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Quantum Fractals and the Casimir-Dark Energy Duality—The Road to a Clean Quantum Energy Nano Reactor

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Abstract

Based on Witten's T-duality and mirror symmetry we show, following earlier work, the fundamental complimentarity of the Casimir energy and dark energy. Such a conclusion opens new vistas in cold fusion technology in the wider sense of the word which we tackle via fractal nano technologies leading to some design proposals for a nano Casimir-dark energy reactor.

Keywords

Casimir Energy, Zero Point Energy, Dark Energy, E-Infinity Theory, Quantum Set Theory, Algebraic Quantum Field, Cantorian Spacetime, Fractal Quantum Phase Space, Mirror Symmetry, Witten's T-Duality

1. Introduction

Modern theoretical physics has a truly marvellous story to tell about its development [1] [2] and we may start the story with Newton although to make a long story short, it is advisable to start with relativity [1]-[8]. Einstein's theory of relativity [1]-[40] was the first major modern revolution in theoretical physics since Newton and Maxwell [2]. Not only that but the work of Einstein was somehow in between relativity and the next revolution, namely quantum mechanics and that despite Einstein's reluctance to embrace quantum entanglement [99]-[101] and the fundamental changes in our philosophy which this new theory implied [80] [81]. Ironically the present author is of a rather firm opinion that Einstein's iconic $E = mc^2$ is in fact the sum of two quantum parts [20]-[28], namely that of the quantum particle energy $E(O) = mc^2/22$ and that of the quantum wave energy $E(D) = mc^2$ (21/22) so that Einstein did indeed hit the nail on its head, quantumly speaking [29]. It is fair to say that only a few would place the field of deterministic chaos and fractals as the next mile stone or if you want, revolution af-

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ter quantum mechanics [102]-[112]. However the present author is of the opinion that the work of G. Cantor and his transfinite theory is by far the most fundamental mathematics which quantum physics requires and this fact at long last becomes known via the work of the pioneer of nonlinear dynamics, chaos and fractals, notably Lorenz, Raulle, Feigenbaum, Mandelbrot, Takens and York to mention only a few [2] [102]-[107]. However apart of the immensely important work of L. Hardy [99]-[101], the quantum mechanics connection to chaos and fractals was another intensive effort which took much longer to bare fruit due to the genius of people like R. Feynman and the dedicated efforts of numerous scientists, particularly the quantum chaos pioneers Chirikov, Casati, Ford, Gutzwillen and Berry as well as the fractal spacetime pioneers Ord and Nottale in addition to E-infinity Cantorian spacetime theory proposed by the present author [30]-[97]. Stripped to the bare core, nonlinear dynamics is about the discovery of Cantor sets for mechanics so we are justified in asking what does fractals bring to physics in general and quantum mechanics in particular? Without going into the discussion of why we will restrict our explanation to a specific fundamental fractal, namely random Cantor set [60] [69] for which, by a well known theorem, the Hausdorff dimension is the most irrational number $\phi = \left(\sqrt{5}-1\right)/2$ [6] [7] we will look next at one of the most important aspects of quantum mechanics, namely the Heisenberg uncertainty principle [110]. Now this principle precludes the use of one of the most powerful tools of mechanics, namely the phase space method of analysing dynamics and stability of mechanical sets [2] [108]. Needless to say the method proposed by Wiegner to overcome this limitation is not anywhere as widely used as the Hilbert space approach or as the path integral method [108]. That is where Cantor sets come to the rescue. With a Cantor set phase space, *i.e.* a "point less" phase space with non-standard points, there is no problem to do quantum calculations without violating the uncertainty principles [108] [110]. Of course we did similar things in the past in real spacetime and the author integrated Hilbert space into the E-infinity larger picture [98] but we think we did treat phase space a little bit step-motherly and it is time to point out our own unjustified shortcomings [117]-[119]. From here we can then point in a systematic way to the undreamed of possibilities of chaotic fractals and random Cantor set to tackle quantum physics starting from a comprehensive picture up to an exact solution [113]-[115]. This brings us to the next important point in the present discussion, namely Hardy's quantum entanglement. Hardy was seeking an exact solution to a basic particles entanglement [99]-[101]. Using orthodox quantum mechanics in ket and bra formalism of Dirac he found the quantum probability for entanglement of two quantum particles to be about 9 percent [100]. However what he really did not suspect was what Prof. D. Mermin published a little later showing that this 9 percent was an exact value equal to ϕ to the power of five [101]. That was probably the first exact result linking without a trace of a doubt the random Cantor set with quantum mechanics and the fundamental dimensional function of von Neumann-Connes continuous and noncommutative geometry [71] of which Penrose fractal tiling [2] [6] [7] is a generic space mimicking E-infinity theory [69] [70] [73]. Remembering that Penrose tiling depends crucially upon a golden mean proportionality, we see that the hunch that this golden mean is fundamental is far more than a hunch as, as we will see in the present work, it is a fact discussed on numerous previous occasions [38]-[85]. Maybe it is at this point that we should stress the marvel hidden in a golden mean based number system and E-infinity is indeed embedded in a golden mean binary constituting de facto a golden mean computer rivalling even a transfinite version of Turing's machine without hardware save for a modern pocket calculator on the side to perform the elementary manipulation of the adding, multiplying and dividing the golden mean and its power [41] [66]. Having solved the measure technical computational problem as well as the fundamental contradiction between the discrete and the continuous by building a method which unites both opposed concepts into a transfinite discretum which has the cardinality of the continuum, we realize that we have not only a much better understanding of the vacuum fluctuation but we found a handle on it which can be used to the extent of building mini fractal universes in the laboratory from which we can extract clean energy in the form of a Casimir energy reactor [91]. Before that however we show that ordinary energy is identical to Casimir energy and that the cosmological dark energy is the complimentary energy of the Casimir energy. For an instructive simple and exact picture of quantum spacetime, the reader is invited to examine Figure 2 of Ref. [113]. How this all fits together is the subject of the coming sections.

2. Building Elements of E-Infinity Diagrams

The main two elements or building blocks of E-infinity diagrams is the zero set and the empty set [64]. From these irreducibly simple set theoretical elements we can virtually build an entire spacetime and more. Thus we have [33]

(a) The zero set $D(H) = \phi$

(b) The empty set $D(H) = \phi^2$

where $D(H)$ is the Hausdorff dimension and $\phi = \left(\sqrt{5}-1\right)/2$. From the above we obtain the latent Casimir spacetime set representing the latent topological energy of spacetime as the difference of ϕ and ϕ^2 in symbolic diagram (see **Figure 1**) reflecting the essence of the famous Casimir experiment with two uncharged but perfecting conducting plates [89] [91].

From the diagrams of **Figure 1** we can generalize to two limiting cases (see **Figure 1** and **Figure 2**).

1) When the distance between the two plates of **Figure 1** tends to real zero, this is then the totally empty set $\left(\phi\right)^{\infty}$ = zero. In the surrounding space we have a non-empty set with the average latent pressure everywhere equal ϕ^3 while the average spacetime density is basically the fractal five dimensional average $5+\phi^3$. Consequently the density of the latent Casimir pressure is simply $\left(\phi^3\right)/\left(5+\phi^3\right)$. This is exactly equal $\phi^5/2$ which is our well known ordinary measurable energy of the cosmos and in astounding agreement with cosmic measurements and observation [108]. The immediate rather profound conclusion is that the measured real energy density of the cosmos is nothing more but nothing less than the Casimir latent energy of spacetime and at this scale it is

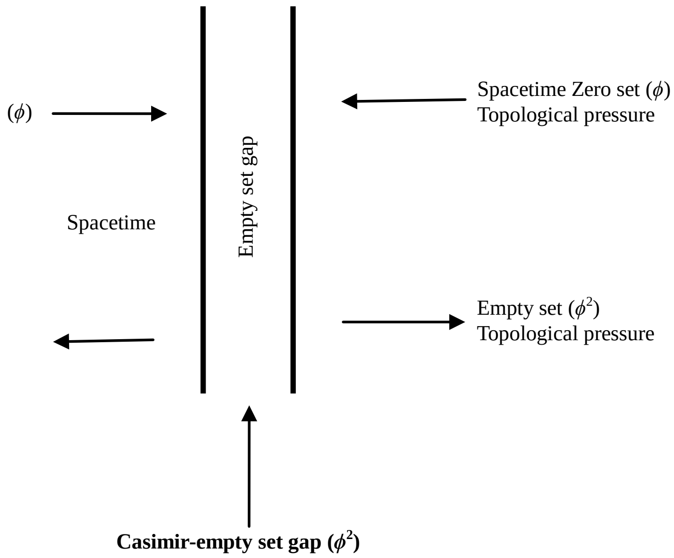

Figure 1. Symbolic representation of the E-infinity Casimir diagram for nano scales. The difference between ϕ and ϕ^2 given the latent spacetime topological pressure ϕ^3. The inverse of ϕ^3 on the other hand is the average Hausdorff dimension of spacetime 4 + ϕ.

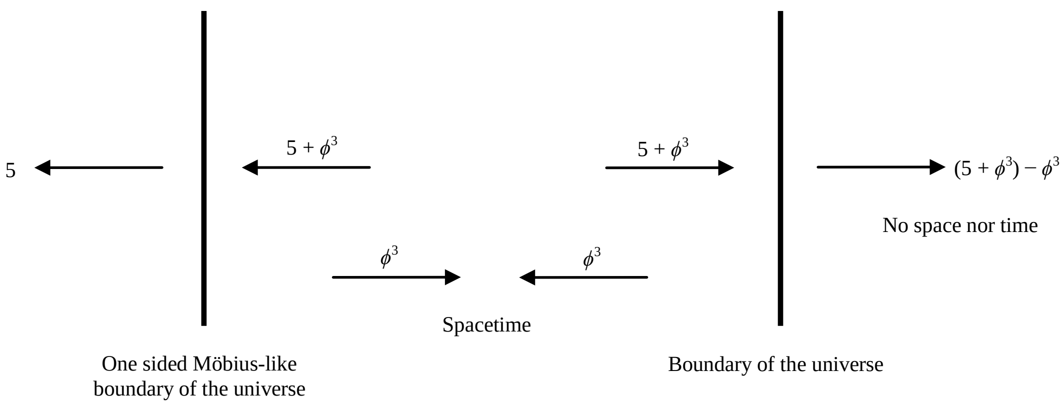

Figure 2. Symbolic representation of the E-infinity Casimir diagram for Hubble scales. The boundary of the holographic boundary of the universe is one sided Möbius-like so that we have no outside. In other words the outside is the totally empty set. This is the mathematical definition of the philosophical concept of non-existence or nothingness.

not related in any way to the Riemann curvature of spacetime but to the chaotic fractality of spacetime in full agreement with our picture which we adopted based on Feynman's conjecture that gravity is similar to van der Waals fluctuation of micro spacetime frequently termed Feynman-El Naschie van der Waals quantum gravity conjecture [6] [7] [38] [69] [70].

2) Now we look at the other extreme where the distance between the Casimir plates is equal to the diameter of our universe as shown schematically in **Figure 2**. The repulsive Casimir-like pressure is in this case equal $5+\phi^3$ minus the latent Casimir topologic pressure ϕ^3 which is working in the opposite direction so that the net repulsive topological pressure pushing the boundary of the universe outwardly is given by $\left(5+\phi^3\right)-\phi^3=5$. Therefore the density of this repulsive topological pressure is $(5)/\left(5+\phi^3\right)$. This happens to be equal to $1-\left[\phi^3/\left(5+\phi^3\right)\right]$ which means it is simply identical to what we calculated for the dark energy of the cosmos $5\phi^2/2$. We recall that $5\phi^2/2$ was interpreted as the energy of the pre-quantum wave, *i.e.* the energy of the empty set albeit in five dimensional Kaluza-Klein space [31]. The corresponding diagram is shown in **Figure 2**.

3. Determining the Exact Topological Diameter of the Universe and the Casimir Cosmic Ordinary Energy Density

We start from our exact picture of quantum spacetime [114]. This picture consists of three concentric circles. The first is the zero set of the quantum pre-particle $(0\,;\phi)$. Around this pre-particle we have a topological surface representing the empty set pre-quantum wave $\left(-1\,;\phi^3\right)$. Finally we have the third layer which happens to be an expectation value $\left\langle\left(-2\,;\phi^3\right)\right\rangle$ representing the average value of an infinite number of empty sets constituting the pre-quantum spacetime. Now ϕ, ϕ^2 and ϕ^3 are not only Hausdorff dimensions but they can also be understood as a topological frequency or critical parameters of corresponding limit cycles. Therefore we can use the well known comparison theorem of eigenvalue due to Dunkerley and figure the combined critical value or joint Hausdorff dimension as follows:

$$\frac{1}{p}=\frac{1}{\phi}+\frac{1}{\phi^2}+\frac{1}{\phi^3}. \quad (1)$$

Consequently we have

$$\frac{1}{p}=\frac{\phi+\phi^2+\phi^3}{\phi^6}=\frac{1+\phi^3}{\phi^6}=(2)(1/\phi)^5=(2)\left(11+\phi^5\right)=22+k \quad (2)$$

where ϕ^5 is Hardy's quantum entanglement, 11 + ϕ^5 is the dimension of Witten's fractal M-theory and $k=\phi^3\left(1-\phi^3\right)$ [69] [70] [110]-[116]. This is a remarkable result on more than one count. On the first count we found the density of ordinary measurable energy to be obviously equal p and to be at the same time the energy density of the latent Casimir energy of the cosmos [113]-[116]

$$E(O)=1/p=\frac{1}{22+k}\cong 95.5\% \quad (3)$$

Second it is clear that 11 + ϕ^5 is the isomorphic length of the Penrose fractal universe and therefore 22 + k = (2) (11 + ϕ^5) is the corresponding diameter of the universe [30] [31]. It follows then that 22 + k is the topological diameter of our fractal M-theory like universe. We recall that this result corresponds to a super symmetric space where the isomorphic length is the result of combining intersectionally the bosonic isomorphic length $\ell=\left(4+\phi^3\right)/2$ with the Fermionic isomorphic length $\ell=\left(5+\phi^3\right)/2$ and finding that [32]-[39]

$$\ell_{ss}=\left(4+\phi^3\right)\left(5+\phi^3\right)/2=22+k/2=11+\phi^5. \quad (4)$$

This is the dimensionality 11 of Witten's M-theory plus a Hardy quantum entanglement part, namely ϕ^5.

4. The E-Infinity Casimir-Dark Energy Diagram for Topological Interactions in Cantorian Spacetime

Similar to the quasi probability of Wigner's quantum mechanics in phase space [108], our topological probability approach to quantum mechanics as enshrined in E-infinity [110]-[116] represents an alternative extremely simple fourth formulation for quantum mechanics which is not directly connected to Hilbert space, path integral nor the density matrix approach [108]. Naturally it bears a resemblance to all the afore mentioned formulations but is by no means identical and in fact it adds a few new points to our understanding of the deep mathematical structure of orthodox quantum mechanics hiding behind complex numbers and analytical continuation [2] [6] [7] [69] [70].

The internal logic of our approach rivals that of Weyl-Wigner quantum mechanics in phase space and its logical underpinning by Groenewold and Moyal [108]. For instance as we enter into the negative dimensions regime and find that the Hausdorff dimension ϕ^n is getting increasingly small, it is clear that this means the "Cantor set" is becoming thinner and tending to real nothingness or the absolute empty set [6] [9] [70]. The situation is thus paralleling the density of the density matrix. Similarly when we are summing in E-infinity theory over all dimensions to reach our expectation Hausdorff dimension of the core of E-infinity spacetime $\langle dc^{(n)} \rangle = 4 + \phi^3$ then we are using essentially a similar mathematical concept as that of Feynman path integral but exchanging "paths" for "dimensions" [2]. We could go on describing our approach but could never explain it as good as by treating a fundamental example and nothing could be better and more fundamental than looking at the Casimir effect and dark energy from the perspective of our E-infinity topological probability quantum field theory [69] [70].

5. The Topological Mass

For want of better words, ϕ^3 was called with equal justification universal fluctuation [69] [70] [110]-[116] and since fluctuation produces "mass" it could be called "topological mass" [110]-[116]. Now mass is also an abstract word so that one could speak of mass charge as we speak of electrical charge or magnetic charge or "criminal charge". Thus in legal language you charge someone of a property called criminality and here we have this "stuff" property that we call mass. Now for Einstein's mass multiplied with high velocity squared is energy.

In E-infinity the topological speed of light is a velocity charge being between zero and infinity in a circular world interval where the golden mean is a topological average. Thus the topological energy is [110]-[116]

$$\mathrm{E}_{\text{topology}} = \left(\phi^3\right)\langle c\rangle_m^2 = \left(\phi^3\right)\left(\phi^2\right) = \phi^5 . \tag{5}$$

This is Hardy's topological energy. To get "real" energy we use Newton [6] [7] [110]-[116]

$$E = \frac{1}{2}\left(mv^2\right) = \left(\frac{1}{2}\right)\left(\phi^3\phi^2\right)mc^2 \tag{6}$$

$$\mathrm{E}_{\text{real}} = \mathrm{E}_{\text{topology}}\left(\frac{1}{2}m\left(v \to c\right)^2\right) = \left(\phi^5/2\right)\left(mc^2\right) \tag{7}$$

This is thus what we call mass in the quantum small world. In the large cosmic world on the other hand, mass is $\phi \to 5$ and consequently the mass of both worlds is simply the sum, namely [6] [7] [110]-[116]

$$\boldsymbol{m} = 5 + \boldsymbol{\phi}^{\mathbf{3}}. \tag{8}$$

Consequently we have

$$\begin{aligned} E &= \underbrace{\left(5+\phi^3\right)\left(\phi^2\right)\frac{1}{2}}_{\text{topological}}\, mc^2 \\ &= \left(5\phi^2/2\right)mc^2 + \left[\left(\phi^3\right)\left(\phi^2\right)/2\right]mc^2 \end{aligned} . \tag{9}$$

From the above the complimentarity of $E(O)$ and $E(D)$ follow:

$$\begin{aligned} 1 - E(O) &= E(D) \\ 1 - E(D) &= E(O) . \\ E(O) + E(D) &= 1 \end{aligned} \tag{10}$$

Similar considerations apply to the Mageuijo-Smolin formula for quantum gravity energy.

6. The Mageuijo-Smolin Quantum Gravity [42]

To obtain $E(O)$ and $E(D)$ and visa versa we have the following transformation [42]

$$\frac{\text{fractal part}}{\text{solid part}} = \frac{\phi^3}{5} \rightarrow \frac{\phi^3}{5}+1 \rightarrow \frac{1}{\frac{\phi^3}{5}+1} = E(D) = 95.5\% = \frac{\text{solid part}}{\text{fractal part}} \tag{11}$$

$$\frac{\text{solid part}}{\text{fractal part}} = \frac{5}{\phi^3} \rightarrow \frac{5}{\phi^3}+1 \rightarrow \frac{1}{\frac{5}{\phi^3}+1} = E(O) = 4.5\% = \frac{\text{fractal part}}{\text{solid part}} \tag{12}$$

The same applies to a fractal de Sitter universe [2] [33]

$$D = 5+\phi^3 = (\text{solid part} = 5) + (\text{fractal part} = \phi^3) \tag{13}$$

That means ordinary energy is

$$\frac{\text{fractal part}}{\text{total}} = \frac{\phi^3}{5+\phi^3} \tag{14}$$

and dark energy is

$$\frac{\text{solid part}}{\text{total}} = \frac{5}{5+\phi^3}. \tag{15}$$

7. The Advantage of Being Transfinite Cantorian for Quantum Mechanics

Let us start with Heisenberg's uncertainty [2] [110]. In E-infinity spacetime [69] [70] there is no ordinary points. It is exactly as in von Neumann's space [6] [7] [69] [70] which he describes in jest as "pointless". Consequently in Cantorian spacetime this fundamental principle is taken care of automatically *ab initio*. In fact writing ϕ^3 as $(\phi)(\phi^2)$ reveals that neither ϕ as a quasi coordinate nor ϕ^2 as its derivative are fixed numbers which can be written in decimal expansion in a finite way nor the quasi phase space spanned by ϕ and ϕ^2. The same is true for the product ϕ^3 which is ambiguous in a fundamental way because ϕ could be seen as a distance or a mean velocity while ϕ^2 could be interpreted as acceleration or mean velocity squared. However for the union of the two sets, the zero set ϕ and the empty set ϕ^2 we have the remarkable collapse in a simple integer [66] [71] [91]

$$(\phi = 0.618033989\cdots) + (\phi^2 = 0.3819660\cdots) = \text{one}. \tag{16}$$

It would be a mistake to think this result is trivial. However, written in symbolic form $\left(\sqrt{5}-1\right)/2 = \phi$ and $4/\left(\sqrt{5}+1\right)^2 = \phi^2$ this is of course trivial computation but not when written in the chaotic decimal expansion. In other words E-infinity transfinite Cantorian quantum mechanics has at its disposal and in a natural way, a transfinite golden computer [41] [66] with a real efficiency which borders on the surreal because simple finite arithmetic operations can handle uncountably infinite numbers and find at the end a neat compact answer [66]. In fact nothing can attest to this coincidentia oppositorum like that first rate orderly character of the continued fraction expansion of ϕ, namely [102]-[107] [122]-[126]

$$\phi = \frac{1}{1+\frac{1}{1+\frac{1}{1+\cdots}}} \rightarrow 0.618033 \tag{17}$$

or

$$\phi = 1\Big/\sqrt{1+\sqrt{1+\sqrt{1}}} \tag{18}$$

when compared to its totally chaotic decimal expansion [102]-[107]

$$\phi = 0.61803398925 \tag{19}$$

and find on the top of that a well known fact, namely that ϕ is the Hausdorff dimension of a random triadic Cantor set.

8. The Disadvantage of Being Transfinite Cantorian-Quantum Theory

At the risk of appearing facetious, we would like to seriously propose that in no minor measure a draw back of E-infinity Cantorian spacetime theory is that it is excessively simple. A theory should be simple but not excessively so. Being excessively simple puts a theory at risk of being called trivial as an easy shot by members of the "voluntary opposition". In his early days working in applied mechanics, the author was in awe vis-à-vis the work of a great scholar Prof. Cliff Trusdell who coined the word rational mechanics [120] [121]. The competence of Trusdell is beyond any doubt. However his mathematical vanity was equally beyond doubt. He was able to smooth anything mathematically to the point that the author felt he was facing a superman of mechanical science. This is what impressed the author as a naïve young post graduate student until he grew out of it with the help of real engineering scientists like W.T. Koiter, J.M.T. Thompson, J. Croll, S. Nemat Nasser and A.W. Walker to mention only a few [120] [121]. There is science and there is selling science within science politics and the part related to funding is more frequently than not unrelated to science and depends on salesmanship and media more than on logic. The problem with E-infinity is that it fulfils the criteria, which says that maximal accuracy is attained in the ideal limit at maximal simplicity. When simplicity is interpreted wrongly as triviality, then it is tragic. We could find nothing better to close this section with than the wise words of a Socrates of modern science, Prof. J.A. Wheeler [69] [70] in his classic book "Information, Physics, Quantum: The Search for Links" where he wrote "*Surely some day we can believe, we will grasp the central idea of it all as so simple, so beautiful, so compelling that we will say to each other*, '*Oh, how could it have been otherwise*!'. *How could we all have been so blind so long*!".

9. The VAK and Quantum Phase Space from the View Point of E-Infinity

In the following we show a simple deep connection between a few fundamental aspects of E-infinity and Weyl-Wigner theory [90].

a) If we take ϕ to mean a coordinate then ϕ^2 is the quasi derivative and $\phi - \phi^2$ becomes a phase space with $(\phi)(\phi^2) = \phi^3$ being simply a cell of this phase space.

b) Seen that way, then $(\phi)(\phi)(\phi)(\phi^2) = \phi^5$ of Hardy may be recognized as the unit cell of 3D Cantorian-quantum phase space.

c) Since the VAK is a Hamiltonian "strange" attractors conjectured by Rene Thom to represent the equilibrium states of quantum mechanics [99] [100], we see how the VAK, E-infinity and Wigner quantum phase quasi probability are intimately connected. We could proceed further to black hole information [2] and the Hawking and Beckenstein theory [2] but space limitation will not permit that.

10. Conclusion

We presented in this relatively short paper a general theory for quantum spacetime and zero point energy fluctuation based on E-infinity and related mathematical concepts. Our main results and conclusions may be summarized in the following rather important points [116]-[126]:

1) Casimir energy and ordinary energy density of the cosmos are not only identical conceptually but identical numerically.

2) Casimir energy and cosmic dark energy are complimentary in the most strict mathematical and physical meaning.

3) The difference between Casimir energy and dark energy is a difference of boundary condition where the boundary of the holographic boundary of the universe is a one sided Möbius-like manifold (see Figure 2 of Ref. [122].

4) Using a heap of space filling fractal nano spheres, we can build in principle a mini universe and use it as an energy reactor (see Figure 1 of Ref. [116]).

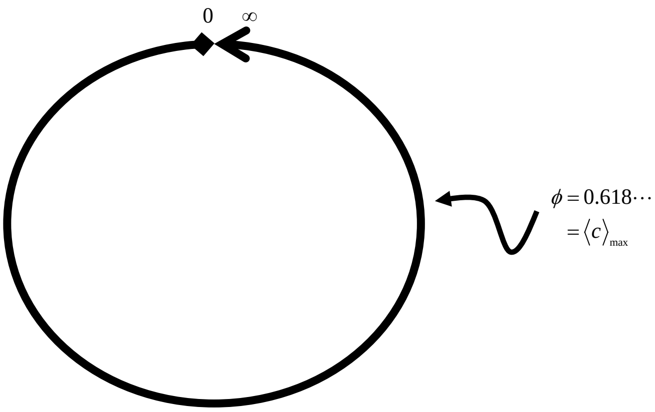

Figure 3. Universe unit circular interval (see Ref. [6] [7] [11] [39] [94]). The point is that self similarity leading to the counter intuitive topological fact that every point in a Cantor set when "magnified" reveals itself as a complete Cantor set leads to the logical conclusion that the number of "points" in the unit interval shown as a closed ring above it is equal to the number of points in our "actual" universe. The weighted centre of "everything" in this unit circular universe is the golden mean. That is how we can reason that ϕ is the weighted speed of light which varies between zero and infinity but because of mirror symmetry and T-duality, this formally zero plus infinity divided by two leads to a finite expectation value, namely $\phi = 0.618033989 = (0.5) + (k + 1)/10$ where $k = \phi^3(1-\phi^3)$ and ϕ^3 is the latent Casimir topological pressure as explained in **Figure 1** and **Figure 2** as well as the main body of the present paper. For a more instructive elaboration and explanation of this point, see Figures 2-4 of Ref. [94].

5) The main conclusion is a natural consequence of mirror symmetry and Witten's T-duality (see Figures 2-4 of Ref. [94]) in addition to Dvoretzky's theorem which explains why energy is concentrated at the edge of the universe similar to electrical charge of a Faraday cage and information of a black hole.

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