&Q_{\rm F}^2 = 0,\ Q_{\rm F}^{\dagger 2} = 0,\ \left\{Q_{\rm F},Q_{\rm F}^\dagger\right\} = N_A,\ \left[N_A,Q_{\rm F}\right] = 0,\ \left[N_A,Q_{\rm F}^\dagger\right] = 0,\\ &\left[N_B,Q_{\rm F}\right] = -2Q_{\rm F},\ \left[N_B,Q_{\rm F}^\dagger\right] = 2Q_{\rm F}^\dagger,\ \left[N_A,N_B\right] = 0.\end{aligned} \tag{60}$$

The above charges are generators of global $U(1)$ and fermionic transformations such that

$$\tilde{\delta}_A\mathcal{O} = i\left[\epsilon_0 N_A,\mathcal{O}\right],\ \tilde{\delta}_B\mathcal{O} = i\left[\xi_0 N_B,\mathcal{O}\right],\ \tilde{\delta}_{\rm F}\mathcal{O} = i\left[\zeta_0 Q_{\rm F},\mathcal{O}\right],\ \tilde{\delta}_{\rm F}^\dagger\mathcal{O} = i\left[Q_{\rm F}^\dagger\zeta_0^\dagger,\mathcal{O}\right], \tag{61}$$

where ϵ_0 and ξ_0 are real parameters, and ζ_0 and $\zeta_0^\dagger$ are Grassmann parameters. Note that $\tilde{\boldsymbol{\delta}}_{\rm F}$ and $\tilde{\boldsymbol{\delta}}_{\rm F}^\dagger$ in (20) and (21) are related to $\tilde{\delta}_{\rm F}$ and $\tilde{\delta}_{\rm F}^\dagger$ as $\tilde{\delta}_{\rm F} = \zeta_0\tilde{\boldsymbol{\delta}}_{\rm F}$, $\tilde{\delta}_{\rm F}^\dagger = \zeta_0^\dagger\tilde{\boldsymbol{\delta}}_{\rm F}^\dagger$.

The system contains negative norm states originated from c_φ, $c_\varphi^\dagger$ and $C_i^\pm$. In the presence of negative norm states, the probability interpretation cannot be endured. To formulate our model in a consistent manner, we use a feature that *conserved charges can be, in general, set to be zero as subsidiary conditions*. We impose the following subsidiary conditions on states by hand,

$$N_A\left|\mathrm{phys}\right\rangle = 0,\ N_B\left|\mathrm{phys}\right\rangle = 0,\ Q_{\rm F}\left|\mathrm{phys}\right\rangle = 0,\ Q_{\rm F}^\dagger\left|\mathrm{phys}\right\rangle = 0. \tag{62}$$

In the Appendix, we point out that subsidiary conditions corresponding to (62) can be realized as remnants of local symmetries in a specific case.

3.2. Unitarity

Let us study the unitarity of physical $\boldsymbol{S}$ matrix in our system, using the Lagrangian density of free fields,

$$\mathcal{L}_0 = \partial_\mu \varphi^\dagger \partial^\mu \varphi - m^2 \varphi^\dagger \varphi + \partial_\mu c_\varphi^\dagger \partial^\mu c_\varphi - m^2 c_\varphi^\dagger c_\varphi - 2\partial_\mu A_i \partial^\mu B^i - \partial_\mu C_i^+ \partial^\mu C^{-i}, \tag{63}$$

where the gauge fixing conditions (49) are imposed on. The $\mathcal{L}_0$ describes the behavior of asymptotic fields of Heisenberg operators in $\mathcal{L} = \mathcal{L}_{\mathrm{M}} + \mathcal{L}_{\mathrm{G}}$.

From (63), free field equations for φ, $\varphi^\dagger$, c_φ, $c_\varphi^\dagger$, A_i, B_i and $C_i^\pm$ are derived. By solving the Klein-Gordon equations, we obtain the solutions

$$\varphi(x) = \int \frac{\mathrm{d}^3 k}{\sqrt{(2\pi)^3 2k_0}} \left(a(\boldsymbol{k}) \mathrm{e}^{-ikx} + b^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} \right), \tag{64}$$

$$\varphi^\dagger(x) = \int \frac{\mathrm{d}^3 k}{\sqrt{(2\pi)^3 2k_0}} \left(a^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} + b(\boldsymbol{k}) \mathrm{e}^{-ikx} \right), \tag{65}$$

$$\pi(x) = i\int \mathrm{d}^3 k \sqrt{\frac{k_0}{2(2\pi)^3}} \left(a^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} - b(\boldsymbol{k}) \mathrm{e}^{-ikx} \right), \tag{66}$$

$$\pi^\dagger(x) = -i\int \mathrm{d}^3 k \sqrt{\frac{k_0}{2(2\pi)^3}} \left(a(\boldsymbol{k}) \mathrm{e}^{-ikx} - b^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} \right), \tag{67}$$

$$c_\varphi(x) = \int \frac{\mathrm{d}^3 k}{\sqrt{(2\pi)^3 2k_0}} \left(c(\boldsymbol{k}) \mathrm{e}^{-ikx} + d^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} \right), \tag{68}$$

$$c_\varphi^\dagger(x) = \int \frac{\mathrm{d}^3 k}{\sqrt{(2\pi)^3 2k_0}} \left(c^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} + d(\boldsymbol{k}) \mathrm{e}^{-ikx} \right), \tag{69}$$

$$\pi_{c_\varphi}(x) = i\int \mathrm{d}^3 k \sqrt{\frac{k_0}{2(2\pi)^3}} \left(c^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} - d(\boldsymbol{k}) \mathrm{e}^{-ikx} \right), \tag{70}$$

$$\pi_{c_\varphi}^\dagger(x) = -i\int \mathrm{d}^3 k \sqrt{\frac{k_0}{2(2\pi)^3}} \left(c(\boldsymbol{k}) \mathrm{e}^{-ikx} - d^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} \right), \tag{71}$$

where $k_0 = \sqrt{\boldsymbol{k}^2 + m^2}$ and $kx = k^\mu x_\mu$.

In the same way, by solving the free Maxwell equations, we obtain the solutions,

$$A_i(x) = \int \frac{\mathrm{d}^3 k}{\sqrt{(2\pi)^3 2k_0}} \left(\epsilon_i^\alpha a_\alpha(\boldsymbol{k}) \mathrm{e}^{-ikx} + \epsilon_i^{*\alpha} a_\alpha^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} \right), \tag{72}$$

$$B_i(x) = \int \frac{\mathrm{d}^3 k}{\sqrt{(2\pi)^3 2k_0}} \left(\epsilon_i^\alpha b_\alpha(\boldsymbol{k}) \mathrm{e}^{-ikx} + \epsilon_i^{*\alpha} b_\alpha^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} \right), \tag{73}$$

$$C_i^+(x) = \int \frac{\mathrm{d}^3 k}{\sqrt{(2\pi)^3 2k_0}} \left(\epsilon_i^\alpha c_\alpha(\boldsymbol{k}) \mathrm{e}^{-ikx} + \epsilon_i^{*\alpha} d_\alpha^\dagger(\boldsymbol{k}) \mathrm{e}^{ikx} \right), \tag{74}$$

$$C_i^-(x)=\int\frac{\mathrm{d}^3k}{\sqrt{(2\pi)^3 2k_0}}\left(\epsilon_i^{*\alpha}c_\alpha^\dagger(\boldsymbol{k})\mathrm{e}^{ikx}+\epsilon_i^\alpha d_\alpha(\boldsymbol{k})\mathrm{e}^{-ikx}\right), \tag{75}$$

$$\Pi_A^i(x)=2i\int\mathrm{d}^3k\sqrt{\frac{k_0}{2(2\pi)^3}}\left(\epsilon_i^{*\alpha}b_\alpha^\dagger(\boldsymbol{k})\mathrm{e}^{ikx}-\epsilon_i^\alpha b_\alpha(\boldsymbol{k})\mathrm{e}^{-ikx}\right), \tag{76}$$

$$\Pi_B^i(x)=2i\int\mathrm{d}^3k\sqrt{\frac{k_0}{2(2\pi)^3}}\left(\epsilon_i^{*\alpha}a_\alpha^\dagger(\boldsymbol{k})\mathrm{e}^{ikx}-\epsilon_i^\alpha a_\alpha(\boldsymbol{k})\mathrm{e}^{-ikx}\right), \tag{77}$$

$$\Pi_C^{+i}(x)=i\int\mathrm{d}^3k\sqrt{\frac{k_0}{2(2\pi)^3}}\left(\epsilon_i^{*\alpha}c_\alpha^\dagger(\boldsymbol{k})\mathrm{e}^{ikx}-\epsilon_i^\alpha d_\alpha(\boldsymbol{k})\mathrm{e}^{-ikx}\right), \tag{78}$$

$$\Pi_C^{-i}(x)=-i\int\mathrm{d}^3k\sqrt{\frac{k_0}{2(2\pi)^3}}\left(\epsilon_i^\alpha c_\alpha(\boldsymbol{k})\mathrm{e}^{-ikx}-\epsilon_i^{*\alpha}d_\alpha^\dagger(\boldsymbol{k})\mathrm{e}^{ikx}\right), \tag{79}$$

where $k_0=|\boldsymbol{k}|$ and ϵ_i^α are polarization vectors satisfying the relations,

$$k_i\epsilon_i^\alpha=0,\ \epsilon_i^\alpha\epsilon^{*i\alpha'}=\delta^{\alpha\alpha'},\ \sum_\alpha\epsilon_i^\alpha\epsilon^{*j\alpha}=\delta_i^j-\frac{k_ik^j}{\boldsymbol{k}^2}. \tag{80}$$

The index α represents the helicity of gauge fields.

By imposing the same type of relations as (50) - (55), we have the relations,

$$\left[a(\boldsymbol{k}),a^\dagger(\boldsymbol{l})\right]=\delta^3(\boldsymbol{k}-\boldsymbol{l}),\ \left[b(\boldsymbol{k}),b^\dagger(\boldsymbol{l})\right]=\delta^3(\boldsymbol{k}-\boldsymbol{l}), \tag{81}$$

$$\left\{c(\boldsymbol{k}),c^\dagger(\boldsymbol{l})\right\}=\delta^3(\boldsymbol{k}-\boldsymbol{l}),\ \left\{d(\boldsymbol{k}),d^\dagger(\boldsymbol{l})\right\}=-\delta^3(\boldsymbol{k}-\boldsymbol{l}), \tag{82}$$

$$\left[a_\alpha(\boldsymbol{k}),b_{\alpha'}^\dagger(\boldsymbol{l})\right]=-\frac{1}{2}\delta_{\alpha\alpha'}\delta^3(\boldsymbol{k}-\boldsymbol{l}),\ \left[b_\alpha(\boldsymbol{k}),a_{\alpha'}^\dagger(\boldsymbol{l})\right]=-\frac{1}{2}\delta_{\alpha\alpha'}\delta^3(\boldsymbol{k}-\boldsymbol{l}), \tag{83}$$

$$\left\{c_\alpha(\boldsymbol{k}),c_{\alpha'}^\dagger(\boldsymbol{l})\right\}=\delta_{\alpha\alpha'}\delta^3(\boldsymbol{k}-\boldsymbol{l}),\ \left\{d_\alpha(\boldsymbol{k}),d_\alpha^\dagger(\boldsymbol{l})\right\}=-\delta_{\alpha\alpha'}\delta^3(\boldsymbol{k}-\boldsymbol{l}), \tag{84}$$

and others are zero.

The states in the Fock space are constructed by acting the creation operators $a^\dagger(\boldsymbol{k})$, $b^\dagger(\boldsymbol{k})$, $c^\dagger(\boldsymbol{k})$, $d^\dagger(\boldsymbol{k})$, $a_\alpha^\dagger(\boldsymbol{k})$, $b_\alpha^\dagger(\boldsymbol{k})$, $c_\alpha^\dagger(\boldsymbol{k})$ and $d_\alpha^\dagger(\boldsymbol{k})$ on the vacuum state $|0\rangle$, where $|0\rangle$ is defined by the conditions $a(\boldsymbol{k})|0\rangle=0$, $b(\boldsymbol{k})|0\rangle=0$, $c(\boldsymbol{k})|0\rangle=0$, $d(\boldsymbol{k})|0\rangle=0$, $a_\alpha(\boldsymbol{k})|0\rangle=0$, $b_\alpha(\boldsymbol{k})|0\rangle=0$, $c_\alpha(\boldsymbol{k})|0\rangle=0$ $d_\alpha(\boldsymbol{k})|0\rangle=0$.

We impose the following subsidiary conditions on states to select physical states,

$$N_A|\mathrm{phys}\rangle=0,\ N_B|\mathrm{phys}\rangle=0,\ Q_{\mathrm{F}}|\mathrm{phys}\rangle=0,\ Q_{\mathrm{F}}^\dagger|\mathrm{phys}\rangle=0. \tag{85}$$

Note that $Q_{\mathrm{F}}^\dagger|\mathrm{phys}\rangle=0$ means $\langle\mathrm{phys}|Q_{\mathrm{F}}=0$. We find that all states, except for the vacuum state, are unphysical because they do not satisfy (85). This feature is understood as a counterpart of the quartet mechanism [13] [14]. The projection operator $P^{(n)}$ on the states with n particles is given by

$$P^{(n)}=\frac{1}{n}\left\{a^\dagger P^{(n-1)}a+b^\dagger P^{(n-1)}b+c^\dagger P^{(n-1)}c-d^\dagger P^{(n-1)}d+\sum_\alpha\left(-2a_\alpha^\dagger P^{(n-1)}b_\alpha-2b_\alpha^\dagger P^{(n-1)}a_\alpha+c_\alpha^\dagger P^{(n-1)}c_\alpha-d_\alpha^\dagger P^{(n-1)}d_\alpha\right)\right\}, \tag{86}$$

where $n\geq1$ and we omit $\boldsymbol{k}$, for simplicity. Using the transformation properties,

$$\begin{aligned}
&\tilde{\delta}_{\mathrm{F}} a=-c,\ \tilde{\delta}_{\mathrm{F}} a^{\dagger}=0,\ \tilde{\delta}_{\mathrm{F}} b=0,\ \tilde{\delta}_{\mathrm{F}} b^{\dagger}=-d^{\dagger}, \\
&\tilde{\delta}_{\mathrm{F}} c=0,\ \tilde{\delta}_{\mathrm{F}} c^{\dagger}=a^{\dagger},\ \tilde{\delta}_{\mathrm{F}} d=b,\ \tilde{\delta}_{\mathrm{F}} d^{\dagger}=0, \\
&\tilde{\delta}_{\mathrm{F}} a_{\alpha}=-i d_{\alpha},\ \tilde{\delta}_{\mathrm{F}} a_{\alpha}^{\dagger}=-i c_{\alpha}^{\dagger},\ \tilde{\delta}_{\mathrm{F}} b_{\alpha}=0,\ \tilde{\delta}_{\mathrm{F}} b_{\alpha}^{\dagger}=0, \\
&\tilde{\delta}_{\mathrm{F}} c_{\alpha}=2 i b_{\alpha},\ \tilde{\delta}_{\mathrm{F}} c_{\alpha}^{\dagger}=0,\ \tilde{\delta}_{\mathrm{F}} d_{\alpha}=0,\ \tilde{\delta}_{\mathrm{F}} d_{\alpha}^{\dagger}=2 i b_{\alpha}^{\dagger},
\end{aligned} \tag{87}$$

$P^{(n)}$ is written in a simple form as

$$P^{(n)}=i\left\{Q_{\mathrm{F}}, R^{(n)}\right\}, \tag{88}$$

where $R^{(n)}$ is given by

$$R^{(n)}=\frac{1}{n}\left\{c^{\dagger} P^{(n-1)} a+b^{\dagger} P^{(n-1)} d+i \sum_{\alpha}\left(a_{\alpha}^{\dagger} P^{(n-1)} c_{\alpha}+d_{\alpha}^{\dagger} P^{(n-1)} a_{\alpha}\right)\right\}. \tag{89}$$

From (88), we find that any state with $n \geq 1$ is unphysical from $\langle\text{phys}| P^{(n)}|\text{phys}\rangle=0$. Then, we understand that every field becomes unphysical, and only $|0\rangle$ remains as the physical state. This is also regarded as a field theoretical version of the Parisi-Sourlas mechanism [20].

The system is also formulated using hermitian fermionic charges defined by $Q_1 \equiv Q_{\mathrm{F}}+Q_{\mathrm{F}}^{\dagger}$ and $Q_2 \equiv i\left(Q_{\mathrm{F}}-Q_{\mathrm{F}}^{\dagger}\right)$. They satisfy the relations $Q_1 Q_2+Q_2 Q_1=0$, $Q_1^2=N_A$ and $Q_2^2=N_A$. Though Q_1, Q_2 and N_A form elements of the $N=2$ (quantum mechanical) SUSY algebra [21], our system does not possess the space-time SUSY because N_A is not our Hamiltonian but the $U(1)$ charge N_A. Only the vacuum state is selected as the physical states by imposing the following subsidiary conditions on states, in place of (85),

$$N_A|\text{phys}\rangle=0,\ N_B|\text{phys}\rangle=0,\ Q_1|\text{phys}\rangle=0,\ Q_2|\text{phys}\rangle=0. \tag{90}$$

It is also understood that our fermionic symmetries are different from the space-time SUSY, from the fact that Q_1 and Q_2 are scalar charges. They are also different from the BRST symmetry, as seen from the algebraic relations among charges.

The system with spinor and gauge fields described by $\mathcal{L}^{sp}=\mathcal{L}_{\mathrm{M}}^{sp}+\mathcal{L}_{\mathrm{G}}$ is also quantized, in a similar way. We find that the theory becomes harmless but empty leaving the vacuum state alone as the physical state, after imposing subsidiary conditions corresponding to (62).

3.3. BRST Symmetry

Our system has local symmetries, and it is quantized by the Faddeev-Popov (FP) method. In order to add the gauge fixing conditions to the Lagrangian, several fields corresponding to FP ghost and anti-ghost fields and auxiliary fields called Nakanishi-Lautrup (NL) fields are introduced. Then, the system is described on the extended phase space and has a global symmetry called the BRST symmetry. We present the gauge-fixed Lagrangian density and study the BRST transformation properties.

According to the usual procedure, the Lagrangian density containing the gauge fixing terms and FP ghost terms is constructed as

$$\begin{aligned}
&\mathcal{L}_{\mathrm{T}}=\mathcal{L}_{\mathrm{M}}+\mathcal{L}_{\mathrm{G}}+\mathcal{L}_{\mathrm{gf}}+\mathcal{L}_{\mathrm{FP}}, \\
&\mathcal{L}_{\mathrm{gf}}=-\partial_{\mu} b_A A^{\mu}-\partial_{\mu} b_B B^{\mu}+C^{+\mu} \partial_{\mu} \phi_c+\partial_{\mu} \phi_c^{\dagger} C^{-\mu}+\frac{1}{2} \alpha\left(b_A^2+b_B^2+2 \phi_c^{\dagger} \phi_c\right),
\end{aligned} \tag{91}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{FP}}= & -i \partial_{\mu} \bar{c}_A\left(\partial^{\mu} c_A-i g \phi C^{-\mu}+i g \phi^{\dagger} C^{+\mu}\right)-i \partial_{\mu} \bar{c}_B \partial^{\mu} c_B \\
& +i\left(\partial^{\mu} \phi-2 i g c_B C^{+\mu}+2 i g \phi B^{\mu}\right) \partial_{\mu} \bar{\phi}-i \partial_{\mu} \bar{\phi}^{\dagger}\left(\partial^{\mu} \phi^{\dagger}-2 i g c_B C^{-\mu}-2 i g \phi^{\dagger} B^{\mu}\right),
\end{aligned} \tag{92}$$

where c_A, c_B, ϕ and $\phi^{\dagger}$ are FP ghosts, $\bar{c}_A$, $\bar{c}_B$, $\bar{\phi}$ and $\bar{\phi}^{\dagger}$ are FP anti-ghosts, b_A, b_B, ϕ_c and $\phi_c^{\dagger}$ are NL fields, and α is a gauge parameter. These fields are scalar fields. c_A, c_B, $\bar{c}_A$ and $\bar{c}_B$ are fermionic, and b_A and b_B are bosonic. In contrast, ϕ, $\phi^{\dagger}$, $\bar{\phi}$ and $\bar{\phi}^{\dagger}$ are bosonic, and ϕ_c and $\phi_c^{\dagger}$ are fermionic because the relevant symmetries are fermionic.

The $\mathcal{L}_{\mathrm{T}}$ is invariant under the BRST transformation,

$$\begin{aligned}
&\boldsymbol{\delta}_{\mathrm{BRST}}\varphi = -igc_A\varphi - igc_B\varphi - g\phi c_\varphi,\ \boldsymbol{\delta}_{\mathrm{BRST}}\varphi^\dagger = igc_A\varphi^\dagger + igc_B\varphi^\dagger - g\varphi^\dagger c_\varphi^\dagger,\\
&\boldsymbol{\delta}_{\mathrm{BRST}}c_\varphi = -igc_A c_\varphi + igc_B c_\varphi - g\phi^\dagger\varphi,\ \boldsymbol{\delta}_{\mathrm{BRST}}c_\varphi^\dagger = igc_A c_\varphi^\dagger - igc_B c_\varphi^\dagger + g\phi\varphi^\dagger,\\
&\boldsymbol{\delta}_{\mathrm{BRST}}c_A = -ig\phi^\dagger\phi,\ \boldsymbol{\delta}_{\mathrm{BRST}}c_B = 0,\ \boldsymbol{\delta}_{\mathrm{BRST}}\phi = -2igc_B\phi,\ \boldsymbol{\delta}_{\mathrm{BRST}}\phi^\dagger = 2igc_B\phi^\dagger,\\
&\boldsymbol{\delta}_{\mathrm{BRST}}A_\mu = \partial_\mu c_A - ig\phi C_\mu^- + ig\phi^\dagger C_\mu^+,\ \boldsymbol{\delta}_{\mathrm{BRST}}B_\mu = \partial_\mu c_B,\\
&\boldsymbol{\delta}_{\mathrm{BRST}}C_\mu^+ = -2igc_B C_\mu^+ + 2ig\phi B_\mu + \partial_\mu\phi,\ \boldsymbol{\delta}_{\mathrm{B}}C_\mu^- = 2igc_B C_\mu^- + 2ig\phi^\dagger B_\mu - \partial_\mu\phi^\dagger,\\
&\boldsymbol{\delta}_{\mathrm{BRST}}\overline{c}_A = ib_A,\ \boldsymbol{\delta}_{\mathrm{BRST}}\overline{c}_B = ib_B,\ \boldsymbol{\delta}_{\mathrm{BRST}}\overline{\phi} = i\phi_c,\ \boldsymbol{\delta}_{\mathrm{BRST}}\overline{\phi}^\dagger = -i\phi_c^\dagger,\\
&\boldsymbol{\delta}_{\mathrm{BRST}}b_A = 0,\ \boldsymbol{\delta}_{\mathrm{BRST}}b_B = 0,\ \boldsymbol{\delta}_{\mathrm{BRST}}\phi_c = 0,\ \boldsymbol{\delta}_{\mathrm{BRST}}\phi_c^\dagger = 0,
\end{aligned} \tag{93}$$

where the transformations for φ, $\varphi^\dagger$, c_φ, $c_\varphi^\dagger$, A_μ, B_μ, C_μ^+ and C_μ^- are obtained by regarding the sum of transformations $\delta_A + \delta_B + \delta_{\mathrm{F}} + \delta_{\mathrm{F}}^\dagger$ as $\boldsymbol{\delta}_{\mathrm{BRST}}$ and replacing ε, ξ, ζ and $\zeta^\dagger$ with gc_A, gc_B, $g\phi$ and $-g\phi^\dagger$, and those for c_A, c_B, ϕ and $\phi^\dagger$ are determined by the requirement that $\boldsymbol{\delta}_{\mathrm{BRST}}$ has a nilpotency property, *i.e.*, $\boldsymbol{\delta}_{\mathrm{BRST}}^2\mathcal{O} = 0$.

The sum of the gauge fixing terms and FP ghost terms is simply written as

$$\begin{aligned}
&\mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{FP}}\\
&= i\boldsymbol{\delta}_{BRST}\left\{\partial_\mu\overline{c}_A A^\mu + \partial_\mu\overline{c}_B B^\mu + C^{+\mu}\partial_\mu\overline{\phi} + \partial_\mu\overline{\phi}^\dagger C^{-\mu} - \frac{1}{2}\alpha\left(\overline{c}_A b_A + \overline{c}_B b_B - \varphi_c^\dagger\overline{\phi} - \overline{\phi}^\dagger\phi_c\right)\right\}.
\end{aligned} \tag{94}$$

According to the Noether procedure, the BRST current J_{BRST}^μ and the BRST charge Q_{BRST} are obtained as

$$\begin{aligned}
J_{\mathrm{BRST}}^\mu =\ & b_A\left(\partial^\mu c_A - ig\varphi C^{-\mu} + ig\varphi^\dagger C^{+\mu}\right) - c_A\partial^\mu b_A + b_B\partial^\mu c_B - c_B\partial^\mu b_B\\
& - \phi_c\left(\partial^\mu\phi - 2igc_B C^{+\mu} + 2ig\phi B^\mu\right) + \phi\partial^\mu\phi_c\\
& - \phi_c^\dagger\left(\partial^\mu\phi^\dagger - 2igc_B C^{-\mu} - 2ig\phi^\dagger B^\mu\right) + \phi^\dagger\partial^\mu\phi_c^\dagger\\
& - 2gc_B\phi\partial^\mu\overline{\phi} - 2gc_B\phi^\dagger\partial^\mu\overline{\phi}^\dagger - g\varphi^\dagger\phi\partial^\mu\overline{c}_A\\
& - 2\partial_\nu\left(c_A B^{\mu\nu}\right) - 2\partial_\nu\left(c_B A^{\mu\nu}\right) - \partial_\nu\left(\phi C^{-\mu\nu}\right) - \partial_\nu\left(\phi^\dagger C^{+\mu\nu}\right)
\end{aligned} \tag{95}$$

and

$$\begin{aligned}
Q_{\mathrm{BRST}} \equiv \int\mathrm{d}^3x J_{\mathrm{BRST}}^0 = \int\mathrm{d}^3x\Big\{& b_A\left(\partial^0 c_A - ig\phi C^{-0} + ig\phi^\dagger C^{+0}\right) - c_A\partial^0 b_A\\
& + b_B\partial^0 c_B - \partial^0 b_B\, c_B - \phi_c\left(\partial^0\phi - 2igc_B C^{+0} + 2ig\phi B^0\right) + \phi\partial^0\phi_c\\
& - \phi_c^\dagger\left(\partial^0\phi^\dagger - 2igc_B C^{-0} - 2ig\phi^\dagger B^0\right) + \phi^\dagger\partial^0\phi_c^\dagger\\
& - 2gc_B\varphi\partial^0\overline{\phi} - 2gc_B\varphi^\dagger\partial^0\overline{\phi}^\dagger - g\phi^\dagger\phi\partial^0\overline{c}_A\Big\},
\end{aligned} \tag{96}$$

respectively. Here we use the field equations. The BRST charge is a conserved charge ($\mathrm{d}Q_{\mathrm{BRST}}/\mathrm{d}t = 0$), and it has the nilpotency property such as $Q_{\mathrm{BRST}}^2 = 0$.

By imposing the following subsidiary condition on states,

$$Q_{\mathrm{BRST}}\left|\mathrm{phys}\right\rangle = 0, \tag{97}$$

it is shown that any negative norm states originated from time and longitudinal components of gauge fields as well as FP ghost and anti-ghost fields and NL fields do not appear on the physical subspace, through the quartet mechanism. There still exist negative norm states come from c_φ, $c_\varphi^\dagger$ and $C_\mu^\pm$, and it is necessary to impose additional conditions corresponding to (62) on states in order to project out such harmful states.

4. Conclusions and Discussions

We have studied the quantization of systems with local particle-ghost symmetries. The systems contain ordinary particles including gauge bosons and their counterparts obeying different statistics. There exist negative norm states come from fermionic scalar fields (or bosonic spinor fields) and transverse components of fermionic gauge fields, even after reducing the phase space due to the first class constraints and the gauge fixing conditions or imposing the subsidiary condition concerning the BRST charge on states. By imposing additional subsidiary conditions on states, such negative norm states are projected out on the physical subspace and the unitarity of systems hold. The additional conditions can be originated from constraints in case that gauge fields have no dynamical degrees of freedom.

The systems considered are unrealistic if this goes on, because they are empty leaving the vacuum state alone as the physical state. Then, one might think that it is better not to get deeply involved them. Although they are still up in the air at present, there is a possibility that a formalism or concept itself is basically correct and is useful to explain phenomena of elementary particles at a more fundamental level. It is necessary to fully understand features of our particle-ghost symmetries, in order to appropriately apply them on a more microscopic system.

We make conjectures on some applications. We suppose that particle-ghost symmetries exist and the system contains only a few states including the vacuum as physical states at an ultimate level. Most physical particles might be released from unphysical doublets that consist of particles and their ghost partners. A release mechanism has been proposed based on the dimensional reduction by orbifolding [17].

After the appearance of physical fields, Q_{F}-singlets and Q_{F}-doublets coexist with exact fermionic symmetries. The Lagrangian density is, in general, written in the form as $\mathcal{L}_{\mathrm{Total}} = \mathcal{L}_{\mathrm{S}} + \mathcal{L}_{\mathrm{D}} + \mathcal{L}_{\mathrm{mix}} = \mathcal{L}_{\mathrm{S}} + \tilde{\boldsymbol{\delta}}_{\mathrm{F}}\tilde{\boldsymbol{\delta}}_{\mathrm{F}}^{\dagger}\left(\Delta\mathcal{L}\right)$. Here, $\mathcal{L}_{\mathrm{S}}$, $\mathcal{L}_{\mathrm{D}}$ and $\mathcal{L}_{\mathrm{mix}}$ stand for the Lagrangian density for Q_{F}-singlets, Q_{F}-doublets and interactions between Q_{F}-singlets and Q_{F}-doublets. Under the subsidiary conditions $N_A\left|\mathrm{phys}\right\rangle = 0$, $N_B\left|\mathrm{phys}\right\rangle = 0$, $Q_{\mathrm{F}}\left|\mathrm{phys}\right\rangle = 0$ and $Q_{\mathrm{F}}^{\dagger}\left|\mathrm{phys}\right\rangle = 0$ on states, all Q_{F}-doublets become unphysical and would not give any physical effects on Q_{F} singlets. Because Q_{F} singlets would not receive any radiative corrections from Q_{F} doublets, the theory is free from the gauge hierarchy problem if all heavy fields form Q_{F} doublets [15].

The system seems to be same as that described by $\mathcal{L}_{\mathrm{S}}$ alone, and to be impossible to show the existence of Q_{F}-doublets. However, in a very special case, an indirect proof would be possible through fingerprints left by symmetries in a fundamental theory. The fingerprints are specific relations among parameters such as a unification of coupling constants, reflecting on underlying symmetries [15] [22].

In most cases, our ghost fields require non-local interactions [15] and the change of degrees of freedom can occur in systems with infinite numbers of fields [17]. Then, they might suggest that fundamental objects are not point particles but extended objects such as strings and membranes. Hence, it would be interesting to explore systems with particle-ghost symmetries and their applications in the framework of string theories[1].

Acknowledgements

The author thanks the Editor and the referee for their comments. This work was supported in part by scientific grants from the Ministry of Education, Culture, Sports, Science and Technology under Grant No. 22540272.

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Appendix

A1. System with Auxiliary Gauge Fields

Let us study the system without $\mathcal{L}_{\mathrm{G}}$ described by

$$\begin{aligned}\mathcal{L}_{\mathrm{M}} &= \left\{\left(\partial_\mu - igA_\mu - igB_\mu\right)\varphi^\dagger - gC_\mu^- c_\varphi^\dagger\right\}\left\{\left(\partial^\mu + igA^\mu + igB^\mu\right)\varphi + gC^{+\mu}c_\varphi\right\} \\ &+ \left\{\left(\partial_\mu - igA_\mu + igB_\mu\right)c_\varphi^\dagger - gC_\mu^+\varphi^\dagger\right\}\left\{\left(\partial^\mu + igA^\mu - igB^\mu\right)c_\varphi - gC^{-\mu}\varphi\right\} \\ &- m^2\varphi\varphi - m^2 c_\varphi^\dagger c_\varphi.\end{aligned} \tag{98}$$

In this case, gauge fields do not have any dynamical degrees of freedom, and are regarded as auxiliary fields. The conjugate momenta of φ, $\varphi^\dagger$, c_φ and $c_\varphi^\dagger$ are same as those obtained in (36) - (39). The conjugate momenta of A_μ, B_μ, C_μ^+ and C_μ^- become constraints,

$$\Pi_A^\mu = 0,\ \Pi_B^\mu = 0,\ \Pi_C^{+\mu} = 0,\ \Pi_C^{-\mu} = 0. \tag{99}$$

Using the Legendre transformation, the Hamiltonian density is obtained as

$$\begin{aligned}\mathcal{H}_{\mathrm{M}} &= \pi\dot{\varphi} + \dot{\varphi}^\dagger\pi^\dagger + \pi_{c_\varphi}\dot{c}_\varphi + \dot{c}_\varphi^\dagger\pi_{c_\varphi}^\dagger + \dot{A}_\mu\Pi_A^\mu + \Pi_B^\mu\dot{B}_\mu + \dot{C}_\mu^+\Pi_C^{+\mu} + \Pi_C^{-\mu}\dot{C}_\mu^- - \mathcal{L} \\ &+ \lambda_{A\mu}\Pi_A^\mu + \Pi_B^\mu\lambda_{B\mu} + \lambda_{C\mu}^+\Pi_C^{+\mu} + \Pi_C^{-\mu}\lambda_{C\mu}^- \\ &= \pi\pi^\dagger + \pi_{c_\varphi}\pi_{c_\varphi}^\dagger + \left(D_i\Phi\right)^\dagger\left(D^i\Phi\right) + m^2\Phi^\dagger\Phi \\ &- igA_0\left(\pi\varphi - \varphi^\dagger\pi^\dagger + \pi_{c_\varphi}c_\varphi - c_\varphi^\dagger\pi_{c_\varphi}^\dagger\right) - igB_0\left(\pi\varphi - \varphi^\dagger\pi^\dagger - \pi_{c_\varphi}c_\varphi + c_\varphi^\dagger\pi_{c_\varphi}^\dagger\right) \\ &- gC_0^+\left(\pi c_\varphi - \varphi^\dagger\pi_{c_\varphi}^\dagger\right) - g\left(c_\varphi^\dagger\pi^\dagger - \pi_{c_\varphi}\varphi\right)C_0^- \\ &+ \lambda_{A\mu}\Pi_A^\mu + \Pi_B^\mu\lambda_{B\mu} + \lambda_{C\mu}^+\Pi_C^{+\mu} + \Pi_C^{-\mu}\lambda_{C\mu}^-,\end{aligned} \tag{100}$$

where $\lambda_{A\mu}$, $\lambda_{B\mu}$, $\lambda_{C\mu}^+$ and $\lambda_{C\mu}^-$ are Lagrange multipliers.

Secondary constraints are obtained as

$$\frac{\mathrm{d}\Pi_A^\mu}{\mathrm{d}t} = \left\{\Pi_A^\mu, H_{\mathrm{M}}\right\}_{\mathrm{PB}} = gj_A^\mu = 0, \tag{101}$$

$$\frac{\mathrm{d}\Pi_B^\mu}{\mathrm{d}t} = \left\{\Pi_B^\mu, H_{\mathrm{M}}\right\}_{\mathrm{PB}} = gj_B^\mu = 0, \tag{102}$$

$$\frac{\mathrm{d}\Pi_C^{+\mu}}{\mathrm{d}t} = \left\{\Pi_C^{+\mu}, H_{\mathrm{M}}\right\}_{\mathrm{PB}} = gj_C^{+\mu} = 0, \tag{103}$$

$$\frac{\mathrm{d}\Pi_C^{-\mu}}{\mathrm{d}t} = \left\{\Pi_C^{-\mu}, H_{\mathrm{M}}\right\}_{\mathrm{PB}} = gj_C^{-\mu} = 0, \tag{104}$$

where H_{M} is the Hamiltonian $H_{\mathrm{M}} = \int\mathcal{H}_{\mathrm{M}}\mathrm{d}^3x$, and j_A^μ, j_B^μ, $j_C^{+\mu}$ and $j_C^{-\mu}$ are the currents of $U(1)$ and fermionic symmetries given by

$$\begin{aligned}j_A^0 &= i\left(\pi\varphi - \varphi^\dagger\pi^\dagger + \pi_{c_\varphi}c_\varphi - c_\varphi^\dagger\pi_{c_\varphi}^\dagger\right), \\ j_A^i &= i\Big[\left\{\left(\partial^i - igA^i - igB^i\right)\varphi^\dagger - gC^{-i}c_\phi^\dagger\right\}\varphi - \varphi^\dagger\left\{\left(\partial^i + igA^i + igB^i\right)\varphi + gC^{+i}c_\varphi\right\} \\ &+ \left\{\left(\partial^i + igA^i - igB^i\right)c_\varphi - gC^{-i}\varphi\right\}c_\varphi - c_\varphi^\dagger\left\{\left(\partial^i + igA^i - igB^i\right)c_\varphi - gC^{-i}\varphi\right\}\Big],\end{aligned} \tag{105}$$

$$
\begin{aligned}
j_B^0 &= i\left(\pi\varphi - \varphi^\dagger\pi^\dagger - \pi_{c_\varphi}c_\varphi + c_\varphi^\dagger\pi_{c_\varphi}^\dagger\right),\\
j_B^i &= i\Big[\left\{\left(\partial^i - igA^i - igB^i\right)\varphi^\dagger - gC^{-i}c_\phi^\dagger\right\}\varphi - \varphi^\dagger\left\{\left(\partial^i + igA^i + igB^i\right)\varphi + gC^{+i}c_\varphi\right\}\\
&\quad -\left\{\left(\partial^i + igA^i - igB^i\right)c_\varphi - gC^{-i}\varphi\right\}c_\varphi + c_\varphi^\dagger\left\{\left(\partial^i + igA^i - igB^i\right)c_\varphi - gC^{-i}\varphi\right\}\Big],
\end{aligned}
\tag{106}
$$

$$
\begin{aligned}
j_C^{+0} &= \pi c_\varphi - \varphi^\dagger\pi_{c_\varphi}^\dagger,\\
j_C^{+i} &= \left\{\left(\partial^i - igA^i - igB^i\right)\varphi^\dagger - gC^{-i}c_\varphi^\dagger\right\}c_\varphi - \varphi^\dagger\left\{\left(\partial^i + igA^i - igB^i\right)c_\varphi - gC^{-i}\varphi\right\},
\end{aligned}
\tag{107}
$$

$$
\begin{aligned}
j_C^{-0} &= c_\varphi^\dagger\pi^\dagger - \pi_{c_\varphi}\varphi,\\
j_C^{-i} &= c_\varphi^\dagger\left\{\left(\partial^i + igA^i + igB^i\right)\varphi + gC^{+i}c_\varphi\right\} - \left\{\left(\partial^i - igA^i + igB^i\right)c_\varphi^\dagger - gC^{+i}\varphi^\dagger\right\}\varphi.
\end{aligned}
\tag{108}
$$

In the same way, tertiary constraints are obtained as

$$\frac{\mathrm{d}j_A^0}{\mathrm{d}t} = \left\{j_A^0, H_\mathrm{M}\right\}_\mathrm{PB} = -\partial_i j_A^i = 0, \tag{109}$$

$$\frac{\mathrm{d}j_B^0}{\mathrm{d}t} = \left\{j_B^0, H_\mathrm{M}\right\}_\mathrm{PB} = -\partial_i j_B^i = 0, \tag{110}$$

$$\frac{\mathrm{d}j_C^{+0}}{\mathrm{d}t} = \left\{j_C^{+0}, H_\mathrm{M}\right\}_\mathrm{PB} = -\partial_i j_C^{+i} = 0, \tag{111}$$

$$\frac{\mathrm{d}j_C^{-0}}{\mathrm{d}t} = \left\{j_C^{-0}, H_\mathrm{M}\right\}_\mathrm{PB} = -\partial_i j_C^{-i} = 0, \tag{112}$$

from the invariance under the time evolution of $j_A^0 = 0$, $j_B^0 = 0$, $j_C^{+0} = 0$ and $j_C^{-0} = 0$. On the other hand, the conditions $\mathrm{d}j_A^i/\mathrm{d}t = \left\{j_A^i, H_\mathrm{M}\right\}_\mathrm{PB} = 0$, $\mathrm{d}j_B^i/\mathrm{d}t = \left\{j_B^i, H_\mathrm{M}\right\}_\mathrm{PB} = 0$, $\mathrm{d}j_C^{+i}/\mathrm{d}t = \left\{j_C^{+i}, H_\mathrm{M}\right\}_\mathrm{PB} = 0$ and $\mathrm{d}j_C^{-i}/\mathrm{d}t = \left\{j_C^{-i}, H_\mathrm{M}\right\}_\mathrm{PB} = 0$ are not new constraints but the relations to determine λ_{Ai}, λ_{Bi}, λ_{Ci}^+ and λ_{Ci}^-. Furthermore, new constraints do not appear from the conditions $\mathrm{d}\left(\partial_i j_A^i\right)/\mathrm{d}t = 0$, $\mathrm{d}\left(\partial_i j_B^i\right)/\mathrm{d}t = 0$, $\mathrm{d}\left(\partial_i j_C^{+i}\right)/\mathrm{d}t = 0$ and $\mathrm{d}\left(\partial_i j_C^{-i}\right)/\mathrm{d}t = 0$.

The constraints are classified into the first class ones

$$\Pi_A^0 = 0,\ \Pi_B^0 = 0,\ \Pi_C^{+0} = 0,\ \Pi_C^{-0} = 0 \tag{113}$$

and the second class ones

$$
\begin{aligned}
&\Pi_A^i = 0,\ \Pi_B^i = 0,\ \Pi_C^{+i} = 0,\ \Pi_C^{-i} = 0,\\
&j_A^i = 0,\ j_B^i = 0,\ j_C^{+i} = 0,\ j_C^{-i} = 0,\\
&j_A^0 = 0,\ j_B^0 = 0,\ j_C^{+0} = 0,\ j_C^{-0} = 0,\\
&\partial_i j_A^i = 0,\ \partial_i j_B^i = 0,\ \partial_i j_C^{+i} = 0,\ \partial_i j_C^{-i} = 0.
\end{aligned}
\tag{114}
$$

The determinant of Poisson bracket between second class ones does not vanish on constraints.

Using j_A^0, j_B^0, j_C^{+0} and j_C^{-0}, the conserved $U(1)$ and fermionic charges are constructed as

$$N_A \equiv -i\int \mathrm{d}^3x j_A^0 = -i\int \mathrm{d}^3x\left(\pi\varphi - \varphi^\dagger\pi^\dagger + \pi_{c_\varphi}c_\varphi - c_\varphi^\dagger\pi_{c_\varphi}^\dagger\right), \tag{115}$$

$$N_B \equiv -i\int \mathrm{d}^3x j_B^0 = -i\int \mathrm{d}^3x\left(\pi\varphi - \varphi^\dagger\pi^\dagger - \pi_{c_\varphi}c_\varphi + c_\varphi^\dagger\pi_{c_\varphi}^\dagger\right), \tag{116}$$

$$Q_\mathrm{F} \equiv -\int \mathrm{d}^3x\, j_C^{+0} = -\int \mathrm{d}^3x\left(\pi c_\varphi - \varphi^\dagger\pi_{c_\varphi}^\dagger\right), \tag{117}$$

$$Q_{\mathrm{F}}^{\dagger} \equiv -\int \mathrm{d}^3 x j_C^{-0} = -\int \mathrm{d}^3 x \left(c_{\varphi}^{\dagger} \pi^{\dagger} - \pi_{c_{\varphi}} \varphi \right). \tag{118}$$

The same algebraic relations hold as those in (60).

The above charges are conserved and generators of global $U(1)$ and fermionic transformations for scalar fields. They satisfy the relations,

$$\left\{ N_A, \phi^{\hat{a}} \right\}_{\mathrm{PB}} = 0,\ \left\{ N_B, \phi^{\hat{a}} \right\}_{\mathrm{PB}} = 0,\ \left\{ Q_{\mathrm{F}}, \phi^{\hat{a}} \right\}_{\mathrm{PB}} = 0,\ \left\{ Q_{\mathrm{F}}^{\dagger}, \phi^{\hat{a}} \right\}_{\mathrm{PB}} = 0, \tag{119}$$

where $\phi^{\hat{a}}$ are first class constraints (113) and the Hamiltonian H_{M}. From (101) - (104) and (119), following relations can be considered as first class constraints,

$$N_A = 0,\ N_B = 0,\ Q_{\mathrm{F}} = 0,\ Q_{\mathrm{F}}^{\dagger} = 0. \tag{120}$$

After taking the following gauge fixing conditions for the first class ones (113),

$$A^0 = 0,\ B^0 = 0,\ C^{+0} = 0,\ C^{-0} = 0, \tag{121}$$

the system is quantized by regarding variables as operators and imposing the same type of relations (50) and (51) on the canonical pairs. From (120), it is reasonable to impose the following subsidiary conditions on states,

$$N_A \left| \mathrm{phys} \right\rangle = 0,\ N_B \left| \mathrm{phys} \right\rangle = 0,\ Q_{\mathrm{F}} \left| \mathrm{phys} \right\rangle = 0,\ Q_{\mathrm{F}}^{\dagger} \left| \mathrm{phys} \right\rangle = 0. \tag{122}$$

Then, they guarantee the unitarity of our system, though it contains negative norm states originated from c_{φ} and $c_{\varphi}^{\dagger}$.